Reconsidering the Market Size Effect in Innovation and Growth*

Hélène Latzer
Paris-I, CNRS, France
Université Saint-Louis, Belgium
helene.latzer@usaintlouis.be

Kiminori Matsuyama
Northwestern University, USA
k-matsuyama@northwestern.edu

Mathieu Parenti
Université Libre de Bruxelles, Belgium
mathieu.parenti@ulb.ac.be

First Version: November 23, 2019
This version: July 6, 2020

Abstract

In the standard horizontal innovation model of endogenous growth, larger economies innovate more and grow faster. Due to the homotheticity of preferences, however, it does not matter whether the large market size comes from a large population or a high per capita expenditure. In this paper, we extend the standard model to allow for nonhomothetic preferences, while preserving its balanced growth property. Among others, we show that, holding the size fixed, economies with higher per capita expenditure and smaller populations innovate more and grow faster (slower) in the case of increasing (decreasing) relative love for variety.

Keywords: Endogenous growth, Balanced growth, Horizontal innovation, Nonhomothetic preferences, Directly explicitly additive (DEA) preferences, Demand composition, Competition and growth

JEL classification: O11, O31, O33

*We thank Daron Acemoglu, Philippe Aghion, Gino Gancia, Gene Grossman, Chad Jones, Xavier Vives, the workshop participants at University of Tokyo, Keio University, Chicago Fed, as well as those at joint seminars in Saint Peterburg and Vienna, and at the conferences at Hitotsubashi University and Peking University for their feedback. Matsuyama is grateful for ECARES for its hospitality during his visit. The usual disclaimer applies.

1. Introduction

The standard horizontal innovation models of endogenous growth—Romer (1990), Grossman and Helpman (1993, ch.3), Gancia and Zilibotti (2005), and Acemoglu (2008; ch.13.4) just to name a few—, predict that countries with larger market sizes innovate more and hence grow faster. However, due to the assumption of consumer demand homotheticity, the composition of the aggregate demand has no effect. In particular, it does not matter whether the large market size comes from a larger population size or a higher per capita expenditure. Once the aggregate market size is controlled for, poorer countries with larger population sizes innovate as much as richer countries with smaller population sizes. Furthermore, without any effects of the demand composition, the (demand-side) *market size effect* becomes indistinguishable from the (supply-side) *scale effect,* for which there exists little supporting evidence.

In this paper, we extend textbook horizontal innovation models of endogenous growth to allow for *nonhomothetic* preferences to examine the effect of the demand composition on innovation and growth. We consider an economy, populated by N identical agents, each endowed with h units of labor.[1] For the preferences, we follow the footsteps of Dixit and Stiglitz (1977). Virtually all the existing horizontal innovation models of endogenous growth build on their well-known model of monopolistic competition with homothetic CES preferences in Dixit-Stiglitz (1977; Section I). Instead, we build on their lesser-known model of monopolistic competition with directly explicitly additive (DEA) nonhomothetic preferences in Dixit-Stiglitz (1977; Section II), which contains homothetic CES as a knife-edge case.[2]

A distinctive feature of monopolistic competition model with DEA is that the price elasticity of demand each firm faces, which inversely affects the markup rate charged by the firm, is a function of per capita consumption of its product *only*. Furthermore, at the symmetric

---

[1]Thus, h measures the worker efficiency, and can be interpreted as the level of human capital or the quality of the labor force. The total labor endowment of the economy is hence equal to $L = hN$. With labor being the only factor of production, L is also the size of the economy.

[2]See also Zhelobodko et al. (2012). Although Dixit and Stiglitz (1977) called Section II "Variable Elasticity Case," the well-known Bergson's Law states that, within this class of preferences, they are homothetic if and only if they are CES. In other words, any departure from CES within this class introduces nonhomotheticity. Hence, one could equally call Section II "Nonhomothetic Case." Of course, nonhomotheticity and non-CES are generally distinct properties of preferences. Indeed, it is possible to have homothetic non-CES in a broader class of symmetric preferences. (It is also possible to have nonhomothetic CES if we allow for asymmetric preferences.) However, the demand composition effect would be absent under homothetic non-CES. Moreover, homothetic non-CES would be incompatible with the balanced growth property (see footnote 3). To put it differently, it is nonhomotheticity, *not* variable elasticity *per se*, that matters. What is crucial for the results in this paper is that the elasticity of substitution varies *across* indifferent curves, *not along* an indifferent curve.

equilibrium in which all products are consumed by the same amount, this price elasticity, which is now a function of per capita consumption of *each* product, is also equal to the elasticity of substitution between every pair of products. Obviously, homothetic CES is a special case where this function is constant. But, whenever this elasticity function is not constant, it introduces nonhomotheticity. For example, if it is decreasing in per capita consumption of each product, the elasticity of substitution between products is *smaller* on higher indifferent curves, which means that richer consumers are *more* willing to pay for additional variety. This is the case Zhelobodko et.al. (2012) called the case of "increasing relative love for variety (RLV)." Alternatively, if this function is increasing, the elasticity of substitution between products is *larger* on higher indifferent curves, which means that richer consumers are *less* willing to pay for additional variety. This is the case Zhelobodko et.al. (2012) called the case of "decreasing relative love for variety (RLV)."

