Limited Farsightedness in R&D Network Formation

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Abstract

We adopt the horizon-K farsighted set of Herings, Mauleon and Vannetelbosch (2019) to study the R&D networks that will emerge in the long run when firms are neither myopic nor fully farsighted but have some limited degree of farsightedness. We find that a singleton set consisting of a pairwise stable network is a horizon-K farsighted set for any degree of farsightedness $K \geq 2$. That is, each R&D network consisting of two components of nearly equal size satisfies both horizon-K deterrence of external deviations and horizon-K external stability for $K \geq 2$. On the contrary, each R&D network consisting of two components with the largest one comprising three-quarters of firms, predicted when all firms are fully farsighted, violates horizon-K deterrence of external deviations. Thus, when firms are homogeneous in their degree of farsightedness, pairwise stable R&D networks consisting of two components of nearly equal size are robust to limited farsightedness.

Key words: Limited farsightedness, Stability, R&D Networks.
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1 Introduction

R&D bilateral collaborations are agreements that allow firms to access to new knowledge providing them a competitive advantage. Hence, such collaborative agreements could induce competitors to look for their own R&D alliances. Since innovation enhances both growth and welfare, it is important to analyse the formation of bilateral R&D agreements between firms.

Different ways of characterizing which network structures are stable have been proposed in the literature depending on whether (and how far) agents anticipate that their action may also induce others to change the network relations they maintain.[1] On the one hand pairwise stability (Jackson and Wolinsky, 1996) involves fully myopic agents in the sense that they do not anticipate that others might react to their actions: agents are able to modify the network one link at a time, and choose to change the network if the resulting network implies higher payoffs for the deviating agents. On the other hand, a number of solution concepts involve perfectly farsighted agents: agents fully anticipate the complete sequence of reactions that results from their own actions in the network.[2] However, Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016) provide experimental evidence suggesting that subjects are consistent with an intermediate degree of farsightedness: agents only anticipate a limited number of reactions by the other agents to the actions they take themselves.

Similarly to Goyal and Moraga-Gonzalez (2001), Goyal and Joshi (2003), and Mauleon, Sempere-Monerris and Vannetelbosch (2014), we consider a n-firm industry, where initially firms produce an homogeneous good at a given equal and constant marginal cost. Each firm is able to reduce its marginal cost by forming a R&D collaborative link with another competitor. The marginal cost of production reduction for one firm is proportional to the number of firms it is connected to. When a link is formed between two firms that are not connected, all firms connected to those two firms benefit from that link while others become less competitive. The collection of R&D collaborative links define the R&D network which in turn deter-

[1] Mauleon and Vannetelbosch (2016) provide a comprehensive overview of the solution concepts for solving network formation games.

[2] Various approaches to farsightedness can be found in Chwe (1994), Xue (1998), Herings, Mauleon and Vannetelbosch (2004, 2009), Mauleon and Vannetelbosch (2004), Dutta, Ghosal and Ray (2005), Page, Wooders and Kamat (2005), Page and Wooders (2009), Mauleon, Vannetelbosch and Vergote (2011), and Ray and Vohra (2015).

mines the marginal cost profile for the n firms. Once the R&D network is formed, firms compete in quantities.

When all firms are myopic, Mauleon, Sempere-Monerris and Vannetelbosch (2014) show that a R&D network is pairwise stable if and only if it consists of two "symmetric" components of nearly equal size (close to $n/2+1$ and $n/2-1$). In contrast to myopia, once all firms are fully farsighted, farsighted stability leads to R&D networks consisting of two "asymmetric" components of different sizes (close to $3n/4$ and $n/4$). Recently, Mauleon, Sempere-Monerris and Vannetelbosch (2018) allows for a mixed population of firms: myopic firms interacting together with farsighted firms.[3] When the majority of firms are myopic, the myopic-farsighted stable set consists of R&D networks having either two "asymmetric" components of different sizes (close to $3n/4$ and $n/4$) with farsighted firms mostly occupying key positions in the largest component, or two "symmetric" components of nearly equal size (close to $n/2+1$ and $n/2-1$) with the largest component having only myopic firms. However, when the majority of firms becomes farsighted, networks having two components of nearly equal size are now unstable. The myopic-farsighted stable set consists only of the networks having two components of different sizes.[4]

Which are the R&D networks that will emerge in the long run when firms have a limited degree of farsightedness? We adopt the horizon-K farsighted set of Herings, Mauleon and Vannetelbosch (2019) to answer this question. The concept encompasses both the pairwise farsightedly stable set and the pairwise myopically stable set introduced by Herings, Mauleon and Vannetelbosch (2009). A set of networks G_K is a horizon-K farsighted set if three conditions are satisfied. First, deviations outside the set should be horizon-K deterred. Second, horizon-K external stability is required. That is, from any network outside of G_K there is a sequence of farsighted improving paths of length smaller than or equal to K leading to some network in G_K. Third, a minimality condition is required. That is, there is no proper subset of G_K satisfying the first two conditions.

[3] Herings, Mauleon and Vannetelbosch (2020) define the myopic-farsighted stable set for two-sided matching problems, while Luo, Mauleon and Vannetelbosch (2021) investigate the myopic-farsighted stable set in general network formation problems.

[4] Petrakis and Tskas (2018) investigate the effect of potential entry on the formation and stability of R&D networks when firms are farsighted while Roketskiy (2018) studies collaboration between farsighted firms competing in a tournament and finds that stable networks consist of two asymmetric mutually disconnected complete components.

In this paper, we show that a singleton set consisting of a pairwise stable network is a horizon-K farsighted set for any degree of farsightedness greater or equal than 2. That is, each R&D network having two components of nearly equal size satisfies both horizon-K deterrence of external deviations and horizon-K external stability for $K \geq 2$. On the contrary, we find that each R&D network having two "asymmetric" components with the largest one comprising three-quarters of firms violates horizon-K deterrence of external deviations. Hence, R&D networks having two components of nearly equal size are not only stable when the majority of firms are myopic but also when all firms are limitedly farsighted. To sum up, when firms are homogeneous in their degree of farsightedness, pairwise stable R&D networks consisting of two "symmetric" components of nearly equal size are robust to limited farsightedness.

The formation of research collaborations is also studied using the group formation approach where collaborations are modelled in terms of a coalition structure which is a partition of the set of firms (i.e. each firm can only belong to one coalition). Bloch (1995) proposes a sequential game for forming associations of firms. In equilibrium, firms form two asymmetric associations, with the largest one comprising roughly three-quarters of industry members. So, the sizes of the two associations coincide with those Mauleon, Sempere-Monerris and Vannetelbosch (2014) obtain when firms are farsighted. In fact, by assuming that all connected firms in a network fully benefit from a new link, Mauleon, Sempere-Monerris and Vannetelbosch (2014) recover the assumption in Bloch (1995) where the benefits from cooperation increase linearly in the size of the association. The network approach differs from the group formation approach by focusing on bilateral relationships and allowing for a richer class of collaborations. It also differs in the decision making for establishing R&D collaborations. Mutual consent is needed for forming a new link between two firms,[5] whereas the consent of all members of the association is required when a firm joins the association.[6] Both approaches lead to similar conclusions only if firms are farsighted and anticipate the reactions of other firms to the decisions they take.[7]

[5]Caulier, Mauleon and Vannetelbosch (2013) and Caulier, Mauleon, Sempere-Monerris and Vannetelbosch (2013) propose the concept of contractual stability to predict the stable networks when the consent of coalition partners is needed for adding or deleting links.

[6]An exception is the open membership game. Yi (1997) finds that only the grand coalition is stable, but this result is not always robust when firms are not identical (see Yi and Shin, 2000). See Bloch (2005) for a survey on group and network formation in industrial organization.

[7]Mauleon, Sempere-Monerris and Vannetelbosch (2016) show that if firms are myopic (Δ-

The paper is organized as follows. In Section 2 we introduce some notation and basic properties of R&D networks. In Section 3 we define the notion of a horizon-K farsighted set. In Section 4 we identify the horizon-K farsighted set of R&D networks. Finally, in Section 5 we conclude.

2 R&D Networks

We consider a two-stage game in a setting with n competing identical firms that produce some homogenous good. In the first stage, firms decide the bilateral R&D collaborations they establish to maximize their respective profits. Let $N = \{1, 2, ..., n\}$ be the set of firms.[8] A network g of R&D collaborations is a list of which pairs of firms are linked to each other and $ij \in g$ indicates that i and j are linked under g. The complete network on the set of firms $S \subseteq N$ is denoted by g^S and is equal to the set of all subsets of S of size 2. The empty network is denoted by $g^{\emptyset}$. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The cardinality of $\mathcal{G}$ is denoted by $n' = 2^{n(n-1)/2}$.

