Minimally Farsighted Unstable Networks

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Abstract

We propose the notion of minimal instability to determine the networks that are more likely to emerge in the long run when agents are farsighted. A network is minimally farsighted unstable if there is no other network which is more farsightedly stable. To formulate what it means to be more farsightedly stable, we compare networks by comparing (in the set inclusion or cardinal sense) their sets of farsighted defeating networks. We next compare networks in terms of their absorbtiveness by comparing both their sets of farsighted defeating networks (i.e. in terms of their stability) and their sets of farsighted defeated networks (i.e. in terms of their reachability). A network is maximally farsighted absorbing if there is no other network which is more farsightedly absorbing. We provide general results for characterizing minimally farsighted unstable networks and maximally farsighted absorbing networks, and we study their relationships with alternative notions of farsightedness. Finally, we use experimental data to show the relevance of the new solution concepts.

Key words: networks; stability comparisons; farsighted players.
JEL Classification: A14, C70, D20.

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1 Introduction

The organization of agents into networks plays an important role in the determination of the outcome of many social and economic interactions (e.g. R&D collaborations, bilateral free-trade agreements, manufacturer-retailer relationships). Networks often do not emerge randomly but rather through the decisions taken by the agents for forming links with whom they want. Our aim is to better understand the formation of social and economic networks when building a link between two agents requires the consent of both agents involved, while deleting a link between two agents can be done unilaterally by one of the agents involved.[1] A simple notion to analyse the networks that one might expect to emerge in the long run is the pairwise stability notion introduced by Jackson and Wolinsky (1996). A network is pairwise stable if no agent benefits from severing one of their links and no other two agents benefit from adding a link between them, with one benefiting strictly and the other at least weakly. This stability notion presumes that agents are myopic since they do not anticipate that other agents may react to their changes. However, farsighted agents may add a link that is not valuable to them given the current network, as that may in turn lead to the formation of other links and ultimately increase the payoffs of the original deviating agents.

A network farsightedly defeats another network if there is a farsighted improving path from the latter to the former network. A farsighted improving path is a sequence of networks that can emerge when agents form or delete links based on the improvement the end network offers relative to the current network. Jackson (2008) defines the notion of farsightedly pairwise stable network: a network is a farsightedly pairwise stable network if and only if it is not farsightedly defeated. But, there are many network formation games where a farsightedly pairwise stable network fails to exist. In fact, requiring a network to be immune to any profitable farsighted deviations is very demanding. Indeed, farsighted deviations are not limited to a specific horizon and so they even include myopic deviations. Hence, we propose the notion of minimal farsighted instability to predict the networks that are most likely to emerge in the long run when agents are farsighted.

A network is said to be minimally farsighted unstable if there is no other network which is more farsightedly stable. To formulate what it means to be more farsightedly stable, we consider comparing networks by comparing their sets of farsighted defeating networks.[2] A network is more farsightedly stable than another network if the set of farsighted defeating networks in the former is a proper subset of the set of farsighted defeating networks in the latter network. We also consider a corresponding cardinal version such that a network is cardinally more farsightedly stable than another network if the number of farsighted defeating networks in the former is less than the number of farsighted defeating networks

[1] Mauleon and Vannetelbosch (2016) provide a comprehensive overview of the solution concepts for solving network formation games.

[2] The set of farsighted defeating networks of a given network is the set of all networks that farsightedly defeat this network.

in the latter network. A network is said to be cardinally minimally farsighted unstable if there is no other network which is cardinally more farsightedly stable.

Clearly, if a network is more farsightedly stable than another network, then it is also cardinally more farsightedly stable. Hence, the set of cardinally minimally farsighted unstable networks refines the set of minimally farsighted unstable networks. We also provide results that are helpful for characterizing the minimally farsighted unstable networks. If there is a farsighted improving path from one network to another one, then the former cannot be more farsightedly stable than the latter network. Hence, if there is a farsighted improving path from all other networks to some network, then this network is minimally farsighted unstable. Moreover, if the set of networks that defeat a network is a proper subset of the set of networks that defeat any other network, then the former network is the unique minimally farsighted unstable network.

Our method to compare networks by their farsighted stability is inspired by methods used in recent matching studies. Doğan and Ehlers (2020) compare assignments in school choice in terms of (myopic) stability by comparing (in the set inclusion or cardinal sense) their sets of blocking pairs or comparing (in the set inclusion or cardinal sense) their sets of blocking students involved in at least one blocking pair. Recently, Doğan and Ehlers (2021) use similar methods to compare assignments in the context of priority-based allocation of objects.[3] Here, we adapt their methods for matching problems with myopic agents to network games with farsighted agents.