We use this class of nonhomothetic preferences, because replacing CES with DEA in the standard horizontal innovation model of endogenous growth does not destroy its balanced growth property, which we would like to preserve in order to keep our departure from the standard model to the minimum. The reader may be surprised that the balanced growth property is preserved in the presence of nonhomotheticity. To see why, consider the case of increasing RLV. For a fixed measure of the existing product variety, growing per capita real income would increase the per capita consumption of each product, causing the price elasticity to go down and the markup rate to go up. However, there is an offsetting force. For a fixed per capita real income, expanding product variety would reduce the per capita consumption of each product, causing the price elasticity to go up and the markup rate to go down. Along the equilibrium path, these two forces exactly cancel out each other, because the measure of product variety and the per capita real income grow at the same rate, so that the per capita consumption of each product stays constant. Consequently, the price elasticity and the markup rate stay constant, preserving the balanced growth property.[3]

Here are our main results. First, after controlling for the size $L = hN$, a richer country with a smaller population (a higher h with a smaller N) innovates more and hence grows faster

---

[3]This also explains why the balanced growth property *cannot* be preserved under homothetic non-CES. Under homotheticity, growing per capita real income cannot affect the markup rate. Furthermore, the price elasticity of demand for each product can depend solely on the measure of the existing variety in a symmetric setting. Thus, expanding variety would have to change the price elasticity along the equilibrium path.

under increasing RLV. Under decreasing RLV, the effect goes in the opposite direction. Hence, the composition of the aggregate demand matters, and the demand-side market size effect can be distinguished from the supply-side scale effect. Second, even though both the markup rate charged by the firms and the innovation (and growth) rate stay constant over time along the balanced growth path, they are both endogenous. Thus, they could either be positively or negatively correlated as a change in the parameters shifts the balanced growth path. We show under increasing RLV that the markup and innovation rates move in the same direction if changes are caused by exogenous variations in production cost or per capita expenditure, while they move in the opposite directions if changes are caused by variations in the discount rate, the innovation cost or the population size. This implies, in particular, that the measure of competitiveness and the growth rate are positively correlated in cross-sections of countries, if countries differ mostly in the innovation (or firm entry) cost.

It should be noted that the (demand-side) *market size effect* on innovation and growth we study is conceptually distinct from the (supply-side) *scale effect*, whose empirical validity has been questioned by Jones (1995) and many others. Nonhomotheticity provides one natural way of distinguishing these two effects.[4] Indeed, our results suggest that the difference in per capita income across countries could potentially be one reason why there is little supporting evidence for the scale effect. We also show that correlations between competition and growth across countries depend on the sources of variations across countries. As such, our results suggest that horizontal innovation models of growth can also contribute to the debate regarding competition and growth.

Some recent studies have explored the role of the demand composition on innovation under nonhomotheticity. See, e.g., Fajgelbaum, Grossman, and Helpman (2011), Foellmi and Zweimueller (2006), Latzer (2018), and Matsuyama (2019a). All these studies, however, consider the type of nonhomotheticity, where higher per capita real income causes the demand composition to shift away from low income elastic products/sectors toward high income elastic products/sectors. Here, we instead focus on the type of nonhomotheticity, under which per capita real income affects the consumer's willingness to pay for innovation. For this reason, we

[4] An alternative approach to distinguish the two has been pursued in the directed technological change literature, using multi-sector, multi-factor extensions of endogenous growth models, see, e.g., Acemoglu (2008, ch.15) and Gancia and Zilibotti (2009).

consider a one-sector model, where all products enter symmetrically in preferences, and hence products do not differ in their income elasticity. To the best of our knowledge, this paper is the first to investigate the role of this type of nonhomotheticity in the balanced growth framework.

Finally, two of us have previously investigated the implications of departing from CES preferences on the markup and growth rates in a model of horizontal innovation in Boucekkine et. al. (2017). However, that model is too restrictive to study the effect of the demand composition. Moreover, even though the balanced growth path exists in that model, the equilibrium path may differ from the balanced growth path.

2. Back to the Basics: Innovation and Growth under CES

We start with a benchmark balanced growth model with homothetic CES, which encompasses two versions of the textbook models (Grossman and Helpman, 1993; Gancia and Zilibotti, 2005) in order to facilitate comparisons with our model with DEA preferences.

2.1. Intratemporal problem

Labor is the only factor of production. We consider an economy populated by N identical agents, each supplying inelastically h units of labor measured in efficiency units. Hence, the total labor supply measured in efficiency units is $L = hN$, which is also the size of the economy.

Time is continuous and extends from $t = 0$ to infinity. Intertemporal preferences of each agent take the following form:

$$\mathcal{U}_0 = \int_0^\infty log\big(U(\mathbf{x}_t)\big)e^{-\rho t}\,dt,$$

where $U(\mathbf{x}_t)$ is the intratemporal utility, $\mathbf{x}_t = \{x_t(\omega);\ \omega \in [0, V_t]\}$ is the consumption profile, with V_t being the range of the products that have been innovated by time t, and $x_t(\omega)$ denoting the consumption of product $\omega \in [0, V_t]$.

At time t, each agent earns a wage income equal to $w_t h$, and spends E_t, and chooses $\mathbf{x}_t$ to maximize $U(\mathbf{x}_t)$ subject to the intratemporal budget constraint,

$$\int_0^{V_t} p_t(\omega)x_t(\omega)d\omega = E_t, \tag{1}$$

where $p_t(\omega)$ denotes the price of product $\omega \in [0, V_t]$. When $U(\mathbf{x}_t)$ is a CES with the elasticity of substitution $\sigma > 1$:

$$U(\mathbf{x}_t) = \int_0^{V_t} \big(x_t(\omega)\big)^{1-\frac{1}{\sigma}}d\omega,$$

this intratemporal maximization problem yields the per capita demand curve for each product:

$$x_t(\omega) = \frac{[p_t(\omega)]^{-\sigma}}{(P_t)^{1-\sigma}} E_t, \tag{2}$$

with the price elasticity being constant and equal to $\sigma > 1$, where

$$(P_t)^{1-\sigma} \equiv \int_0^{V_t} [p_t(\omega')]^{1-\sigma} d\omega'.$$

The total demand for product ω is simply given by $q_t(\omega) = Nx_t(\omega)$.