The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \text{there is } j \text{ such that } ij \in g\}$ be the set of firms who have at least one link in the network g. A path in a network g between i and j is a sequence of firms $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for all $i \in N$ and $j \in N \setminus \{i\}$, there exists a path in g connecting i and j. A non-empty network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $C(g)$. A component h of g is minimally connected if h has $\#N(h) - 1$ links.[9] Knowing the components of a network, we can partition the firms into coalitions within which firms are connected. Let $\mathcal{P}(g)$ denote the partition of N into components and singletons induced by the network g. That is, $S \in \mathcal{P}(g)$ if and only if either there exists $h \in C(g)$ such that $S = N(h)$ or there

stability) there is no stable association structure for $n \geq 8$.

[8]Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subset$ for strict inclusion. Finally, $\#$ will refer to the notion of cardinality.

[9]In a minimally connected component, every pair of firms belonging to the component **is** connected by exactly one path.

exists $i \notin N(g)$ such that $S = \{i\}$. We denote by $S(i)$ the coalition $S \in \mathcal{P}(g)$ such that $i \in S$. We denote by $\bar{S}(g)$ the largest coalition of g. That is, $\bar{S}(g) \in \mathcal{P}(g)$ is such that $\#\bar{S}(g) \geq \#S(g)$ for all $S(g) \in \mathcal{P}(g)$.

R&D collaborations reduce marginal costs of production as in Mauleon, Sempere-Monerris and Vannetelbosch (2014).[10] Each firm benefits fully from its own R&D and from the R&D done by the firms it is connected to.[11] Given a network g, the marginal cost for firm i is given by

$$c_i(g) = c_0 - \#S(i)$$

where c_0 is a firm's initial marginal cost and $\#S(i)$ is the number of firms in the component of firm i. Since knowledge flows perfectly throughout the component, firms bear an infinitesimally small cost for maintaining redundant (or superfluous) links.[12]

In the second stage, firms compete in quantities in the oligopolistic market, taking as given the costs of production. Let $p = a - \sum_{i \in N} q_i$ with $a > 0$ be the linear inverse demand function. For any given R&D network g, one can easily show that there exists a unique Cournot equilibrium on the market, and that each firm's profit $\pi_i(g)$ is a monotonically increasing function of the following valuation or payoff function,[13]

$$\Pi_i(g) = a - c_0 + (n+1)\#S(i) - \sum_{S \in \mathcal{P}(g)} (\#S)^2. \tag{1}$$

In fact, $\Pi_i(g) = (n+1)\sqrt{\pi_i(g)} = (n+1)q_i(g)$ where $q_i(g)$ is the equilibrium output.

[10]Firms collaborate in R&D but do not cooperate on R&D effort choices. For a general background on R&D cooperation in oligopoly the reader is directed to Amir (2000), Kamien, Muller, and Zang (1992) and Katz (1986), among others.

[11]In Mauleon, Sempere-Monerris and Vannetelbosch (2008), the reduction in marginal costs depends on the total number of connected firms, but decreases with the distance. In Goyal and Joshi (2003), the reduction in marginal costs only depends on the number of direct links. In Goyal and Moraga-Gonzalez (2001), firms also benefit imperfectly from public spillovers, i.e. the research done by firms to whom they are not connected.

[12]In a network g, a component $h \in C(g)$ has no redundant links if and only if h is minimally connected. It reflects the idea that firms avoid wasting resources. When a firm deletes a redundant or superfluous link, it remains connected to the same set of firms and so still benefits from the same reduction in marginal costs.

[13]Excluding infinitesimally small costs for maintaining redundant links.

We focus our analysis on the case where there are at least eight firms.[14] Notice that R&D networks connecting all firms are the ones that maximize social welfare, i.e. the sum of the profits and the consumer surplus. As in Bloch (1995), for $n \geq 8$, this payoff function in (1) satisfies some general properties (**P1-P5**) that are useful for characterizing the networks that will emerge in the long run. **P1** states that, in any R&D network, linking two components decreases the payoffs of the firms that do not belong to those components. **P2** states that, in any R&D network, firms belonging to bigger components obtain greater payoffs. **P3** states that, in any R&D network, firms belonging to the two smallest components obtain greater payoffs by bridging the two components. **P4** states that, firms prefer to belong to the largest component of a R&D network with one component encompassing all firms except one rather than to belong to the largest component of a R&D network with two nearly symmetric coalitions. **P5** states that, firms prefer to belong to the single component of a R&D network that includes less firms rather than to belong to the single component of a R&D network that includes more firms, provided that more than half of the firms belong to the single component. Formally, the payoff function satisfies the following five properties.

P1 In any R&D network g, we have that $\Pi_i(g+jk) < \Pi_i(g)$ if $S(i) \neq S(j) \neq S(k)$ and $S(i), S(j), S(k) \in \mathcal{P}(g)$.

P2 In any R&D network g, we have that $\Pi_i(g) > \Pi_j(g)$ if and only if $\#S(i) > \#S(j)$.

P3 In any R&D network g with $\#\mathcal{P}(g) \geq 3$, we have that $\Pi_i(g+ij) > \Pi_i(g)$ and $\Pi_j(g+ij) > \Pi_j(g)$ if $S(i) \neq S(j)$, $S(i), S(j) \in \mathcal{P}(g)$, and $\#S \geq \max\{\#S(i), \#S(j)\}$ for all $S \in \mathcal{P}(g)$, $S \neq S(i), S(j)$.

P4 In any R&D networks g, g' such that $\#\mathcal{P}(g) = \#\mathcal{P}(g') = 2$ with $\#\bar{S}(g) = n-1$ and $\bar{S}(g') = \lceil (n+1)/2 \rceil$, we have that $\Pi_i(g) > \Pi_i(g')$ if $i \in \bar{S}(g)$ and $i \in \bar{S}(g')$.[15]

P5 In any R&D networks g, g' such that $\#C(g) = \#C(g') = 1$ and $\#\bar{S}(g') > \#\bar{S}(g) > \lceil n/2 \rceil$, we have $\Pi_i(g) > \Pi_i(g')$ if $i \in \bar{S}(g)$ and $i \in \bar{S}(g')$.

[14] Petrakis and Tsakas (2018) consider a setup where R&D effort is costly and endogenous but in an environment with only three farsighted firms that could differ in the initial marginal cost and the levels of substitutability between products.

[15] $\lceil x \rceil$ is the function that takes as input a real number x and gives as output the lowest integer greater than or equal to x.

Throughout the paper we illustrate our main results by means of an example with eight firms. In Table 1 we give the firms' equilibrium payoffs in several networks for $a - c_0 = 42$. We make a slight abuse of notation. For instance, $\{5, 2, 1\}$ should be interpreted as a network, composed of three "components" of size 5, 2 and 1, that can be formed by eight firms. Firms in the component of size 5 obtain a payoff of 57, firms in the component of size 2 obtain a payoff of 30, and the (isolated) firm in the "component" of size 1 obtains a payoff of 21.

Networks:	$\{8\}$	$\{5,3\}$	$\{5,2,1\}$	$\{3,3,2\}$	$\{3,3,1,1\}$
Payoffs:	(50)	$(53,35)$	$(57,30,21)$	$(47,47,38)$	$(49,49,31,31)$
Networks:	$\{7,1\}$	$\{4,4\}$	$\{4,3,1\}$	$\{5,1,1,1\}$	$\{3,2,2,1\}$
Payoffs:	$(55,1)$	$(46,46)$	$(52,43,25)$	$(59,23,23,23)$	$(51,42,42,33)$
Networks:	$\{6,2\}$	$\{6,1,1\}$	$\{4,2,2\}$	$\{4,2,1,1\}$	$\{2,2,2,2\}$
Payoffs:	$(56,20)$	$(58,13,13)$	$(54,36,36)$	$(56,38,29,29)$	$(44,44,44,44)$

Table 1: Payoffs for the 8-firm case with $a - c_0 = 42$.

3 Horizon-K Farsighted Set

We propose the notion of horizon-K farsighted set introduced by Herings, Mauleon and Vannetelbosch (2019) to determine the R&D networks that emerge in the long run when firms are neither fully myopic nor completely farsighted but have some limited degree of farsightedness.