In addition of comparing networks in terms of their stability, one may also require to compare them in terms of their reachability. Indeed, selecting a network that is more farsightedly stable than another one might be a more robust prediction if at the same time the former is more likely to be reached than the latter one. A network is said to be maximally farsighted absorbing if there is no other network which is more farsightedly absorbing. To formulate what it means to be more farsightedly absorbing, we consider comparing networks by comparing both their sets of farsighted defeating networks (i.e. in terms of their stability) and their sets of farsighted defeated networks (i.e. in terms of their reachability).[4] A network is more farsightedly absorbing than another network if (i) the set of farsighted defeating networks in the former is a proper subset of the set of farsighted defeating networks in the latter network and (ii) the set of farsighted defeated networks in the latter is a proper subset of the set of farsighted defeated networks in the former network. We also consider a corresponding cardinal version such that a network is cardinally more farsightedly absorbing than another network if (i) the number of farsighted defeating networks in the former is less than the number of farsighted defeating networks in the latter network and (ii) the number of farsighted defeated networks in the latter is

[3]Minimally unstable assignments (in the set inclusion sense) are considered by Abdulkadiroğlu, Che, Pathak, Roth and Tercieux (2020) and Tang and Zhang (2021) for school choice problems and by Combe, Tercieux and Terrier (2020) for teacher assignment problems.

[4]The set of farsighted defeated networks of a given network is the set of all networks that this network farsightedly defeats.

less than the number of farsighted defeated networks in the former network. A network is said to be cardinally maximally farsighted absorbing if there is no other network which is cardinally more farsightedly absorbing. Finally, we consider an additive version of the cardinality version such that a network is cardinally$^+$ more farsightedly absorbing than another network if the number of farsighted defeating networks minus the number of farsighted defeated networks in the former is less than in the latter network. A network is said to be cardinally$^+$ maximally farsighted absorbing if there is no other network which is cardinally$^+$ more farsightedly absorbing.

Obviously, if a network is more farsightedly absorbing than another network, then it is also cardinally more farsightedly absorbing. Moreover, if a network is cardinally more farsightedly absorbing than another network, then it is also cardinally$^+$ more farsightedly absorbing. Hence, the set of cardinally$^+$ maximally farsighted absorbing networks refines the set of cardinally maximally farsighted absorbing networks that itself refines the set of maximally farsighted absorbing networks. Moreover, we show that, if there is a farsighted improving path from all other networks to some network, then this network is maximally farsighted absorbing. Finally, we provide a sufficient condition such that a network is both Pareto efficient, minimally farsighted unstable and maximally farsighted absorbing.

Another common approach for analysing networks that emerge in the long run when agents are farsighted are set-valued concepts like the vNM farsighted stable set (Herings, Mauleon and Vannetelbosch, 2009; Mauleon, Vannetelbosch and Vergote, 2011; Ray and Vohra, 2015), the largest consistent set (Chwe, 1994; Page, Wooders and Kamat, 2005), the pairwise farsightedly stable set (Herings, Mauleon and Vannetelbosch, 2009) or the horizon-K farsighted set (Herings, Mauleon and Vannetelbosch, 2019).[5] We study the relationships between those set-valued concepts and our newly defined concepts. If a singleton set is a vNM farsighted stable set then this single network in the set is both minimally farsighted unstable and maximally farsighted absorbing. Similarly, if a singleton set is a pairwise farsighted stable set then this single network is both minimally farsighted unstable and maximally farsighted absorbing. Moreover, if there is a unique pairwise farsightedly stable set, then this set coincides with the set of (cardinally) minimally farsighted unstable networks.

We use experimental data from Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016) to test the relevance of the new solution concepts. We find that the set of minimally farsighted unstable networks is not significantly different than the set of maximally farsighted absorbing networks. Moreover, the set of cardinally$^+$ maximally farsighted absorbing networks performs better than the vNM farsighted stable set. Finally, the set of minimally farsighted unstable networks predicts most networks that occur with positive probability while the set of cardinally minimally farsighted unstable networks is good at

[5] Alternative notions of farsightedness are suggested by Bloch and van den Nouweland (2020), Diamantoudi and Xue (2003), Dutta, Ghosal and Ray (2005), Dutta and Vohra (2017), Ray and Vohra (2019), Herings, Mauleon and Vannetelbosch (2004), Kimya (2020), Mauleon and Vannetelbosch (2004), Page and Wooders (2009), Xue (1998) among others.

selecting the network with the highest frequency.

The paper is organized as follows. In Section 2 we introduce some notation. In Section 3 we define the notion of (cardinally) minimally unstable network. In Section 4 we define the notion of (additive / cardinally) maximally absorbing network. In Section 5 we study the relationships with set-valued concepts. In Section 6 we test the relevance of the new solution concepts using experimental data. Finally, in Section 7 we conclude.