2.2. Firms' intratemporal problem

Each product $\omega \in [0, V_t]$ is produced and sold exclusively by a single firm, which is also indexed by $\omega \in [0, V_t]$. Producing one unit of each product requires ψ_t efficiency units of labor. Each firm chooses its price, $p_t(\omega)$ or the quantity, $q_t(\omega) = Nx_t(\omega)$, to maximize the profit,

$$\pi_t(\omega) \equiv (p_t(\omega) - w_t\psi_t)q_t(\omega) = (p_t(\omega) - w_t\psi_t)Nx_t(\omega)$$

subject to eq.(2) taking w_t, ψ_t, P_t, and E_t as given. This profit maximization problem has a unique solution, and hence all firms adopt the same pricing rule:

$$p_t(\omega)\left(1 - \frac{1}{\sigma}\right) = w_t\psi_t \Leftrightarrow p_t(\omega) = Mw_t\psi_t \equiv p_t,$$

where $M \equiv \sigma/(\sigma - 1)$ is the markup rate, which is exogenously constant under CES.

Because all firms set the same price, firm symmetry entails that all products are produced and consumed by the same amount, and all firms earn the same level of profits:

$$q_t(\omega) = Nx_t(\omega) = Nx_t = q_t; \pi_t(\omega) = \pi_t$$

and the intratemporal budget constraint, eq.(1), becomes simplified to:

$$p_t x_t V_t = E_t.$$

The above mark-up rule also implies that the share of the aggregate expenditure that goes to the firms' profits is also exogenously constant and given by:

$$\frac{\pi_t V_t}{NE_t} = \frac{(p_t - w_t\psi_t)q_t V_t}{p_t q_t V_t} = \frac{p_t - w_t\psi_t}{p_t} = \frac{1}{\sigma} \tag{3}$$

Likewise, the share of the aggregate expenditure that goes to the wage payment in the production sector is also exogenously constant, and given by:

$$\frac{w_t\, L_{Xt}}{NE_t} = 1 - \frac{1}{\sigma} = \frac{1}{M} \tag{4}$$

where $L_{Xt} = \psi_t q_t V_t = \psi_t N x_t V_t$ denotes the total number of efficiency units of labor employed in the production of the existing products.

2.3. R&D and resource constraints

Because firms are symmetric, the market value of each firm is the same and equal to $B_t \equiv \int_t^{\infty} \pi_s e^{-(R_s - R_t)}\, ds$, where $R_s \equiv \int_0^s r_\tau d\tau$ is the cumulative interest rate and r_τ is the instantaneous one. Log-differentiating this expression of B_t with respect to *t,* we obtain:

$$\frac{\dot{B}_t + \pi_t}{B_t} = r_t. \tag{5}$$

Innovating per unit of new products requires F_t efficiency units of labor:

$$F_t \dot{V}_t = L_{Rt},$$

where L_{Rt} is the number of units of labor being employed in the R&D sector at time t. There is free entry in the R&D sector. Hence, whenever the R&D sector is active, net returns from R&D, $B_t \dot{V}_t - w_t L_{Rt} = (B_t - w_t F_t)\dot{V}_t$, are equal to zero, which means that the cost of creating a product (the R&D cost) and the value of creating a product (the value of a firm) are equalized:

$$B_t = w_t F_t. \tag{6}$$

Finally, the labor resource constraint, or the labor market equilibrium condition, is given by:

$$hN = L = L_{Rt} + L_{Xt} = F_t \dot{V}_t + \psi_t N x_t V_t. \tag{7}$$

2.4. Intertemporal problem

To describe the intertemporal maximization problem of the agent, we first derive the intertemporal budget constraint. Each agent holds $1/N$ fraction of the ownership shares of the profit-making firms, hence their asset holding is $a_t = B_t V_t / N$. At time t, an agent earns the wage income $w_t h$ and the profit income $\pi_t V_t / N$, spends $E_t = p_t x_t V_t$ and purchases assets (the ownership shares of the new profit-making firms) by $B_t \dot{V}_t / N$. The flow budget constraint is hence:

$$B_t \dot{V}_t / N + E_t = w_t h + \pi_t V_t / N$$

By adding the capital gains $\dot{B}_t V_t / N$ on both sides, and using eq.(5) and the fact that $a_t = B_t V_t / N$, the above expression can be written as:

$$\dot{a}_t + E_t = w_t h + r_t a_t$$

By integrating this expression from $t = 0$ to infinity, we obtain the intertemporal budget constraint:

$$\int_0^{\infty} E_t e^{-R_t} dt \leq a_0 + \int_0^{\infty} w_t h e^{-R_t} dt, \tag{8}$$

with the no-Ponzi scheme condition, $\lim_{t\to\infty} a_t e^{-R_t} \geq 0$. Subject to this intertemporal budget constraint, eq.(8), agents choose an expenditure path, $\{E_t\}_{t=0}^{\infty}$, so as to maximize:

$$\mathcal{U}_0 = \int_0^{\infty} log\big(U(\mathbf{x}_t)\big)e^{-\rho t}dt = \int_0^{\infty} log\left(V_t(x_t)^{1-\frac{1}{\sigma}}\right)e^{-\rho t}dt = \int_0^{\infty} log\left(V_t\left(\frac{E_t}{p_t V_t}\right)^{1-\frac{1}{\sigma}}\right)e^{-\rho t}dt.$$

The first-order condition is given by

$$\frac{1}{E_t}e^{-\rho t} = \lambda_0 e^{-R_t},$$

where λ_0 is the Lagrange multiplier associated with eq.(8). Log-differentiating this first-order condition with respect to t leads to the familiar Euler equation:

$$\frac{\dot{E}_t}{E_t} = r_t - \rho. \tag{9}$$

2.5. The Balanced Growth Path

The balanced growth path (BGP) is defined as an equilibrium path satisfying the following three conditions:

i) The growth rate of the range of products $g_t \equiv \dot{V}_t/V_t$ is constant and positive.

ii) The allocation of labor between the production and R&D sectors is constant: $L_{Xt} = L_X^*$ and $L_{Rt} = L_R^*$.

iii) The markup rate, $\equiv \sigma/(\sigma-1)$, is constant, satisfied automatically under CES.