A farsighted improving path of length $K \geq 1$ from a network g to a network g' is a finite sequence of networks $g_0, \ldots, g_K$ with $g_0 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either (i) $g_{k+1} = g_k - ij$ for some ij such that $\Pi_i(g_K) > \Pi_i(g_k)$ or $\Pi_j(g_K) > \Pi_j(g_k)$, or (ii) $g_{k+1} = g_k + ij$ for some ij such that $\Pi_i(g_K) > \Pi_i(g_k)$ and $\Pi_j(g_K) > \Pi_j(g_k)$. If there exists a farsighted improving path of length K from g to g', then we write $g \to_K g'$. For a given network g and some $K' \geq 1$, let $\phi_{K'}(g)$ be the set of networks that can be reached from g by a farsighted improving path of length $K \leq K'$. That is, $\phi_{K'}(g) = \{g' \in \mathcal{G} \mid \exists K \leq K' \text{ such that } g \to_K g'\}$. Let

$\phi_\infty(g) = \{g' \in \mathcal{G} \mid \exists K \in \mathbb{N}$ such that $g \to_K g'\}$ denote the set of networks that can be reached from g by some farsighted improving path. Lemma 1 in Herings, Mauleon and Vannetelbosch (2019) shows that for every $K \geq 1$, for every $g \in \mathcal{G}$, it holds that $\phi_K(g) \subseteq \phi_{K+1}(g)$, and that for $K \geq n' - 1$, for every $g \in \mathcal{G}$, it holds that $\phi_K(g) = \phi_{K+1}(g) = \phi_\infty(g)$.

A network g' is *adjacent* to g if either $g' = g + ij$ or $g' = g - ij$ for some ij. A network g' *defeats* g either if $g' = g - ij$ and $\Pi_i(g') > \Pi_i(g)$ or $\Pi_j(g') > \Pi_j(g)$, or if $g' = g + ij$ with $\Pi_i(g') > \Pi_i(g)$ and $\Pi_j(g') > \Pi_j(g)$. A network is *pairwise stable* (Jackson and Wolinsky, 1996) if and only if it is not defeated by another network. Notice that $g' \in \phi_1(g)$ if and only if g' defeats g. We can therefore define the pairwise stable networks P_1 as those $g \in \mathcal{G}$ for which $\phi_1(g) = \emptyset$. For $K \geq 1$, let $P_K = \{g \in \mathcal{G} \mid \phi_K(g) = \emptyset\}$ denote the set of horizon-K pairwise stable networks.

A refinement of pairwise stability is obtained when we require the network g to defeat every other adjacent network, so $g \in \phi_1(g')$ for every network g' adjacent to g. We call such a network g *pairwise dominant.* For $K \geq 1$, a network $g \in \mathcal{G}$ is horizon-K pairwise dominant if for every g' adjacent to g it holds that $g \in \phi_K(g')$. The set of horizon-K pairwise dominant networks is denoted by D_K.

The set $\phi_K^2(g) = \phi_K(\phi_K(g)) = \{g'' \in \mathcal{G} \mid \exists g' \in \phi_K(g)$ such that $g'' \in \phi_K(g')\}$ consists of those networks that can be reached by a composition of two farsighted improving paths of length at most K from g. For $m \in \mathbb{N}$, let $\phi_K^m(g)$ be the networks that can be reached from g by means of m compositions of farsighted improving paths of length at most K. Let ϕ_K^∞ denote the set of networks that can be reached from g by means of any number of compositions of farsighted improving paths of length at most K. Lemma 2 in Herings, Mauleon and Vannetelbosch (2019) shows that for every $K \geq 1$, for every $g \in \mathcal{G}$, it holds that $\phi_K^\infty(g) \subseteq \phi_{K+1}^\infty(g)$, and that for $K \geq n' - 1$, for every $g \in \mathcal{G}$, it holds that $\phi_K^\infty(g) = \phi_{K+1}^\infty(g) = \phi_\infty^\infty(g)$.

The notion of a horizon-K farsighted set is based on two main requirements: horizon-K deterrence of external deviations and horizon-K external stability. A set of networks G satisfies *horizon-K deterrence of external deviations* if all possible deviations from any network $g \in G$ to a network outside G are deterred by a threat of ending worse off or equally well off.[16]

Definition 1 (Herings, Mauleon and Vannetelbosch, 2019)**.** For $K \geq 1$, a set of networks $G \subseteq \mathcal{G}$ satisfies horizon-K deterrence of external deviations if for every

[16] We use the notational convention that $\phi_{-1}(g) = \emptyset$ for every $g \in \mathcal{G}$.

$g \in G$,

(a) $\forall\, ij \notin g$ such that $g+ij \notin G$, $\exists g' \in [\phi_{K-2}(g+ij) \cap G] \cup [\phi_{K-1}(g+ij) \setminus \phi_{K-2}(g+ij)]$ such that $\Pi_i(g') \leq \Pi_i(g)$ or $\Pi_j(g') \leq \Pi_j(g)$,

(b) $\forall\, ij \in g$ such that $g - ij \notin G$, $\exists g', g'' \in [\phi_{K-2}(g - ij) \cap G] \cup [\phi_{K-1}(g - ij) \setminus \phi_{K-2}(g - ij)]$ such that $\Pi_i(g') \leq \Pi_i(g)$ and $\Pi_j(g'') \leq \Pi_j(g)$.

Condition (a) in Definition 1 captures that adding a link ij to a network $g \in G$ that leads to a network $g + ij$ outside of G, is deterred by the threat of ending in g'. Here g' is such that either there is a farsighted improving path of length smaller than or equal to $K - 2$ from $g + ij$ to g' and g' belongs to G or there is a farsighted improving path of length equal to $K - 1$ from $g + ij$ to g' and there is no farsighted improving path from $g + ij$ to g' of smaller length. Condition (b) is a similar requirement, but then for the case where a link is severed.[17]

A set of networks G satisfies *horizon-K external stability* if from any network outside of G there is a sequence of farsighted improving paths of length smaller than or equal to K leading to some network in G.

Definition 2 (Herings, Mauleon and Vannetelbosch, 2019)**.** For $K \geq 1$, a set of networks $G \subseteq \mathcal{G}$ satisfies horizon-K external stability if for every $g' \in \mathcal{G} \setminus G$, $\phi_K^\infty(g') \cap G \neq \emptyset$.

This requirement implies that if we allow players with a degree of farsightedness equal to K to successively create or delete links, they will ultimately reach the set G irrespective of the initial network.

Definition 3 (Herings, Mauleon and Vannetelbosch, 2019)**.** For $K \geq 1$, a set of networks $G_K \subseteq \mathcal{G}$ is a horizon-K farsighted set if it is a minimal set satisfying horizon-K deterrence of external deviations and horizon-K external stability.

[17] Since the degree of farsightedness of players is equal to K, Herings, Mauleon and Vannetelbosch (2019) distinguish farsighted improving paths of length less than or equal to $K-2$ after a deviation from g to $g + ij$ and farsighted improving paths of length equal to $K - 1$. In the former case, the reasoning capacity of the players is not yet reached, and the threat of ending in g' is only credible if it belongs to the set G. In the latter case, the only way to reach g' from g requires K steps of reasoning or even more; one step in the deviation to $g + ij$ and at least $K - 1$ additional steps in any farsighted improving path from $g + ij$ to g'. Since this exhausts the reasoning capacity of the players, the threat of ending in g' is credible, irrespective of whether it belongs to G or not.

Herings, Mauleon, and Vannetelbosch (2019) prove that a horizon-K farsighted set of networks exists. For $K = 1$, Theorem 3 of Herings, Mauleon, and Vannetelbosch (2019) show that there is a unique horizon-1 farsighted set consisting of all networks that belong to a closed cycle.[18] This result does not carry over to higher levels of K.

As shown by Herings, Mauleon, and Vannetelbosch (2019), the collection of horizon-K farsighted sets is independent of K when $K \geq n' + 1$. Moreover, for every pairwise farsightedly stable set G_∞ defined by Herings, Mauleon and Vannetelbosch (2009), there is a set $G' \subseteq G_\infty$ such that G' is a horizon-$(n' + 1)$ farsighted set.[19]

4 Horizon-K Farsighted Set of R&D Networks

Mauleon, Sempere-Monerris and Vannetelbosch (2014) show that the set $G^{1/2} = \{g \in \mathcal{G} \mid C(g) = (h_1, h_2),\ h_1$ and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N, \#N(h_1) = \lceil (n+1)/2 \rceil\}$ is the set of pairwise stable networks. That is, a network $g \in \mathcal{G}$ is pairwise stable if and only if g consists of two minimally connected components with the cardinality of the largest component equal to $\lceil (n + 1)/2 \rceil$. Thus, in the case all firms are myopic, stability leads to R&D networks consisting of nearly symmetric components. However, in the case all firms are farsighted, Mauleon, Sempere-Monerris and Vannetelbosch (2014) show that the set $G^{3/4}$ is a pairwise farsightedly stable set, where $G^{3/4} = \{g \in \mathcal{G} \mid C(g) = (h_1, h_2),\ h_1$ and h_2 are minimally connected, $N(h_1) \cup N(h_2) = N$, and $\#N(h_1) = \lfloor (3n + 1)/4 \rfloor\}$ is the set of all R&D networks that consist of two minimally connected components of different size close to $3n/4$ and $n/4$, respectively.[20]

Which R&D networks are stable when firms are neither myopic nor fully farsighted but rather have a limited degree of farsightedness?