2 Networks

The set of players is denoted by $N = \{1, 2, ..., n\}$, where n is the total number of players. A network g is a list of which pairs of players are linked to each other and $ij \in g$ indicates that i and j are linked under g. The complete network on the set of players $S \subseteq N$ is denoted by g^S and is equal to the set of all subsets of S of size 2. It follows in particular that the empty network is denoted by $g^{\emptyset}$. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \text{there is } j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g. Let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbours of player i in g. A path in a network g between i and j is a sequence of players $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for all $i \in N$ and $j \in N \setminus \{i\}$, there exists a path in g connecting i and j. A non-empty network $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. The set of components of g is denoted by $H(g)$. The partition of N induced by g is denoted by $\Pi(g)$, where $S \in \Pi(g)$ if and only if either there exists $h \in H(g)$ such that $S = N(h)$ or there exists $i \notin N(g)$ such that $S = \{i\}$.[6]

A network utility function (or payoff function) is a mapping $u : \mathcal{G} \to \mathbb{R}^N$ that assigns to each network g a utility $u_i(g)$ for each player $i \in N$. A network $g \in \mathcal{G}$ is strongly efficient relative to u if it maximizes $\sum_{i \in N} u_i(g)$; i.e. if $\sum_{i \in N} u_i(g) \geq \sum_{i \in N} u_i(g')$ for all $g' \in \mathcal{G}$. A network $g \in \mathcal{G}$ Pareto dominates a network $g' \in \mathcal{G}$ relative to u if $u_i(g) \geq u_i(g')$ for all $i \in N$, with strict inequality for at least one $i \in N$. A network $g \in \mathcal{G}$ is Pareto efficient relative to u if it is not Pareto dominated, and a network $g \in \mathcal{G}$ is Pareto dominant if it Pareto dominates any other network. To determine which networks can be formed in the long run, Jackson and Wolinsky (1996) propose a myopic notion of stability: a network g is pairwise stable with respect to u if and only if (i) for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, and (ii) for all $ij \notin g$, if $u_i(g) < u_i(g + ij)$ then $u_j(g) > u_j(g + ij)$.

[6] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, # will refer to the notion of cardinality.

3 Minimally Farsighted Unstable Networks

A farsighted improving path is a sequence of networks that can emerge when farsighted players form or delete links based on the improvement the end network offers relative to the current network; see Jackson (2008) or Herings, Mauleon and Vannetelbosch (2009).[7] Since we only allow for pairwise deviations, each network in the sequence differs from the previous one in that either a new link is formed between two players or an existing link is deleted. If a link ij is deleted, then it must be that either player i or player j prefers the end network to the current network. If a link is added between player i and player j, then both player i and player j must prefer the end network to the current network.

Definition 1. A farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \dots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \dots, K-1\}$ either

(i) $g_{k+1} = g_k - ij$ for some ij such that $u_i(g_K) > u_i(g_k)$ or $u_j(g_K) > u_j(g_k)$; or

(ii) $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_K) > u_i(g_k)$ and $u_j(g_K) \geq u_j(g_k)$.

If there exists a farsighted improving path from a network g to a network g', then we write $g \to g'$. For a given network $g \in \mathcal{G}$, let $\phi(g)$ be the set of all networks that can be reached from g by a farsighted improving path. That is,

$$\phi(g) = \{g' \in \mathcal{G} \mid g \to g'\}.$$

Definition 2. A network $g' \in \mathcal{G}$ farsightedly defeats $g \in \mathcal{G}$ $(g \neq g')$ if $g' \in \phi(g)$.

Hence, $\phi(g)$ gives us the set of networks that farsightedly defeat the network $g \in \mathcal{G}$. Jackson (2008) defines the set of farsightedly pairwise stable networks as those that are immune to farsighted pairwise deviations.

Definition 3. A network $g \in \mathcal{G}$ is farsightedly pairwise stable if $\phi(g) = \emptyset$.

Let P_1 be the set of pairwise stable networks and let P_∞ be the set of farsightedly pairwise stable networks. There is no guarantee that the set P^1 is non-empty. Since $P_1 \supseteq P_\infty$, emptiness or instability is more likely to become a problem when players are farsighted.

Example 1. In Bloch and Jackson (2007) or Jackson (2008) distance-based model, if player i is connected to player j by a path of t links, then player i receives a benefit of $b(t)$ from her indirect connection with player j. It is assumed that $b(t) \geq b(t+1) > 0$ for any t. Each direct link $ij \in g$ results in a benefit $b(1)$ and a cost c to both i and j. This

[7] Jackson and Watts (2002) define the notion of improving path in the case that all players are myopic. Herings, Mauleon and Vannetelbosch (2020) and Luo, Mauleon and Vannetelbosch (2021) extend this notion to a mixed population composed of both myopic and farsighted players.

cost can be interpreted as the time a player must spend with another player in order to maintain a direct link. Player i's distance-based payoff from a network g is given by

$$u_i(g) = \sum_{j \neq i} b(t(ij)) - \#N_i(g) \cdot c,$$

where $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j), $c \geq 0$ is a cost per link, and b is a non-increasing function. The symmetric connections model ($b(t) = \delta^t$) and the truncated connections model of Jackson and Wolinsky (1996) are special cases of distance-based payoffs. In Figure 1 we have depicted the 3-player case for $b(1) = 3$, $b(2) = 1.5$, and $c = 2$. We have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_4, g_5, g_6\}$, $\phi(g_2) = \{g_4, g_5, g_6\}$, $\phi(g_3) = \{g_4, g_5, g_6\}$, $\phi(g_4) = \{g_5, g_6\}$, $\phi(g_5) = \{g_4, g_6\}$, $\phi(g_6) = \{g_4, g_5\}$ and $\phi(g_7) = \{g_4, g_5, g_6\}$. Hence, there is no farsightedly pairwise stable network, $P_\infty = \emptyset$. But, are some networks more farsightedly stable than others?