To guarantee the existence of such a BGP, we follow Grossman and Helpman (1993) and Gancia and Zilibotti (2005) and many others by assuming that knowledge spillovers from past R&D experiences reduce the cost of R&D as follows:

$$F_t = \frac{F}{V_t}, \tag{10}$$

which implies that $L_{Rt} = F_t\dot{V}_t = Fg_t$.

Regarding the production cost, ψ_t, Grossman and Helpman (1993) assume $\psi_t = \psi$ so that knowledge spillovers are limited to R&D. In contrast, Gancia and Zilibotti (2005) assume $\psi_t = \psi/V_t$ so that they benefit both R&D and production equally. As will become clear below, however, neither of these assumptions play any role in ensuring the existence of a BGP under CES, so we intentionally leave ψ_t unspecified in this section.

We now derive the law of motion for this economy under the assumption that the R&D sector is active: $L_{Rt} = F_t \dot{V}_t = Fg_t > 0$. By inserting eq.(5) into the Euler equation, eq.(9), we obtain

$$\frac{\dot{E}_t}{E_t} = \frac{\dot{B}_t + \pi_t}{B_t} - \rho.$$

Using eqs. (3), (6) and (10), this can be written as

$$\frac{\dot{E}_t}{E_t} = \frac{\dot{w}_t}{w_t} - g_t + \frac{N}{\sigma F}\frac{E_t}{w_t} - \rho.$$

By defining $\mathcal{E}_t \equiv NE_t/w_t$, this can be simplified to:

$$\frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} = \frac{\mathcal{E}_t}{\sigma F} - g_t - \rho, \tag{11}$$

while the labor market equilibrium condition eq.(7) becomes, using eq.(4):

$$L = L_{Rt} + L_{Xt} = Fg_t + \left(1 - \frac{1}{\sigma}\right)\mathcal{E}_t. \tag{12}$$

By combining eq.(11) and eq.(12), we obtain the law of motion for $\mathcal{E}_t$:

$$\frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} = \frac{\mathcal{E}_t - \mathcal{E}^*}{F}, \text{where } \mathcal{E}^* \equiv L + \rho F. \tag{13}$$

Since $\dot{\mathcal{E}}_t > 0$ for $\mathcal{E}_t > \mathcal{E}^*$ and $\dot{\mathcal{E}}_t < 0$ for $\mathcal{E}_t < \mathcal{E}^*$, eq.(13) would imply divergence, leading to a violation of the equilibrium conditions, unless the economy jumps immediately to $\mathcal{E}_0 = \mathcal{E}^*$ and stays at $\mathcal{E}_t = \mathcal{E}^*$. This in turns implies $L_{Xt} = (1 - 1/\sigma)\mathcal{E}^* \equiv L_X^*$ and $L_{Rt} = Fg_t = L - L_X^*$ are all constant along the only equilibrium path and the economy stays on the balanced growth path, as long as the parameters are such that the R&D sector is active: $L_R^* = L - L_X^* = L - (1 - 1/\sigma)\mathcal{E}^* > 0 \iff M \equiv \sigma/(\sigma - 1) > 1 + \rho F/L$. Hence, we have:

Proposition 1A: Balanced Growth Path under CES

Suppose $M \equiv \sigma/(\sigma - 1) > 1 + \rho F/L$. Then, the economy jumps immediately to the balanced growth path along which

$$L_{Xt} = L_X^* = \left(1 - \frac{1}{\sigma}\right)(L + \rho F) = \frac{L + \rho F}{M} < L;$$

$$L_{Rt} = L_R^* = \frac{L}{\sigma} - \left(1 - \frac{1}{\sigma}\right)\rho F = \left(1 - \frac{1}{M}\right)L - \frac{\rho F}{M} > 0;$$

$$g_t = g^* = \frac{L}{\sigma F} - \left(1 - \frac{1}{\sigma}\right)\rho = \left(1 - \frac{1}{M}\right)\frac{L}{F} - \frac{\rho}{M} > 0.$$

From Proposition 1A, one could immediately show

Proposition 1B: Comparative statics under CES

In the benchmark CES case,

i) Both an increase in the discount rate ρ and in the R&D cost F leave the markup rate M unchanged, increase L_X^*, decrease L_R^*, and decrease the growth rate g^*;

ii) An increase in the total labor supply $L = hN$ leaves the markup rate M unchanged, and increases L_X^*, L_R^*, and g^*;

iii) An increase in the elasticity of substitution σ decreases the markup rate M, increases L_X^*, and decreases L_R^*, and g^*.

Three features of these results under CES deserve special emphasis. First, the per capita labor endowment h and the population size N enter in the law of motion for $\mathcal{E}_t$, eq.(13), as well as the expressions for $L_R^* = L - L_X^*$ and g^* only through their product $L = hN$. In short, what matters is the aggregate market size, not its composition. Once the country size, $L = hN$, is controlled for, a richer country with a higher h and a lower N innovates as much as a poorer country with a lower h and a higher N. This also implies that the (demand-side) market size effect is indistinguishable from the (supply-side) scale effect. This property is due to the homotheticity of preferences and does not hold under nonhomothetic preferences. Second, the production cost, ψ_t, has no effect on the aggregate dynamics. This neutrality of ψ_t is due to the exogeneity of the markup rate, which depends solely on the preference parameter σ. Thus, the share of the aggregate expenditure accruing to the firms' profits never change when the production cost changes. Hence, the production cost never affects incentive to innovate, and hence it has no impact on the labor allocation between the production and R&D sectors. This is the reason why we left it unspecified in this section.[5] Again, this feature will disappear as we depart from CES. Third, the markup rate changes only with a variation in σ, which leads to a positive correlation between the markup and growth rates. This result is at odds with some empirical evidence suggesting that competition fosters innovation and growth, if we use the markup rate as an inverse measure of competition, as commonly done.