[18] Similarly to Jackson and Watts (2002), a set of networks C is a cycle if for any $g' \in C$ and $g \in C \setminus \{g'\}$, $g' \in \phi_1^\infty(g)$. A cycle C is a closed cycle if $\phi_1^\infty(C) = C$. For every pairwise stable network $g \in P_1$, the set $\{g\}$ is a closed cycle.

[19] Herings, Mauleon and Vannetelbosch (2009) define a pairwise farsightedly stable set as a set G_∞ of networks satisfying horizon-∞ deterrence of external deviations and minimality, but with horizon-∞ external stability replaced by the requirement that for every $g' \in \mathcal{G} \backslash G_\infty$, $\phi_\infty(g') \cap G_\infty \neq \emptyset$.

[20] $\lfloor x \rfloor$ is the function that takes as input a real number x and gives as output the greatest integer less than or equal to x.

We say that firm i is a *leaf node* in network g if it only has a single link, i.e. $\#N_i(g) = 1$. If a firm that is a leaf node deletes its single link, it becomes an isolated firm with no R&D collaboration.

We now provide results that turn to be useful for characterizing the set of horizon-1 pairwise dominant networks, the horizon-1 farsighted set and the horizon-K farsighted sets for $K \geq 2$. Lemma 1 tells us that, from any network where some component is not minimally connected there is a sequence of farsighted improving paths of length 1 leading to some network where this component is now minimally connected and all firms belong to the same component or are isolated as in the initial network. Lemma 1 follows from the incentives for deleting superfluous links in components that are not minimally connected.

Lemma 1. *Take any network $g \in \mathcal{G}$ such that some component $h \in C(g)$ is not minimally connected. It holds that there exists some network $g' \in \phi_1^{\infty}(g)$ such that every component $h \in C(g')$ is minimally connected and $\mathcal{P}(g) = \mathcal{P}(g')$.*

Lemma 2 tells us that, from any network with at least three components and/or isolated firms, there is a sequence of farsighted improving paths of length 1 leading to some network with two components. Lemma 2 follows from **P3** and the incentives for adding a link between two firms that belong to the two smallest coalitions.

Lemma 2. *Take any network $g \in \mathcal{G}$ such that every component $h \in C(g)$ is minimally connected and $\#\mathcal{P}(g) \geq 3$. It holds that there exists some network $g' \in \phi_1^{\infty}(g)$ such that every component $h \in C(g')$ is minimally connected and $\#\mathcal{P}(g') = \#C(g') = 2$.*

Lemma 3 tells us that, from any minimally connected network with a single component connecting all firms, there is a sequence of farsighted improving paths of length 1 towards some network belonging to the set of pairwise stable networks, $G^{1/2}$. Starting from a minimally connected network g such that $\mathcal{P}(g) = \{N\}$ we build a sequence of farsighted improving paths of length 1 towards some network $g' \in G^{1/2}$ as follows. First, at each step a firm that is a leaf is isolated until we reach a network $g_1 \subset g$ with $\#\bar{S}(g_1) = \lceil (n+1)/2 \rceil$ and $n - \lceil (n+1)/2 \rceil$ isolated firms. Given **P5**, firms belonging to the largest component have incentives to isolate a firm that is a leaf. Next, at each step isolated firms have incentives to add links one by one between them to form a minimally connected component leading to some g' such that $\mathcal{P}(g') = \{S_1, S_2\}$ with $\#S_1 = \lceil (n+1)/2 \rceil$ and $\#S_2 = n - \lceil (n+1)/2 \rceil$.

Lemma 3. *Take any minimally connected network $g \in \mathcal{G}$ such that $\#\mathcal{P}(g) = 1$. It holds that $\phi_1^\infty(g) \cap G^{1/2} \neq \emptyset$.*

Lemma 4 tells us that, from any non-pairwise stable network with two minimally connected components or one minimally connected component and one isolated firm, there is a sequence of farsighted improving paths of length 1 leading to some pairwise stable network.

Lemma 4. *Take any network $g \in \mathcal{G} \setminus G^{1/2}$ such that every component $h \in C(g)$ is minimally connected and $\#\mathcal{P}(g) = 2$. It holds that $\phi_1^\infty(g) \cap G^{1/2} \neq \emptyset$.*

Proof. Starting from a network $g \notin G^{1/2}$ such that each component is minimally connected and $\#\mathcal{P}(g) = 2$ we build a sequence of farsighted improving paths of length 1 towards some network $g' \in G^{1/2}$ as follows. At each step some firm that is a leaf in the largest component is first isolated. Next, this isolated firm adds a link to some firm that does not belong to the largest component. We proceed so up to the largest component reaches the size $\lceil (n+1)/2 \rceil$. Formally, let $g_0 = g$ and $g_{\bar{k}} = g'$. For $k = 1, ..., \bar{k} = 2(\#\bar{S}(g) - \lceil (n+1)/2 \rceil)$, either $g_k = g_{k-1} - ij$ for k odd with $\#N_j(g_{k-1}) = 1$ and $j \in \bar{S}(g_{k-1})$, or $g_k = g_{k-1} + jl$ for k even with $N_j(g_{k-1}) = \emptyset$ and $l \notin \bar{S}(g_{k-1})$. When k is odd, we have that $\Pi_i(g_k) > \Pi_i(g_{k-1})$ for $i \in \bar{S}(g_{k-1}) \setminus \{j\}$ with $\mathcal{P}(g_k) = \{\bar{S}(g_{k-1}) \setminus \{j\}, \{j\}, N \setminus \bar{S}(g_{k-1})\}$. When k is even, we have that $\Pi_j(g_k) > \Pi_j(g_{k-1})$ and $\Pi_l(g_k) > \Pi_l(g_{k-1})$ for j such that $N_j(g_{k-1}) = \emptyset$ and $l \in N \setminus \bar{S}(g_{k-1})$, with $\mathcal{P}(g_k) = \{\bar{S}(g_{k-1}) \setminus \{j\}, N \setminus \bar{S}(g_{k-1}) \cup \{j\}\}$. Thus, $\phi_1^\infty(g) \cap G^{1/2} \neq \emptyset$ □

We first show that the set of pairwise stable networks, $G^{1/2}$, is the set of pairwise dominant networks.

Proposition 1. *The set $G^{1/2}$ is the set of horizon-*1 *pairwise dominant networks, $D_1 = G^{1/2}$.*

Proof. **(i)** We first show that $G^{1/2} \subseteq D_1$. Each $g \in G^{1/2}$ defeats every adjacent network g' to g, so $g \in \phi_1(g')$. Indeed, for every $g \in G^{1/2}$ we have $\Pi_i(g) > \Pi_i(g - ij)$ for all $ij \in g$. Deleting some $ij \in g$ would split one of the two components in $C(g)$ either in two new components or in one new component with an isolated firm. In both cases, all firms belonging to the component that is split become strictly worse off. Adding a link between two firms belonging to the two different components in

g would strictly decrease the profit of the firm belonging to the largest component in g. That is, $\Pi_i(g) > \Pi_i(g+ij)$ for $g \in G^{1/2}$, $i \in h_1$, $j \in h_2$, $C(g) = (h_1, h_2)$, $\#N(h_1) > \#N(h_2)$; so $g \in \phi_1(g+ij)$. Adding a link $ij \notin g$ between two firms i and j belonging to the same component in $g \in G^{1/2}$ would strictly decrease the profit of both firms i and j since this link ij does not increase the profit and is infinitesimally costly; so again $g \in \phi_1(g+ij)$. Thus, $G^{1/2} \subseteq D_1$.
(ii) We now show that for any $g \notin G^{1/2}$, $\phi_1(g) \neq \emptyset$, and so $g \notin D_1$. From Lemma 1, Lemma 2, Lemma 3 and Lemma 4 it directly follows that for any $g \in \mathcal{G} \setminus G^{1/2}$ it holds that $\phi_1(g) \neq \emptyset$. Thus, from **(i)** and **(ii)** it follows that $D_1 = G^{1/2}$. $\square$

We next show that the set of pairwise stable networks, $G^{1/2}$, is the unique horizon-1 farsighted set.