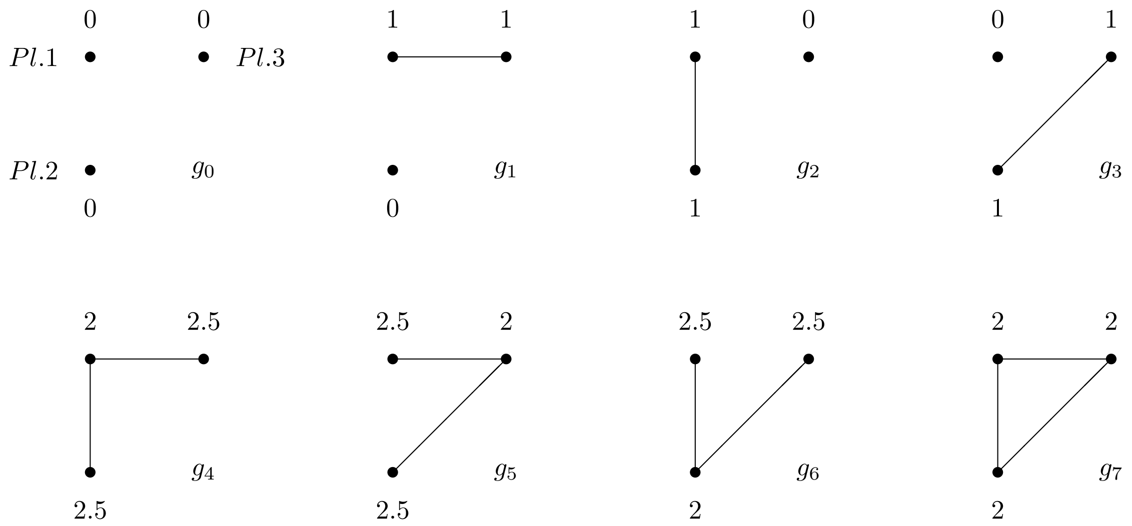

Figure 1: The distance-based model with three players.

A farsighted stability comparison is a function σ associating with each network formation game a binary relation $\gtrsim^\sigma$ over networks, where $g \gtrsim^\sigma g'$ means that g is σ-more farsightedly stable than g'. We write $g \gnsim^\sigma g'$ instead of $[g \gtrsim^\sigma g'$ and not $g' \gtrsim^\sigma g]$. We consider two primitive stability comparisons based on farsighted defeating networks. The farsighted defeating networks inclusion comparison (denoted $\sigma(\subseteq)$) is defined as follows. For each $g, g' \in \mathcal{G}$,

$$g \gtrsim^{\sigma(\subseteq)} g' \Leftrightarrow \phi(g) \subseteq \phi(g').$$

The farsighted defeating networks cardinality comparison (denoted $\sigma(\#)$) is defined as follows. For each $g, g' \in \mathcal{G}$,

$$g \gtrsim^{\sigma(\#)} g' \Leftrightarrow \#\phi(g) \leq \#\phi(g').$$

Any two networks can be compared with respect to farsighted defeating networks cardinality but not necessarily with respect to farsighted defeating networks inclusion. If $g \gtrsim^{\sigma(\subseteq)} g'$, we say that g is more farsightedly stable than g', while if $g \gtrsim^{\sigma(\#)} g'$, we say that g is #-more farsightedly stable than g' (or g is cardinally more farsightedly stable than g').[8]

We now provide the definition of a minimally farsighted unstable network as well as the definition of a cardinally minimally unstable network.

Definition 4. A network $g \in \mathcal{G}$ is minimally farsighted unstable if there is no $g' \neq g$ such that $g' \gtrsim^{\sigma(\subseteq)} g$.

Definition 5. A network $g \in \mathcal{G}$ is #-minimally farsighted unstable if there is no $g' \neq g$ such that $g' \gtrsim^{\sigma(\#)} g$.

Let F be the set of minimally farsighted unstable networks and let $F_{\#}$ be the set of #-minimally farsighted unstable networks (or cardinally minimally unstable networks). For any farsightedly pairwise stable network $g \in P_{\infty}$, there is no other network $g' \neq g$ which is more farsightedly stable (#-more farsightedly stable) than g. Notice that if g is more farsightedly stable than g' then g is also #-more farsightedly stable than g'. Hence, we do have that $F_{\#}$ is a refinement of F, i.e. $F_{\#} \subseteq F$.

Proposition 1. $F_{\#} \subseteq F$.