[5]Note however that the time path of ψ_t affects that of x_t (and hence the welfare), because $L_X^* = \psi_t N x_t V_t$. For example, if we assume $\psi_t = \psi$, as in Grossman and Helpman (1993), x_t must shrink at the rate equal to g^*, so that $x_t V_t$ stays constant. Instead, if we assume $\psi_t = \psi / V_t$, as in Gancia and Zilibotti (2005), x_t stays constant.

3. Innovation and Growth under Directly Explicitly Additive (DEA) Preferences

We now depart from CES preferences and consider a broader class of DEA preferences, which admits CES as a knife-edge case.

3.1. Intratemporal problem

The intratemporal preferences satisfy *direct explicit additivity* (DEA)[6] if the *direct* utility function is *explicitly additive*:

$$U(\mathbf{x}_t) \equiv \int_0^{V_t} u(x_t(\omega))d\omega,$$

where the sub-utility function, $u(\cdot)$, satisfies $u(0) = 0$, $u'(x) > 0$, and $u"(x) < 0$. For a technical reason, it is assumed to be thrice-differentiable. The agents maximize this intratemporal utility subject to their intratemporal budget constraint, eq.(1), which yields the inverse demand curve for each product;

$$p_t(\omega) = \frac{u'\big(x_t(\omega)\big)E_t}{\Delta_t}, \tag{14}$$

where

$$\Delta_t \equiv \int_0^{V_t} u'\big(x_t(\omega')\big)x_t(\omega')d\omega'$$

captures the effects of the competing firms in the market.

The firms choose $p_t(\omega)$ or $q_t(\omega) = Nx_t(\omega)$ to maximize the profit $\pi_t(\omega) = (p_t(\omega) - w_t\psi_t)Nx_t(\omega)$ subject to eq.(14), taking w_t, ψ_t, Δ_t, and E_t as given, which yields the pricing rule:

$$\frac{u'\big(x_t(\omega)\big)E_t}{\Delta_t}\left(1 - \frac{1}{\sigma\big(x_t(\omega)\big)}\right) = p_t(\omega)\left(1 - \frac{1}{\sigma\big(x_t(\omega)\big)}\right) = w_t\psi_t,$$

or

$$\frac{u'\big(x_t(\omega)\big)E_t}{M\big(x_t(\omega)\big)\Delta_t} = w_t\psi_t, \tag{15}$$

where

[6] Even though this class of preferences is often referred to simply as "additive," this term fails to distinguish it from other types of additivity, such as *indirect explicit additivity*, *direct implicit additivity*, and *indirect implicit additivity*, which form different classes of preferences; see Matsuyama (2019b: Appendix A). Hence, we prefer to call it *direct explicit additivity* (DEA) to be precise.

$$\sigma(x) \equiv -\frac{u'(x)}{xu''(x)} > 1$$

is the price elasticity, and $M(x) \equiv \sigma(x)/[\sigma(x)-1] > 1$ is the markup rate.[7]

A distinctive feature of monopolistic competition under DEA is that the price elasticity of demand each firm faces, $\sigma(x)$, is a function of per capita consumption of its product and nothing else. Notice that, for any differentiable price elasticity function $\sigma(\cdot) > 1$, one could define the sub-utility function as $u(x) = \int_0^x exp\left[-\int_{y_0}^{y} \frac{ds}{s\sigma(s)}\right] dy$, which satisfies $u(0) = 0$; $u'(x) > 0$, $u''(x) < 0$, and is thrice-differentiable. Hence, one could also use the price elasticity function $\sigma(\cdot)$ as the primitive of the DEA preferences. Clearly, CES is a special case, where $\sigma(x) = \sigma > 1 \iff u(x) = A(x)^{1-\frac{1}{\sigma}}$, with A being a positive constant.

In what follows, we restrict ourselves to the subclass of DEA preferences that satisfy the following assumption:

(D1):
$$\frac{1}{\sigma(x)} + \frac{xM'(x)}{M(x)} > 0.$$

This inequality is equivalent to assuming that $u'(x)/M(x)$ is decreasing in x. In words, the firm's marginal revenue, the LHS of eq.(15), is decreasing in $x_t(\omega)$. Hence, eq.(15) has a unique solution, $x_t(\omega) = x_t$, which is decreasing in $w_t\psi_t\Delta_t/E_t$. This implies the symmetry of equilibrium across firms and products, $p_t(\omega) = p_t$, $q_t(\omega) = q_t$, and $\pi_t(\omega) = \pi_t$. Furthermore, (D1) ensures that the balanced growth path is the only equilibrium path of the economy: see Proposition 2A below.

In addition, we introduce the following two alternative conditions.

(D2):
$$\sigma'(x) < 0 \Leftrightarrow M'(x) > 0.$$

In words, this condition states that the demand for each product becomes more price elastic as one moves up along the demand curve, eq.(14), (i.e., for a higher price/a lower quantity), which is sometimes called "Marshall's Second Law of Demand". (D2) is also equivalent to stating that the elasticity of substitution between any two products, $\omega_1$and ω_2 evaluated at $x(\omega_1) =$

[7]Without the condition, $\sigma(x) > 1$, the firm's profit-maximization problem, its pricing rule, eq.(15), and its markup rate, $M(x) > 1$ would not be well-defined. It is equivalent to assuming that the firm's revenue, $p_t(\omega)q_t(\omega) = u'\big(x_t(\omega)\big)x_t(\omega)(NE_t/\Delta_t)$ is increasing in $x_t(\omega)$, and hence its marginal revenue, the LHS of eq.(15), is positive.