Proposition 2. *The set $G^{1/2}$ is the unique horizon-1 farsighted set.*

Proof. **(i)** Notice that if $g \in \phi_1^{\infty}(g')$ and $g' \in \phi_1^{\infty}(g'')$ then $g \in \phi_1^{\infty}(g'')$. From Lemma 1, Lemma 2, Lemma 3 and Lemma 4 it directly follows that for any $g \in \mathcal{G} \setminus G^{1/2}$ it holds that $\phi_1^{\infty}(g) \cap G^{1/2} \neq \emptyset$. Hence, $G^{1/2}$ satisfies horizon-1 external stability. **(ii)** From Proposition 1 we have that $D_1 = G^{1/2}$ and each $g \in G^{1/2}$ defeats every adjacent network g' to g, so $g \in \phi_1(g')$. Hence, $G^{1/2}$ satisfies horizon-1 deterrence of external deviations. **(iii)** Since $G^{1/2}$ is the set of pairwise stable networks, it holds that $\phi_1(g) = \phi_1^{\infty}(g) = \emptyset$ for every $g \in G^{1/2}$. Hence, $G^{1/2}$ satisfies the minimality requirement. **(iv)** Suppose that $G' \neq G^{1/2}$ is a horizon-1 farsighted set. Since for every $g \in G^{1/2}$, it holds that $\phi_1^{\infty}(g) = \emptyset$, then $G^{1/2} \subseteq G'$. Otherwise, G' violates horizon-1 external stability. But, if $G^{1/2} \subsetneq G'$ then G' violates the minimality requirement. Hence, G is the unique horizon-1 farsighted set. $\square$

What happens for higher degrees of farsightedness (i.e. for $K \geq 2$)? We show that a singleton set consisting of a pairwise stable network is a horizon-K farsighted set for any degree of farsightedness greater or equal than 2. Hence, pairwise stable networks are not only stable when firms are myopic but also when firms are limitedly farsighted.

Proposition 3. *A singleton set $\{g\}$ such that $g \in G^{1/2}$ is a horizon-K farsighted set for every $K \geq 2$.*

Since $g \in D_1$, each $\{g\}$ with $g \in G^{1/2}$ satisfies horizon-K deterrence of external deviations. Obviously, each $\{g\}$ with $g \in G^{1/2}$ also satisfies the minimality requirement. We now provide Lemma 5 and Lemma 6 that are useful for showing that each $\{g\}$ with $g \in G^{1/2}$ satisfies horizon-K external stability for $K \geq 2$, and then we prove Proposition 3. Take any two pairwise stable networks such that the firms belonging to the largest component are not exactly the same. Lemma 5 tells us that there is a sequence of farsighted improving paths of length at most 2 from one of the two pairwise stable networks to some pairwise stable network where firms in the largest component are the same as the ones in the largest component of the other pairwise stable network.

Lemma 5. *Take any* $g, g' \in G^{1/2}$ *such that* $g \neq g', \bar{S}(g) \neq \bar{S}(g')$*. Then, there exists* $g'' \in \phi_2^{\infty}(g')$ *such that* $g'' \in G^{1/2}$ *and* $\bar{S}(g) = \bar{S}(g'')$*.*

Proof. Take any $g, g' \in G^{1/2}$ such that $g \neq g', \bar{S}(g) \neq \bar{S}(g')$. There is $i^* \in \bar{S}(g')$ and $j^* \in N \setminus \bar{S}(g')$ such that $i^*, j^* \in \bar{S}(g)$. We now build in steps a sequence of farsighted improving paths (of length at most $K = 2$) from g' to some g'' (i.e. $g'' \in \phi_2^{\infty}(g')$) such that $g'' \in G^{1/2}$ and $\bar{S}(g'') = \bar{S}(g)$.

Step 1. We build a farsighted improving path of length $K = 2$ from g' where firms look two steps forward such that: $g' \to g' + i^*j^* \to g' + i^*j^* - kl$ with $k, l \in N \setminus \bar{S}(g')$, $kl \in g'$ with l being a leaf in $g' + i^*j^*$. Looking two steps forward, firms i^* and j^* have incentives to add the first link i^*j^* since $\Pi_{i^*}(g' + i^*j^* - kl) > \Pi_{i^*}(g')$ and $\Pi_{j^*}(g' + i^*j^* - kl) > \Pi_{j^*}(g')$ by **P4**, even tough $\Pi_{i^*}(g' + i^*j^*) < \Pi_{i^*}(g')$ and $\Pi_{j^*}(g' + i^*j^*) > \Pi_{j^*}(g')$. From $g' + i^*j^*$ to $g' + i^*j^* - kl$, firm k that is not a leaf has incentives (looking one step forward) to delete its link with the leaf l since $\Pi_k(g' + i^*j^*) < \Pi_k(g' + i^*j^* - kl)$ by **P5**. Notice that $\bar{S}(g' + i^*j^* - kl) = N \setminus \{l\}$.

Step 2. We build a sequence of farsighted improving paths of length $K = 1$ from $g' + i^*j^* - kl$, where firms that were belonging to $N \setminus \bar{S}(g')$ are isolated one by one with firm j^* being the last one to be isolated. That is, starting from $g' + i^*j^* - kl$, we have a sequence of deviations $g' + i^*j^* - kl \to g' + i^*j^* - kl - k_1l_1 \to g' + i^*j^* - kl - k_1l_1 - k_2l_2 \to ... \to g' + i^*j^* - \{kl \mid kl \in g' \text{ and } k, l \in N \setminus \bar{S}(g')\}$ where at each step m firm k_m deletes its link to firm l_m that is a leaf, with $k_m, l_m \neq j*$, $k_m \neq l_m$, $k_m, l_m \in N \setminus \bar{S}(g')$, $m = 1, ..., \#(N \setminus \bar{S}(g')) - 2$. Firm k_m that is not a leaf has incentives (looking one step forward) to delete its link with the leaf firm l_m since $\Pi_{k_m}(g' + i^*j^* - k_1l_1 - ... - k_{m-1}l_{m-1}) < \Pi_{k_m}(g' + i^*j^* - k_1l_1 - ... - k_{m-1}l_{m-1} - k_ml_m)$ by **P5**. Next firm i^* deletes its link with firm j^* to isolate firm j^* and we reach a network $g_1 = g' - \{ij \mid ij \in g' \text{ and } i, j \in N \setminus \bar{S}(g\prime)\}$ where $\bar{S}(g_1) = \bar{S}(g')$ and all firms that were belonging to $N\backslash\bar{S}(g')$ are isolated. Next those isolated firms add one by one links between them to form a star component with firm j^* in the center. By **P3**, firm j^* and each firm k have incentives to add the link j^*k to the current network looking one step forward. We reach $g_2 = g_1 + \{j^*k \mid k \neq j^*, k \in N \setminus \bar{S}(g')\}$. Notice that $\bar{S}(g_2) = \bar{S}(g')$ and $g_2 \in G^{1/2}$.

Step 3. We build a farsighted improving path of length $K = 2$ from $g_2 = g_1 + \{j^*k \mid k \neq j^*, k \in N \setminus \bar{S}(g')\}$ where, looking two steps forward ($K = 2$), firm i^* and firm j^* first build the link i^*j^* and next firm j^* deletes its link to some firm l with $l \in N \setminus \bar{S}(g_2)$ and $l \notin \bar{S}(g)$ so that firm l becomes isolated. Indeed, looking two steps forward, firms i^* and j^* have incentives to add the link i^*j^* since $\Pi_{i^*}(g_2 + i^*j^* - j^*l) > \Pi_{i^*}(g_2)$ and $\Pi_{j^*}(g_2 + i^*j^* - j^*l) > \Pi_{j^*}(g_2)$ by **P4**, even tough $\Pi_{i^*}(g_2 + i^*j^*) < \Pi_{i^*}(g_2)$ and $\Pi_{j^*}(g_2 + i^*j^*) > \Pi_{j^*}(g_2)$. From $g_2 + i^*j^*$ to $g_2 + i^*j^* - j^*l$, firm j^* that is not a leaf has incentives (looking one step forward) to delete its link with firm l that is a leaf in $g_2 + i^*j^*$ since $\Pi_{j^*}(g_2 + i^*j^*) < \Pi_{j^*}(g_2 + i^*j^* - j^*l)$ by **P5**. Notice that $\bar{S}(g_2 + i^*j^* - j^*l) = N \setminus \{l\}$. Let $g_3 = g_2 + i^*j^* - j^*l$.