For the distance-based model with three players (example of Figure 1) we have that the set of minimally farsighted unstable networks is given by $F = \{g_4, g_5, g_6\}$. Indeed, g_4, g_5 and g_6 are more farsightedly stable than any $g' \in \{g_0, g_1, g_2, g_3, g_7\}$ and no additional comparison among g_4, g_5 and g_6 can be made with respect to farsighted defeating networks inclusion comparison. With respect to the farsighted defeating networks cardinality comparison, we obtain the following ranking: $\#\phi(g_4) = \#\phi(g_5) = \#\phi(g_6) = 2$, $\#\phi(g_1) = \#\phi(g_2) = \#\phi(g_3) = \#\phi(g_7) = 3$, and $\#\phi(g_0) = 6$. Hence, the set of #-minimally farsighted unstable networks leads to the same prediction, $F_{\#} = \{g_4, g_5, g_6\}$.

We now give an example where the inclusion comparison and the cardinality comparison lead to two different sets.

Example 2. Consider the situation where three players can form links and where the payoffs are given in Figure 2. We have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_2, g_4, g_5, g_6\}$, $\phi(g_2) = \{g_3, g_4, g_5, g_6\}$, $\phi(g_3) = \{g_1, g_4, g_5, g_6\}$, $\phi(g_4) = \{g_2, g_5, g_6\}$, $\phi(g_5) = \{g_1, g_4, g_6\}$, $\phi(g_6) = \{g_3, g_4, g_5\}$ and $\phi(g_7) = \{g_5, g_6\}$. Hence, we obtain that $F = \{g_5, g_6, g_7\}$ while $F_{\#} = \{g_7\}$.

We now provide some results that are helpful for characterizing the minimally farsighted unstable networks. If there is a farsighted improving path from one network to another one, then the former cannot be more farsightedly stable than the latter network.

[8]Notice that $\gtrsim^{\sigma(\#)}$ is complete and transitive while $\gtrsim^{\sigma(\subseteq)}$ is transitive but not complete.

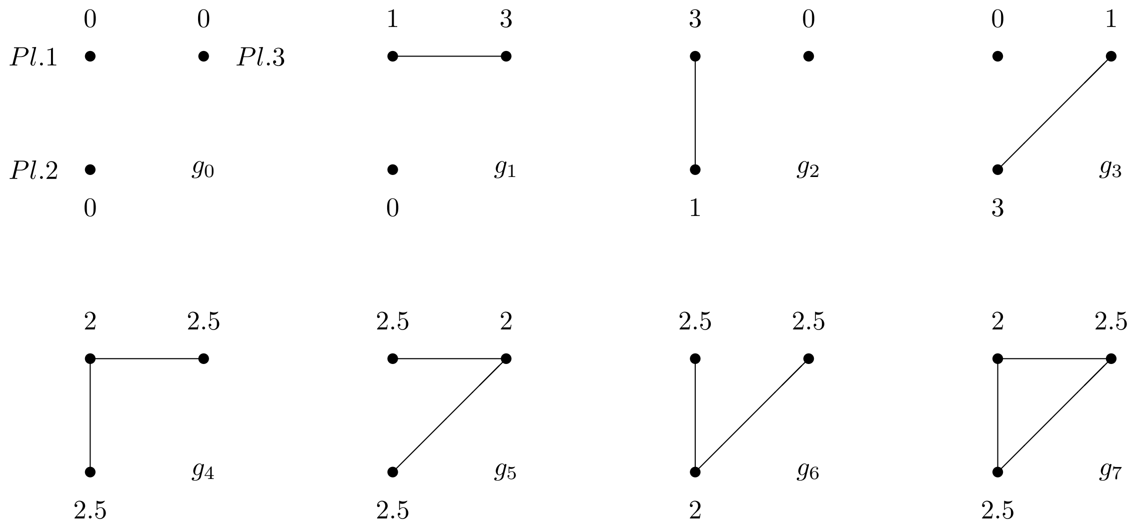

Figure 2: An example with three players where $F_{\#} \subsetneq F$.

Lemma 1. *If $g' \in \phi(g)$ then g cannot be more farsightedly stable than g'.*

Proof. Take g' such that $g' \in \phi(g)$. Since $g' \notin \phi(g')$, we cannot have that $\phi(g) \subseteq \phi(g')$. □

Proposition 2. *If for every $g' \in \mathcal{G} \setminus \{g\}$, either $\phi(g) \cap \phi(g') \neq \phi(g')$ or $\phi(g) = \phi(g')$ holds, then g is a minimally farsighted unstable network.*

Proof. Take $g, g' \in \mathcal{G}$, $g' \neq g$. If g is such that $\phi(g) = \phi(g')$ then g' cannot be more farsightedly stable than g. If g is such that $\phi(g) \cap \phi(g') \neq \phi(g')$, then there is $g'' \in \phi(g')$ such that $g'' \notin \phi(g)$, and so g' cannot be more farsightedly stable than g', $\phi(g') \nsubseteq \phi(g)$. □

Corollary 1 tells us that, if there is a farsighted improving path from all other networks to some network, then this network is minimally farsighted unstable.