$x(\omega_2) = x$ is decreasing in x, or increasing "relative love for variety" (RLV) to use the terminology of Zhelobodkho et.al. (2012).[8] Note that (D2) implies (D1), but not the other way around. More specifically, under the following condition,

(D3):
$$\frac{1}{\sigma(x)} > -\frac{xM'(x)}{M(x)} > 0.$$

(D1) holds, and yet the opposite of (D2) holds. Thus, (D3) implies a violation of Marshall's Second Law of Demand and decreasing RLV. Obviously, CES is a borderline case between the two subclasses of DEA satisfying (D2) or (D3).

Whether (D2) or (D3) holds plays a crucial role in the comparative static results in Proposition 2B below. It is thus important to understand its empirical implications. Note that (D1) implies that the unique solution of eq.(15), $x_t(\omega)$, is strictly decreasing in $\psi_t \Delta_t$. We can thus show:

i) *Imperfect Pass-Through:* A higher production cost, ψ_t, reduces $x_t(\omega)$. This leads to a lower markup rate, $M(x_t(\omega))$ under (D2);

ii) *Strategic Complementarity in Pricing,* If the competitors reduced their prices and increased their sales, Δ_t would go up, which would reduce $x_t(\omega)$. This would lead to a lower markup rate, $M(x_t(\omega))$, and a lower price, $p_t(\omega)$ under (D2);

iii) *Procompetitive Entry;* The presence of more firms, an exogenous increase in V_t, would lead to a higher Δ_t, which would reduce $x_t(\omega)$. This would lead to a lower markup, $M(x_t(\omega))$ under (D2).

Now, suppose instead that (D3) holds. Then, a decline in $x_t(\omega)$, caused by an increase in ψ_t or Δ_t, would lead to an increase in $M(x_t(\omega))$ with $M'(x) < 0$. Thus, it would imply more than 100% pass-through, strategic substitutes in pricing, and anti-competitive entry. As discussed in Latzer, Matsuyama, and Parenti (2019), the empirical evidence is generally in support of imperfect pass-through, strategic complementarity in pricing, and procompetitive entry, which suggests that (D2) might be the more empirically relevant case. However, for the sake of completeness, we will discuss the implications of both (D2) and (D3) below.[9]

[8] Zhelobodkho et.al. (2012) called the *inverse* of $\sigma(\cdot)$ "relative love for variety (RLV)."

[9] It is worth noting that, in other classes of non-CES preferences, assuming Marshall's Second Law of Demand does not necessarily imply strategic complementarity in pricing or procompetitive entry. For example, in a class of preferences used in Boucekkine et.al. (2017), the pricing rule of each firm is *independent* of the pricing behaviors of other firms or the number of firms competing.

Because (D1) ensures the symmetry of equilibrium, the pricing rule now becomes

$$p_t\left(1-\frac{1}{\sigma(x_t)}\right)=w_t\psi_t \Leftrightarrow p_t=M(x_t)w_t\psi_t.$$

Using this expression, and following the same steps as in the CES case, the profit share in the aggregate expenditure, eq.(3) now becomes

$$\frac{\pi_t V_t}{NE_t}=\frac{1}{\sigma(x_t)}\equiv 1-\frac{1}{M(x_t)}, \tag{16}$$

while the share of the wage payment to the production sector in the aggregate expenditure, eq.(4) becomes

$$\frac{w_t\, L_{Xt}}{NE_t}=1-\frac{1}{\sigma(x_t)}\equiv\frac{1}{M(x_t)}.$$

Note that departing from CES to DEA does not change the relations between these shares and the markup rate. However, these shares are endogenous under DEA, because the markup rate is a function of per capita consumption of each product, which is increasing under (D2) and decreasing under (D3).

3.2. Intertemporal problem

Under DEA, the intertemporal utility is now given by

$$\mathcal{U}_0=\int_0^\infty log\left(V_t u\left(\frac{E_t}{p_t V_t}\right)\right)e^{-\rho t}\,dt,$$

which agents maximize subject to the intertemporal budget constraint eq.(8). This leads to the first-order condition given by

$$\frac{\zeta(x_t)}{E_t}e^{-\rho t}=\lambda_0 e^{-R_t},$$

where $\zeta(x)\equiv u'(x)x/u(x)>0$, while λ_0 is again the Lagrange multiplier associated with eq.(8). Log-differentiating this first-order condition with respect to t yields an augmented Euler equation,

$$\frac{\dot{E}_t}{E_t}=r_t-\rho+\frac{\dot{\zeta}(x_t)}{\zeta(x_t)}=r_t-\rho+\left[1-\frac{1}{\sigma(x_t)}-\zeta(x_t)\right]\frac{\dot{x}_t}{x_t}, \tag{17}$$

which features an additional term, which is absent in the original Euler equation, eq.(9).[10]

[10] For CES, $\sigma(x_t)=\sigma$ and $\zeta(x_t)=1-1/\sigma$, so that the last term disappears.

3.3. Balanced Growth Path

Following the same definition for the BGP as before, a constant markup rate requires that x_t must be constant. Furthermore, from $L_{Xt} = \psi_t N x_t V_t$, a constant x_t and a constant L_{Xt} requires that $\psi_t V_t$, must be constant. Thus, in order to ensure the existence of a BGP, it is now necessary to assume along Gancia and Zilibotti (2005) that knowledge spillovers improve productivity not only in R&D but also in production, as $\psi_t = \psi/V_t$.[11]

Again, we derive the law of motion for the economy. Following the same steps as in the CES case, but noticing that the augmented Euler equation, eq.(17) now has the additional term, eq.(11) is now modified to:

$$\frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} = \frac{\mathcal{E}_t}{\sigma(x_t)F} - g_t - \rho + \left[1 - \frac{1}{\sigma(x_t)} - \zeta(x_t)\right]\frac{\dot{x}_t}{x_t},$$

while eq.(12) is modified to

$$L = L_{Rt} + L_{Xt} = Fg_t + \psi N x_t.$$

Combining these two equations, and using eq.(16), $\mathcal{E}_t \equiv NE_t/w_t = N\psi x_t M(x_t)$, we obtain the following law of motion for x_t:

$$\left[\zeta(x_t) + \frac{1}{\sigma(x_t)} + \frac{x_t M'(x_t)}{M(x_t)}\right]\frac{\dot{x}_t}{x_t} = \frac{N\psi x_t M(x_t) - (L + \rho F)}{F}. \tag{18}$$