Step 4. We build a sequence of farsighted improving paths of length $K = 1$ from $g_{30} \to g_{31} \to \ldots \to g_{3\bar{m}}$ with $g_{30} = g_3$, $\bar{m} = \#(N \setminus \bar{S}(g)) - 1$, and $g_{3m} = g_{3m-1} - k_m l_m$ where $l_m \in \{i \notin \bar{S}(g) \mid i \text{ leaf in } g_{3m-1}\}$ if $\{i \notin \bar{S}(g) \mid i \text{ leaf in } g_{3m-1}\} \neq \emptyset$, $l_m \in \{i \in \bar{S}(g) \mid i \text{ leaf in } g_{3m-1}\}$ and $k_m l_m \notin g''$ if $\{i \notin \bar{S}(g) \mid i \text{ leaf in } g_{3m-1}\} = \emptyset$. That is, along this sequence we isolate at each step a firm that is a leaf giving priority to firms in $N \setminus \bar{S}(g)$. We so proceed up to the number of isolated firms is equal to $\#(N\setminus\bar{S}(g))$. If $\bar{S}(g_{3\bar{m}}) = \bar{S}(g)$ we go to Step 6. If $\bar{S}(g_{3\bar{m}}) \neq \bar{S}(g)$ we build a star network component between the isolated firms in $g_{3\bar{m}}$. From $g_{3\bar{m}}$, isolated firms have incentives by **P3** to add one by one links between them to form a star component with some firm $k^* \in \bar{S}(g)$ such that $N_{k^*}(g_{3\bar{m}}) = \emptyset$ in the center. We reach $g_4 = g_{3\bar{m}} + \{k^*l \mid l \neq k^*, N_l(g_{3\bar{m}}) = \emptyset\}$. Let g'' be such that $\{k^*l \mid l \neq k^*, N_l(g_{3\bar{m}}) = \emptyset, l \in \bar{S}(g)\} \subseteq g''$. Links added from $g_{3\bar{m}}$ to g_4 between k_* and other firms belonging $\bar{S}(g)$ are not deleted later on. We next go to Step 5.

Step 5. We build a farsighted improving path of length $K = 2$ from $g_4 = g_{3\bar{m}} + \{k^*l \mid l \neq k^*, N_l(g_{3\bar{m}}) = \emptyset\}$ where, looking two steps forward ($K = 2$), firm j^* and firm k^* first build the link j^*k^* and next firm k^* deletes its link to some firm l

with $l \in N\backslash\bar{S}(g_4)$ and $l \notin \bar{S}(g)$ so that firm l becomes isolated. Indeed, looking two steps forward, firms j^* and k^* have incentives to add the link j^*k^* since $\Pi_{j^*}(g_4+j^*k^*-k^*l) > \Pi_{k^*}(g_4)$ and $\Pi_{k^*}(g_4+j^*k^*-k^*l) > \Pi_{k^*}(g_4)$ by **P4**, even tough $\Pi_{j^*}(g_4+j^*k^*) < \Pi_{j^*}(g_4)$ and $\Pi_{k^*}(g_4+j^*k^*) > \Pi_{k^*}(g_4)$. From $g_4+j^*k^*$ to $g_4+j^*k^*-k^*l$, firm k^* that is not a leaf has incentives (looking one step forward) to delete its link with firm l that is a leaf in $g_4+j^*k^*$ since $\Pi_{k^*}(g_4+j^*k^*) < \Pi_{k^*}(g_4+j^*k^*-k^*l)$ by **P5**. Notice that $\bar{S}(g_4+j^*k^*-k^*l) = N\setminus\{l\}$. We next repeat the process from Step 4 starting with $g_5 = g_4+j^*k^*-k^*l$ instead of g_3.

Step 6. We build a sequence of farsighted improving paths of length $K=1$ from $g_{3\bar{m}}$ with $\bar{S}(g_{3\bar{m}}) = \bar{S}(g)$, where all isolated firms belong to $N\backslash\bar{S}(g)$ and by **P3** they have incentives to add one by one links between them to form a star component with some firm $\hat{k} \in N\setminus\bar{S}(g)$ in the center. We reach the end network $g'' = g_{3\bar{m}} + \{\hat{k}l \mid \hat{k}, l \in N\setminus\bar{S}(g), l \neq \hat{k}, \#N_{\hat{k}}(g'') = \#(N\setminus\bar{S}(g)) - 1, \#N_l(g'') = 1\}$. Thus, we have built a sequence of farsighted improving paths (of length at most $K=2$) from g' to g'' (i.e. $g'' \in \phi_2^\infty(g')$) such that $g'' \in G^{1/2}$ and $\bar{S}(g'') = \bar{S}(g)$.

□

Take any two pairwise stable networks such that the same firms belong to the largest component. Lemma 6 tells us that there is a sequence of farsighted improving paths of length at most 2 from each pairwise stable network to the other one.

Lemma 6. *Take any $g, g' \in G^{1/2}$ such that $g \neq g', \bar{S}(g) = \bar{S}(g')$. Then, $g \in \phi_2^\infty(g')$.*

Proof. Take any $g, g' \in G^{1/2}$ such that $g \neq g', \bar{S}(g) = \bar{S}(g')$. We now build in steps a sequence of farsighted improving paths (of length at most $K=2$) from g' to g, i.e. $g \in \phi_2^\infty(g')$.

Step 1. Let $g_0 = g'$. We build a farsighted improving path of length $K=2$ from g_0 where firms look two steps forward such that: $g_0 \to g_0+ij \to g_0+ij-kl$ with $i \in \bar{S}(g_0)$, $j,k,l \in N\setminus\bar{S}(g_0)$, $kl \in g_0$ with l being a leaf in g_0+ij. Looking two steps forward, firms i and j have incentives to add the first link ij since $\Pi_i(g_0+ij-kl) > \Pi_i(g_0)$ and $\Pi_j(g_0+ij-kl) > \Pi_j(g_0)$ by **P4**, even tough $\Pi_i(g_0+ij) < \Pi_i(g_0)$ and $\Pi_j(g_0+ij) > \Pi_j(g_0)$. From g_0+ij to $g_0+ij-kl$, firm k that is not a leaf has incentives (looking one step forward) to delete its link with the leaf l since $\Pi_k(g_0+ij) < \Pi_k(g_0+ij-kl)$ by **P5**. Notice that $\bar{S}(g_0+ij-kl) = N\setminus\{l\}$.

Step 2. We build a farsighted improving path of length $K = 2$ from $g_0 + ij - kl$ where firms look two steps forward such that: $g_0 + ij - kl \to g_0 + ij - kl + i_1j_1 \to g_0 + ij - kl + i_1j_1 - k_1l_1$ with $i_1, j_1 \in \bar{S}(g_0)$, $i_1j_1 \in g$, $i_1j_1 \notin g_0$, $l_1 \in N \setminus \bar{S}(g_0)$, $k_1l_1 \in g_0 + ij - kl + i_1j_1$ with l_1 being a leaf in $g_0 + ij - kl + i_1j_1$. Looking two steps forward, firms i_1 and j_1 have incentives to add the first link i_1j_1 since $\Pi_{i_1}(g_0+ij-kl+i_1j_1-k_1l_1) > \Pi_i(g_0+ij-kl)$ and $\Pi_{j_1}(g_0+ij-kl+i_1j_1-k_1l_1) > \Pi_j(g_0 + ij - kl)$ by **P5**, even tough $\Pi_{i_1}(g_0 + ij - kl + i_1j_1) < \Pi_{i_1}(g_0 + ij - kl)$ and $\Pi_{j_1}(g_0 + ij - kl + i_1j_1) < \Pi_{j_1}(g_0 + ij - kl)$. From $g_0 + ij - kl + i_1j_1$ to $g_0 + ij - kl + i_1j_1 - k_1l_1$, firm k_1 that is not a leaf has incentives (looking one step forward) to delete its link with the leaf l_1 since $\Pi_{k_1}(g_0 + ij - kl + i_1j_1) < \Pi_{k_1}(g_0 + ij - kl + i_1j_1 - k_1l_1)$ by **P5**.