Corollary 1. *If $g \in \phi(g')$ for all $g' \in \mathcal{G} \setminus \{g\}$, then g is a minimally farsighted unstable network.*

Proposition 3. *If $\phi(g) \subsetneq \phi(g')$ for all $g' \in \mathcal{G}$, $g' \neq g$, then g is the unique minimally farsighted unstable network.*

Proof. Take $g, g' \in \mathcal{G}$, $g' \neq g$. If $\phi(g) \subsetneq \phi(g')$, then we have that g is more farsightedly stable than g': $g \gtrnsim^{\sigma(\subseteq)} g'$. □

4 Maximally Farsighted Absorbing Networks

In addition of comparing networks in terms of their stability, one may also require to compare them in terms of their reachability. For a given network $g \in \mathcal{G}$, let $\phi^{-1}(g)$ be the set of all networks from which there is a farsighted improving path going to g. That is,

$$\phi^{-1}(g) = \{g' \in \mathcal{G} \mid g' \to g\}.$$

Definition 6. A network $g' \in \mathcal{G}$ is farsightedly defeated by $g \in \mathcal{G}$ $(g \neq g')$ if $g' \in \phi^{-1}(g)$.

Hence, $\phi^{-1}(g)$ gives us the set of networks that are farsightedly defeated by the network $g \in \mathcal{G}$.

A farsighted absorbtiveness comparison is a function α associating with each network formation game a binary relation $\gtrsim^{\alpha}$ over networks, where $g \gnsim^{\alpha} g'$ means that g is α-more farsightedly absorbing than g'. We consider three primitive absorbtiveness comparisons based on both farsighted defeating networks and farsighted defeated networks.

- The farsighted defeating/defeated networks inclusion comparison (denoted $\alpha(\subseteq)$) is defined as follows. For each $g, g' \in \mathcal{G}$,

  $$g \gtrsim^{\alpha(\subseteq)} g' \Leftrightarrow \phi(g) \subseteq \phi(g') \text{ and } \phi^{-1}(g') \subseteq \phi^{-1}(g).$$

  That is, g is more farsightedly absorbing than g', or $g \gnsim^{\alpha(\subseteq)} g'$, if and only if $\phi(g) \subseteq \phi(g')$ and $\phi^{-1}(g') \subseteq \phi^{-1}(g)$, with one inclusion holding strictly.

- The farsighted defeating/defeated networks cardinality comparison (denoted $\alpha(\#)$) is defined as follows. For each $g, g' \in \mathcal{G}$,

  $$g \gtrsim^{\alpha(\#)} g' \Leftrightarrow \#\phi(g) \leq \#\phi(g') \text{ and } \#\phi^{-1}(g') \leq \#\phi^{-1}(g).$$

  That is, g is #-more farsightedly absorbing than g', or $g \gnsim^{\alpha(\#)} g'$, if and only if $\#\phi(g) \leq \#\phi(g')$ and $\#\phi^{-1}(g') \leq \#\phi^{-1}(g)$, with one inequality holding strictly.

- The farsighted defeating/defeated networks additivity comparison (denoted $\alpha(+)$) is defined as follows. For each $g, g' \in \mathcal{G}$,

  $$g \gtrsim^{\alpha(+)} g' \Leftrightarrow \#\phi(g) - \#\phi^{-1}(g) \leq \#\phi(g') - \#\phi^{-1}(g').$$

  That is, g is +-more farsightedly absorbing than g', or $g \gnsim^{\alpha(+)} g'$, if and only if $\#\phi(g) - \#\phi^{-1}(g) < \#\phi(g') - \#\phi^{-1}(g')$.

Any two networks can be compared with respect to farsighted defeating/defeated networks additivity but not necessarily with respect to farsighted defeating/defeated networks inclusion (or cardinality).[9] A network is said to be maximally farsighted absorbing if there is no other network which is more farsightedly absorbing. Similarly, a network is said to be cardinally (cardinally^{+}) maximally farsighted absorbing if there is no other network which is cardinally (cardinally^{+}) more farsightedly absorbing.

Definition 7. A network $g \in \mathcal{G}$ is maximally farsighted absorbing if there is no $g' \neq g$ such that $g' \gnsim^{\alpha(\subseteq)} g$.

Definition 8. A network $g \in \mathcal{G}$ is #-maximally farsighted absorbing if there is no $g' \neq g$ such that $g' \gnsim^{\alpha(\#)} g$.

[9]Notice that $\gtrsim^{\alpha(+)}$ is complete and transitive while $\gtrsim^{\alpha(\subseteq)}$ and $\gtrsim^{\alpha(\#)}$ are transitive but not complete.

Definition 9. A network $g \in \mathcal{G}$ is +-maximally farsighted absorbing if there is no $g' \neq g$ such that $g' \gtrsim^{\alpha(+)} g$.

Let A be the set of maximally farsighted absorbing networks, let $A_{\#}$ be the set of #-maximally farsighted absorbing networks (or cardinally maximally absorbing networks), and let A_+ be the set of +-maximally farsighted absorbing networks (or cardinally$^+$ maximally absorbing networks). Obviously, we have that $A_{\#}$ is a refinement of A (i.e. $A_{\#} \subseteq A$) and A_+ is a refinement of $A_{\#}$ (i.e. $A_+ \subseteq A_{\#}$).