(D1) implies that the bracket term in front of $\dot{x}_t/x_t$ on the LHS of eq.(18) is positive. (D1) also implies that $x_t M(x_t)$ is increasing in x_t. Thus, $\dot{x}_t > 0$ for $x_t > x^*$ and $\dot{x}_t < 0$ for $x_t < x^*$, where x^* is defined implicitly by

$$x^* M(x^*) = \frac{L + \rho F}{N\psi} = \frac{h + \rho F/N}{\psi}. \tag{19}$$

Thus, eq.(18) would imply divergence, leading to a violation of the equilibrium conditions, unless the economy jumps immediately to $x_0 = x^*$ and stays at $x_t = x^*$. This in turns implies $L_{Xt} = \psi N x^*$ and $L_{Rt} = Fg_t = L - L_{Xt} = L - \psi N x^*$ are all constant along the only equilibrium path and the economy stays on the balanced growth path, as long as the parameters are such that the R&D sector is active: $L_{Rt} = Fg_t = L - \psi N x^* = N(h - \psi x^*) > 0$. Since (D1) implies that the LHS of eq.(19) is increasing in x^*, this condition can be written as $M(h/\psi) > 1 + \rho F/L$. Hence, we have:

---

[11]This assumption has been made in other horizontal innovation models of endogenous growth, such as Foellmi and Zweimuller (2006). Note also that it is isomorphic to assuming that $F_t = F, \psi_t = \psi$, and $h_t = hV_t$.

Proposition 2A: Balanced Growth Path under DEA with (D1)

Suppose $M(h/\psi) > 1 + \rho F/L$. Then, the economy jumps immediately to the balanced growth path along which

$$L_{Xt} = L_X^* = \left(1 - \frac{1}{\sigma(x^*)}\right)(L + \rho F) = \frac{L + \rho F}{M(x^*)} < L;$$

$$L_{Rt} = L_R^* = \frac{L}{\sigma(x^*)} - \left(1 - \frac{1}{\sigma(x^*)}\right)\rho F = \left(1 - \frac{1}{M(x^*)}\right)L - \frac{\rho F}{M(x^*)} > 0;$$

$$g_t = g^* = \frac{L}{\sigma(x^*)F} - \left(1 - \frac{1}{\sigma(x^*)}\right)\rho = \left(1 - \frac{1}{M(x^*)}\right)\frac{L}{F} - \frac{\rho}{M(x^*)} > 0,$$

where x^* is defined implicitly by eq.(19).

Note that this proposition requires (D1), but neither (D2) nor (D3). Note also that, by comparing Proposition 1A and Proposition 2A, departing from CES to DEA does not alter the functional relations between L_X^*, L_R^*, g^* on one hand and $\sigma(x^*)$ and $M(x^*)$ on the other hand. The *only but significant* difference is that, under DEA, the parameters, $\rho, F, h, N,$ and ψ affect $\sigma(x^*)$ and $M(x^*)$ through eq.(19).

From Proposition 2A, it is straightforward to conduct the comparative statics both under (D2) and under (D3). The following proposition shows the case of (D2).

Proposition 2B: Comparative Statics under DEA with (D2)

Under DEA preferences with (D2),

i) Both an increase in the discount rate ρ and in the R&D cost F increase per capita per product consumption x^* and the markup rate $M(x^*)$, increase L_X^*, and decrease L_R^* and the growth rate g^*;

ii) An increase in the population size N decreases x^* and $M(x^*)$, increases L_X^*, L_R^*, and g^*;

iii) An increase in per capita labor endowment h increases x^* and $M(x^*)$, increases L_X^*, L_R^*, and g^*;

iv) Both an increase in N (a decrease in h) for a fixed $hN = L$, and an increase in ψ decrease x^* and $M(x^*)$, increase L_X^*, and decrease L_R^* and g^*.

In the interest of saving space, we do not present the comparative statics results under (D3) as a proposition. However, Table summarizes the comparative statics results both under (D2) and under (D3). The signs in the shaded boxes are the consequences of (D2). Under (D3) instead, the signs would be opposite in the shaded boxes. Under the borderline case of CES, they would be "0".

Table: Comparative Statics under DEA under (D2)*

	x^*	$M(x^*)$	$L_X^* = \psi N x^*$	L_R^*	$g^* = L_R^*/F$
$\rho \uparrow$	+	+	+	−	−
$F \uparrow$	+	+	+	−	−
$N \uparrow$	−	−	+	+	+
$h \uparrow$	+	+	+	+	+
$N = L/h \uparrow$, fixed L	−	−	+	−	−
$\psi \uparrow$	−	−	+	−	−

*The signs in the shaded boxes would be the opposite under (D3) and "0" under CES.

An immediate corollary of Proposition 2B is:

Corollary: Correlations between the Markup and Growth Rates under DEA with (D2)

Under DEA preferences with (D2),

v) A change in the discount rate $\rho,$ the R&D cost F, or the population size N causes the markup rate $M(x^*)$ and the growth rate g^* to move in the opposite direction.

vi) A change in per capita labor endowment h or the production cost ψ causes the markup rate $M(x^*)$ and the growth rate $g^*$to move in the same direction.

We now discuss the implications of departing from CES within DEA in the direction of (D2), by comparing Proposition 1B with Proposition 2B and its corollary.

Just as in CES, both an increase in ρ and in F discourage R&D, which causes the reallocation of labor from the R&D sector to the production sector. This causes an increase in x^*, per capita consumption of each product. Unlike in CES, this increases the markup rate $M(x^*)$ under (D2). This secondary effect mitigates the impact on labor reallocation, but not enough to overturn it. However, this causes the negative correlations between the markup rate $M(x^*)$ and the innovation and growth rate, g^*. In particular, this implies that the measure of competitiveness and the growth rate are positively correlated in cross-section of countries, if countries differ mostly in the innovation (or firm entry) cost.