Step 3. We build a farsighted improving path of length $K = 1$ from $g_0 + ij - kl + i_1j_1 - k_1l_1$ to $g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2$ with $i_2, j_2 \in \bar{S}(g_0)$, $i_2j_2 \notin g$, $i_2j_2 \in g_0 + ij - kl + i_1j_1 - k_1l_1$, where i_2j_2 is a superfluous link in the largest component and it does not belong to the end network g,[21] and so, $\Pi_{i_2}(g_0 + ij - kl + i_1j_1 - k_1l_1) < \Pi_{i_2}(g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2)$ and $\Pi_{j_2}(g_0 + ij - kl + i_1j_1 - k_1l_1) < \Pi_{j_2}(g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2)$.

Step 4. We repeat the process from Step 2 until **(i)** every firm belonging to $N \setminus \bar{S}(g_0)$ is now isolated, or **(ii)** each link $ij \in g$ such that $i, j \in \bar{S}(g)$ belongs to $g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2 + ... + i_{\bar{m}-1}j_{\bar{m}-1} - k_{\bar{m}-1}l_{\bar{m}-1} - i_{\bar{m}}j_{\bar{m}}$ and each link $ij \notin g$ such that $i, j \in \bar{S}(g)$ does not belong to $g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2 + ... + i_{\bar{m}-1}j_{\bar{m}-1} - k_{\bar{m}-1}l_{\bar{m}-1} - i_{\bar{m}}j_{\bar{m}}$, with $\bar{m} = \#\{ij \in g \mid ij \notin g_0 \text{ and } i, j \in \bar{S}(g)\}$. If $\#(N \setminus \bar{S}(g)) - 1 < \bar{m}$ then **(i)** is first satisfied and go to Step 5. If $\#(N \setminus \bar{S}(g)) - 1 > \bar{m}$ then **(ii)** is first satisfied and go to Step 6. If $\#(N \setminus \bar{S}(g)) - 1 = \bar{m}$ then **(i)** and **(ii)** are both satisfied and go to Step 7.

Step 5. From Step 4 we have reached the network $g_1 = g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2 + ... + i_{\bar{m}-1}j_{\bar{m}-1} - k_{\bar{m}-1}l_{\bar{m}-1} - i_{\bar{m}}j_{\bar{m}}$ where all firms in $N \setminus \bar{S}(g)$ are isolated and $\bar{S}(g) = \bar{S}(g_1)$. Notice that $\#\{ij \in g \mid ij \notin g_0 \text{ and } i, j \in \bar{S}(g)\} = \bar{m} \leq \#\bar{S}(g) - 1$ and $\#\bar{S}(g) - \#(N \setminus \bar{S}(g)) \leq 2$. Hence, $\bar{m} - \#(N \setminus \bar{S}(g)) + 1 \leq 2$. First, we build a sequence of farsighted improving paths of length $K = 1$

[21] By adding the link i_1j_1, there are now two paths connecting i_1 and j_1 in the network, and so there is a superfluous link that does not belong to the end network on the path that connects indirectly i_1 and j_1.

from g_1 to $g_1 + \{k^*l \mid l \in N \setminus \bar{S}(g), l \neq k^*\} \in G^{1/2}$, where each firm $l \neq k^*$ belonging to $N \setminus \bar{S}(g)$ adds successively links to firm $k^* \in N \setminus \bar{S}(g)$ to form a star component with k^* in the center. By **P3**, firm k^* and each firm l have incentives to add the link k^*l to the current network looking one step forward. Next, we repeat the process from Step 1 to Step 4 starting with $g_0 = g_1 + \{k^*l \mid l \in N \setminus \bar{S}(g), l \neq k^*\}$, followed by Step 6.[22]

Step 6. We build a sequence of farsighted improving paths of length $K = 1$ from $g_0 + ij - kl + i_1j_1 - k_1l_1 - i_2j_2 + ... + i_{\bar{m}-1}j_{\bar{m}-1} - k_{\bar{m}-1}l_{\bar{m}-1} - i_{\bar{m}}j_{\bar{m}}$ to $\{ij \in g \mid i, j \in \bar{S}(g)\}$, where each firm $i \in \bar{S}(g_0+ij-kl+i_1j_1-k_1l_1-i_2j_2+...+i_{\bar{m}-1}j_{\bar{m}-1}-k_{\bar{m}-1}l_{\bar{m}-1} - i_{\bar{m}}j_{\bar{m}})$ such that $i \notin \bar{S}(g)$ and i is (or becomes during the process) a leaf is isolated one by one. We reach the network $\{ij \in g \mid i, j \in \bar{S}(g)\}$ where every firm $i \notin \bar{S}(g)$ is isolated. Next, go to Step 8.

Step 7. From Step 4 we have already reached the network $\{ij \in g \mid i, j \in \bar{S}(g)\}$ where all firms belonging to $\bar{S}(g)$ are exactly as in the end network g while all firms belonging to $N \setminus \bar{S}(g)$ are isolated. Next, go to Step 8.

Step 8. Finally, firms belonging to $N \setminus \bar{S}(g)$ have incentives by **P3** to add successively links between them to form the network $g \in G^{1/2}$. Thus, we have built in steps a sequence of farsighted improving paths (of length at most $K = 2$) from g' to g, i.e. $g \in \phi_2^\infty(g')$.

□

Proof of Proposition 3. Take any $g \in G^{1/2}$. We show that a singleton set $\{g\}$ is a horizon-K farsighted set for every $K \geq 2$.

(i) We first show that each $\{g\}$ satisfies horizon-K external stability for $K \geq 2$. Notice that if $g \in \phi_1^\infty(g')$ and $g' \in \phi_2^\infty(g'')$ then $g \in \phi_2^\infty(g'')$. **(i.a)** From Lemma 1, Lemma 2, Lemma 3 and Lemma 4 it directly follows that for any $g \in \mathcal{G} \setminus G^{1/2}$ it holds that $\phi_1^\infty(g) \cap G^{1/2} \neq \emptyset$. **(i.b)** From Lemma 5 and Lemma 6 we have that, for any $g, g' \in G^{1/2}, g \neq g', g' \in \phi_2^\infty(g)$. Hence, from **(i.a)** and **(i.b)** it follows that $g \in \phi_2^\infty(g')$ for all $g' \in \mathcal{G} \setminus \{g\}$.

[22]Since the number of links in the smallest component (i.e. $\#N \setminus \bar{S}(g) - 1$ links) is greater or equal than the maximum number of links that belong to the end network and are still missing in the largest component (i.e. 2 links), we only need to repeat one time the process from Step 1 to Step 4; and so, we go directly from Step 4 to Step 6 after the first repetition.

(ii) We now show that each $\{g\}$ satisfies horizon-K deterrence of external deviations for $K \geq 2$. From Proposition 1 we have that $D_1 = G^{1/2}$ and each $g \in G^{1/2}$ defeats every adjacent network g' to g, so $g \in \phi_1(g') \subseteq \phi_2(g')$. Hence, $\{g\}$ satisfies horizon-K deterrence of external deviations for $K \geq 2$.

(iii) Since $\{g\}$ is a singleton set, it satisfies minimality.