Proposition 4. *$A_+ \subseteq A_{\#} \subseteq A$.*

The next proposition shows that, if there is a farsighted improving path from all other networks to some network, then this network is a maximally farsighted absorbing network.

Proposition 5. *If $g \in \phi(g')$ for all $g' \in \mathcal{G} \setminus g$, then g is maximally farsighted absorbing.*

Proof. From Corollary 1 we know that g is minimally unstable, and so there is no $g' \neq g$ that is more farsightedly stable than g (i.e. no g' such that $\phi(g') \subsetneq \phi(g)$). In addition, $\phi^{-1}(g) = \mathcal{G} \setminus \{g\}$ and so there is no g' such that $\phi^{-1}(g) \subsetneq \phi^{-1}(g')$. □

Example 1 (Continued). We reconsider the distance-based model with three players of Figure 1. Remember that $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_4, g_5, g_6\}$, $\phi(g_2) = \{g_4, g_5, g_6\}$, $\phi(g_3) = \{g_4, g_5, g_6\}$, $\phi(g_4) = \{g_5, g_6\}$, $\phi(g_5) = \{g_4, g_6\}$, $\phi(g_6) = \{g_4, g_5\}$ and $\phi(g_7) = \{g_4, g_5, g_6\}$. In addition, we have that $\phi^{-1}(g_0) = \emptyset$, $\phi^{-1}(g_1) = \{g_0\}$, $\phi^{-1}(g_2) = \{g_0\}$, $\phi^{-1}(g_3) = \{g_0\}$, $\phi^{-1}(g_4) = \{g_0, g_1, g_2, g_3, g_5, g_6, g_7\}$, $\phi^{-1}(g_5) = \{g_0, g_1, g_2, g_3, g_4, g_6, g_7\}$, $\phi^{-1}(g_6) = \{g_0, g_1, g_2, g_3, g_4, g_5, g_7\}$ and $\phi^{-1}(g_7) = \emptyset$. Hence, we obtain that $A = A_{\#} = A_+ = \{g_4, g_5, g_6\}$, and so the minimally farsighted unstable networks are also the maximally farsighted absorbing ones.

Example 2 (Continued). We reconsider the example with three players of Figure 2. Remember that $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_2, g_4, g_5, g_6\}$, $\phi(g_2) = \{g_3, g_4, g_5, g_6\}$, $\phi(g_3) = \{g_1, g_4, g_5, g_6\}$, $\phi(g_4) = \{g_2, g_5, g_6\}$, $\phi(g_5) = \{g_1, g_4, g_6\}$, $\phi(g_6) = \{g_3, g_4, g_5\}$ and $\phi(g_7) = \{g_5, g_6\}$. In addition, we have that $\phi^{-1}(g_0) = \emptyset$, $\phi^{-1}(g_1) = \{g_0, g_3, g_5\}$, $\phi^{-1}(g_2) = \{g_0, g_1, g_4\}$, $\phi^{-1}(g_3) = \{g_0, g_2, g_6\}$, $\phi^{-1}(g_4) = \{g_0, g_1, g_2, g_3, g_5, g_6\}$, $\phi^{-1}(g_5) = \{g_0, g_1, g_2, g_3, g_4, g_6, g_7\}$, $\phi^{-1}(g_6) = \{g_0, g_1, g_2, g_3, g_4, g_5, g_7\}$ and $\phi^{-1}(g_7) = \emptyset$. Hence, we get that $A = \{g_4, g_5, g_6, g_7\}$, $A_{\#} = \{g_5, g_6, g_7\}$ and $A_+ = \{g_5, g_6\}$, while $F = \{g_5, g_6, g_7\}$ and $F_{\#} = \{g_7\}$.

Example 3. In Jackson and Wolinsky (1996) coauthor model, each player is a researcher who spends time writing papers. If two players are connected, then they are working on a paper together. The amount of time researcher i spends on a given project is inversely related to the number of projects, $\#N_i(g)$, that she is involved in. Formally, player i's payoff is given by

$$u_i(g) = \sum_{j:ij\in g} \left(\frac{1}{\#N_i(g)} + \frac{1}{\#N_j(g)} + \frac{1}{\#N_i(g)\#N_j(g)} \right)$$

for $\#N_i(g) > 0$. For $\#N_i(g) = 0$ we assume that $u_i(g) = 0$. In Figure 3 we have depicted the 3-player case. We have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_4, g_5\}$, $\phi(g_2) = \{g_4, g_6\}$, $\phi(g_3) = \{g_5, g_6\}$, $\phi(g_4) = \phi(g_5) = \phi(g_6) = \{g_7\}$ and $\phi(g_7) = \emptyset$. Hence, $F = F_{\#} = \{g_7\}$. In addition, we have $\phi^{-1}(g_0) = \emptyset$, $\phi^{-1}(g_1) = \{g_0\}$, $\phi^{-1}(g_2) = \{g_0\}$, $\phi^{-1}(g_3) = \{g_0\}$, $\phi^{-1}(g_4) = \{g_0, g_1, g_2\}$, $\phi^{-1}(g_5) = \{g_0, g_1, g_3\}$, $\phi^{-1}(g_6) = \{g_0, g_2, g_3\}$ and $\phi^{-1}(g_7) = \{g_4, g_5, g_6\}$. It follows that $A = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$ while $A_{\#} = A_{+} = \{g_7\}$.