Just as in CES, both an increase in N and in h, by increasing the total labor supply, lead to an increase in the labor supply to both the production and the R&D sectors. The latter leads to an increase in the growth rate, due to the familiar scale effect. However, as seen in eq.(19), they have opposite impacts on x^* and hence on $M(x^*)$ under (D2). A higher N leads to a lower x^*, which in turn leads to a lower $M(x^*)$ under (D2), mitigating the effect on the growth rate as well as generating the negative correlations between $M(x^*)$ and g^*. In contrast, a higher h leads to a higher x^*, which in turn leads to a higher $M(x^*)$ under (D2), amplifying the effect on the growth rate as well as generating the positive correlations.

To see such differential effects of changes in h and in N more clearly, consider the effect of increasing N and decreasing h simultaneously to keep the total labor supply $L = hN$ unchanged. This removes the scale effect. Without a change in L, an increase in N would necessitate a decline in per capita consumption of each product. This would have no impact on the markup rate and the allocation of labor between the production and R&D sectors under CES. However, under (D2), this causes the markup rate to decline, reducing the incentive to innovate, which causes the reallocation of labor from the R&D sector to the production sector, and a decline in the innovation and growth rates. This result suggests that, once the aggregate market size is controlled for, richer countries with smaller population sizes innovate more and hence grow faster than poorer countries with larger population sizes.

Indeed, this effect of an increase in N without an increase in L is completely isomorphic to the effect of an increase in ψ, as clearly seen from eq.(19). Without a change in L, a higher ψ, just like a higher N, necessitates per capita consumption of each product to decline. This change would be neutral under CES. Under (D2), however, it leads to a decline in the markup rate through imperfect pass-through, which discourages R&D, causing labor to reallocate to the production sector and the growth rate to decline.

It is worth pointing out that, though Proposition 2A does not require (D2), the comparative statics results reported in Proposition 2B and its corollary depend on (D2). As already discussed, the signs in the shaded boxes in Table would be reversed if we instead depart from CES in the direction of (D3).

4. Concluding Remarks

In the standard horizontal innovation model of endogenous growth, larger economies innovate more and hence grow faster. Due to the homotheticity of consumer demands, however, it does not matter whether the large market size comes from a large population or a high per capita expenditure. In this paper, we extended the standard textbook model by building on the

Dixit-Stiglitz (1977; Section II) model of monopolistic competition with directly explicitly additive (DEA) nonhomothetic preferences. The model preserves the balanced growth property of the standard model in spite of nonhomotheticity.

Among others, it has been shown that, once the aggregate market size is controlled for, richer countries with smaller populations innovate more and grow faster under the (empirically more relevant) case of "increasing relative love for variety", which suggests that the difference in per capita income across countries could potentially be one reason why there is little supporting evidence for the scale effect. It has also been shown that the correlations between the markup and growth rates across the balanced growth paths could be either positive or negative, depending on the source of variations. In particular, the measure of competitiveness and the growth rate are positively correlated in cross-sections of countries, if countries differ mostly in the innovation (or firm entry) cost.

References:

Acemoglu, Daron, Introduction to Modern Economic Growth, Princeton University Press, 2008.

Boucekkine, Raouf, Hélène Latzer, and Mathieu Parenti, "Variable markups in the long-run: A generalization of preferences in growth models," Journal of Mathematical Economics, 2017, 68 (C), 80–86.

Dixit, Avinash K., Joseph E. Stiglitz, "Monopolistic Competition and Optimal Product Diversity," American Economic Review, June 1977, 67(3), 297-308.

Fajgelbaum, P., G.M. Grossman, and E.Helpman, "Income Distribution, Product Quality, and International Trade," Journal of Political Economy, 2011, 721-765.

Foellmi, Reto, and Josef Zweimueller, "Income Distribution and Demand-Induced Innovation," Review of Economic Studies, 2006, 73, 941-960.

Gancia, Gino and Fabrizio Zilibotti, "Horizontal Innovation in the Theory of Growth and Development,"in Philippe Aghion and Steven Durlauf, eds., Handbook of Economic Growth, Vol. 1, Elsevier, 2005, chapter 3, pp. 111–170.

Gancia, Gino and Fabrizio Zilibotti, "Technological Change and the Wealth of Nations," Annual Review of Economics, 2009, 1, 93-120.

Grossman, Gene M and Elhanan Helpman, Innovation and growth in the global economy, MIT press, 1993.

Jones, Charles I, "R&D-Based Models of Economic Growth," Journal of Political Economy, August 1995, 103 (4), 759–784.

Latzer, Hélène, "A Schumpeterian Theory of Multi-Quality Firms", Journal of Economic Theory, 2018, 175, 766-802.

Latzer, Hélène, Kiminori Matsuyama, Mathieu Parenti, "Revisiting the Market Size Effect in Innovation and Growth," (2019), CEPR Discussion Paper, #14250.

Matsuyama, Kiminori, (2019a): "Engel's Law in the Global Economy: Demand-Induced Patterns of Structural Change, Innovation, and Trade," Econometrica, vol.87, no.2, 497-528.

Matsuyama, Kiminori, (2019b): "Supplement to 'Engel's Law in the Global Economy: Demand-Induced Patterns of Structural Change, Innovation, and Trade'," Econometrica *Supplemental Material*, 87, https://doi.org/10.3982/ECTA13765.

Romer, Paul M, "Endogenous Technological Change," Journal of Political Economy, October 1990, 98 (5), 71–102.

Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Jacques-Francois Thisse, "Monopolistic Competition: Beyond the Constant Elasticity of Substitution," Econometrica, November 2012, Vol.80, No.6, 2765-2784.