□

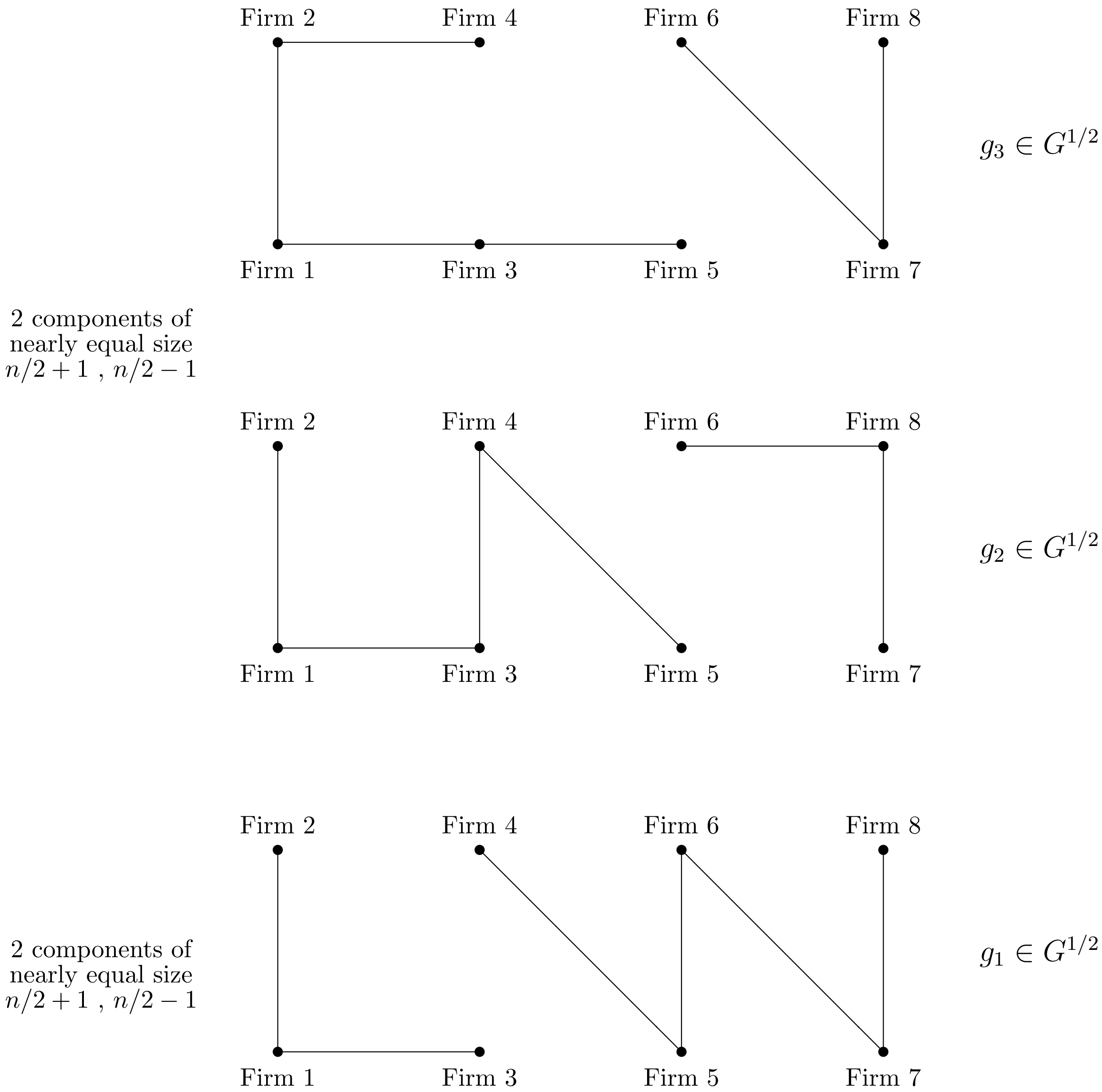

Figure 1: Some R&D networks in $G^{1/2}$ when $n = 8$.

Example 1. We illustrate the proofs of Lemma 5 and Lemma 6 in the case of eight firms ($n = 8$). In Figure 1 we depict three R&D networks, $g_1, g_2, g_3 \in G^{1/2}$ to show that $g_2 \in \phi_2^{\infty}(g_1)$ and $g_3 \in \phi_2^{\infty}(g_2)$ so that $g_3 \in \phi_2^{\infty}(g_1)$. Notice that $\bar{S}(g_2) = \bar{S}(g_3)$

and so $N \setminus \bar{S}(g_2) = N \setminus \bar{S}(g_3)$ but $g_2 \neq g_3$. Starting from $g_1 \in G^{1/2}$ we build a sequence of farsighted improving paths of length at most 2 ending in $g_2 \in G^{1/2}$. First, (looking two steps forward) $g_1 \to g_1 + 34 \to g_1 + 34 - 78 = g_{11}$, followed by (looking one step forward) $g_{11} \to g_{11} - 67 \to g_{11} - 67 - 56 = g_{12}$, followed by (looking one step forward) $g_{12} \to g_{12} + 68 \to g_{12} + 68 + 78 = g_2$; so $g_2 \in \phi_2^\infty(g_1)$. Starting from g_2 we build a sequence of farsighted improving paths of length at most 2 ending in g_3. First, (looking two steps forward) $g_2 \to g_2 + 58 \to g_2 + 58 - 68 = g_{21}$, followed by (looking two steps forward) $g_{21} \to g_{21} + 24 \to g_{21} + 24 - 78 = g_{22}$, followed by (looking one step forward) $g_{22} \to g_{22} - 34 = g_{23}$, followed by (looking two steps forward) $g_{23} \to g_{23} + 35 \to g_{23} + 35 - 58 = g_{24}$, followed by (looking one step forward) $g_{24} \to g_{24} + 67 \to g_{24} + 67 + 78 = g_3$; so $g_3 \in \phi_2^\infty(g_2)$. Thus, $g_3 \in \phi_2^\infty(g_1)$.

Next proposition shows that, meanwhile each singleton set $\{g\}$ with $g \in G^{1/2}$ is a horizon-K farsighted set for every $K \geq 2$, networks that are not pairwise stable cannot be a singleton horizon-K farsighted set. Hence, pairwise stable R&D networks seem robust to limited farsightedness.

Proposition 4. *Take any $g \notin G^{1/2}$ such that $\#\mathcal{P}(g) \leq 2$ and every $h \in C(g)$ is minimally connected. Then, $\{g\}$ is never a horizon-K farsighted set for $K \geq 2$.*

Proof. We show that for any $g \notin G^{1/2}$ such that $\#\mathcal{P}(g) \leq 2$ and every $h \in C(g)$ is minimally connected, the singleton set $\{g\}$ violates horizon-K deterrence of external deviations.

(i) Take a minimally connected network g such that $\mathcal{P}(g) = \{N\}$ and an adjacent network $g' = g - ij$ to g where j is a leaf in g. Then, all firms in $\bar{S}(g')$ are strictly better off in g' than in g, and so they will not participate to any deviation from g' towards g. Hence, $g \notin \phi_{K-1}(g')$, $K \geq 2$ and the deviation to $g' = g - ij$ is not deterred.

(ii) Take a network g such that $\mathcal{P}(g) = \{S_1(g), S_2(g)\}$ with $\#S_1(g) = \#S_2(g)$, each $h \in C(g)$ is minimally connected and an adjacent network $g' = g + ij$ to g where $i \in S_1(g)$ and $j \in S_2(g)$. Then, all firms in $S_1(g)$ and $S_2(g)$ are strictly better off in g' than in g, and so they will not participate to any deviation from g' towards g. Hence, $g \notin \phi_{K-1}(g')$, $K \geq 2$ and the deviation to $g' = g - ij$ is not deterred.

(iii) Take a network g such that $\mathcal{P}(g) = \{S_1(g), S_2(g)\}$ with $\#S_1(g) > \lfloor((n+3)/2\rfloor$ if n even and $\#S_1(g) > (n+1)/2$ if n odd, and each $h \in C(g)$ is minimally connected. Take an adjacent network $g' = g - ij$ to g where $ij \in g$, $i, j \in \bar{S}(g) = S_1(g)$, j is a leaf in g and $\mathcal{P}(g') = \{S_1(g) \setminus \{j\}, \{j\}, S_2(g)\}$. Then, all firms in $S_1(g) \setminus \{j\} = \bar{S}(g')$ and in $S_2(g)$ are strictly better off in g' than in g, and so they will not participate to any deviation from g' towards g. Hence, $g \notin \phi_{K-1}(g')$, $K \geq 2$ and the deviation to $g' = g - ij$ is not deterred. $\square$

When the degree of farsightedness is great enough ($K \geq n' + 1$), then a subset of the networks that are stable when all firms are farsighted become a horizon-K farsighted set. Indeed, Mauleon, Sempere-Monerris and Vannetelbosch (2014) show that $G^{3/4}$ is a pairwise farsightedly stable set while Herings, Mauleon and Vannetelbosch (2019) find that, for every pairwise farsightedly stable set G_∞ there exists some $G \subseteq G_\infty$ such that G is a horizon-$(n' + 1)$ farsighted set where n' is the cardinality of $\mathcal{G}$. Hence, together with Proposition 4, we obtain the following corollary.

Corollary 1. *There is $G \subseteq G^{3/4}$ such that G is a horizon-$(n' + 1)$ farsighted set and $\#G \neq 1$.*

5 Conclusion

We study the R&D networks that will emerge in the long run when firms are neither myopic nor fully farsighted but have some limited degree of farsightedness. We adopt the horizon-K farsighted set of Herings, Mauleon and Vannetelbosch (2019) and we find that a singleton set consisting of a pairwise stable network is a horizon-K farsighted set for any degree of farsightedness greater or equal than 2. That is, each R&D network consisting of two components of nearly equal size satisfies both horizon-K deterrence of external deviations and horizon-K external stability for $K \geq 2$. On the contrary, each R&D network consisting of two components with the largest one comprising three-quarters of firms, predicted when all firms are fully farsighted, violates horizon-K deterrence of external deviations. Thus, when firms are homogeneous in their degree of farsightedness, R&D networks consisting of two components of nearly equal size are robust to limited farsightedness.

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