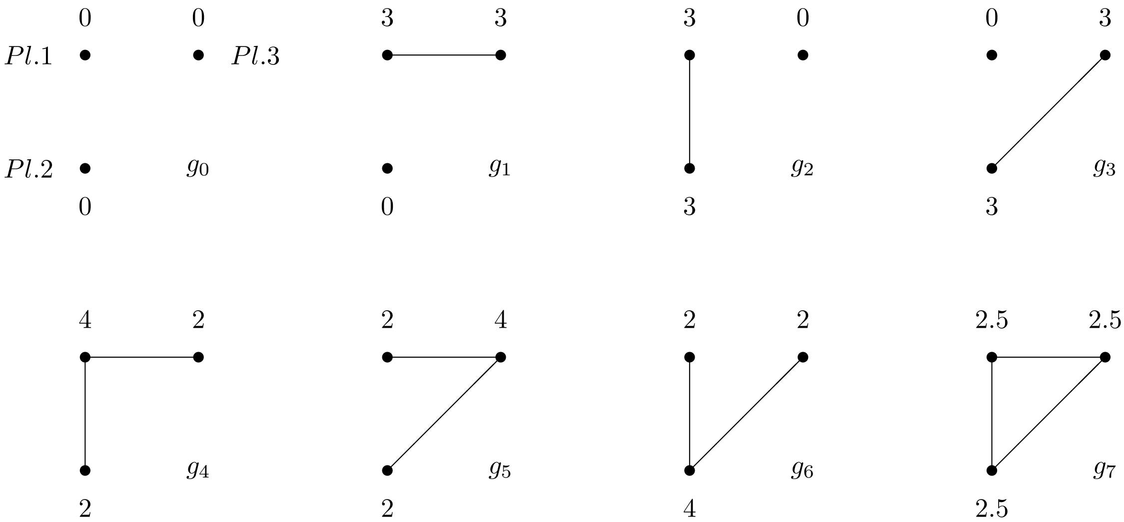

Figure 3: The co-author model with three players.

Example 4. Consider the situation where three players can form links and where the payoffs are given in Figure 4. Wit respect to the defeating networks, we have $\phi(g_0) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$, $\phi(g_1) = \{g_2, g_3\}$, $\phi(g_2) = \{g_3, g_4, g_5\}$, $\phi(g_3) = \{g_4, g_5\}$, $\phi(g_4) = \{g_1, g_5, g_6\}$, $\phi(g_5) = \{g_1, g_6\}$, $\phi(g_6) = \{g_1, g_2, g_3\}$ and $\phi(g_7) = \{g_1, g_2, g_3, g_4, g_5, g_6\}$. With respect to the defeating networks, we have $\phi^{-1}(g_0) = \emptyset$, $\phi^{-1}(g_1) = \{g_0, g_4, g_5, g_6, g_7\}$, $\phi^{-1}(g_2) = \{g_0, g_1, g_6, g_7\}$, $\phi^{-1}(g_3) = \{g_0, g_1, g_2, g_6, g_7\}$, $\phi^{-1}(g_4) = \{g_0, g_2, g_3, g_7\}$, $\phi^{-1}(g_5) = \{g_0, g_2, g_3, g_4, g_7\}$, $\phi^{-1}(g_6) = \{g_0, g_4, g_5, g_7\}$ and $\phi^{-1}(g_7) = \emptyset$. Herings, Mauleon and Vannetelbosch (2009) show that there is no vNM farsighted stable set (a set-valued concept defined in Section 5). However, $F = F_{\#} = \{g_1, g_3, g_5\}$ and $A = A_{\#} = A_{+} = \{g_1, g_3, g_5\}$.

In Table 1 we give the minimally farsighted unstable networks and the maximally farsighted absorbing networks found in Examples 1-4.

Proposition 6. *If $g \in \phi(g')$ for all $g' \in \mathcal{G} \setminus g$, then g is Pareto efficient, minimally farsighted unstable and maximally farsighted absorbing.*

Proof. If $g \in \phi(g')$ for all $g' \in \mathcal{G} \setminus g$, we know from Corollary 1 and Proposition 5 that g is both minimally farsighted unstable and maximally farsighted absorbing. Given the definition of a farsighted improving path, if $g \in \phi(g')$ then there is some player i such that $u_i(g) > u_i(g')$. Hence, g' does not Pareto dominates g. Since $g \in \phi(g')$ for all $g' \in \mathcal{G} \setminus g$, g is Pareto efficient. $\square$

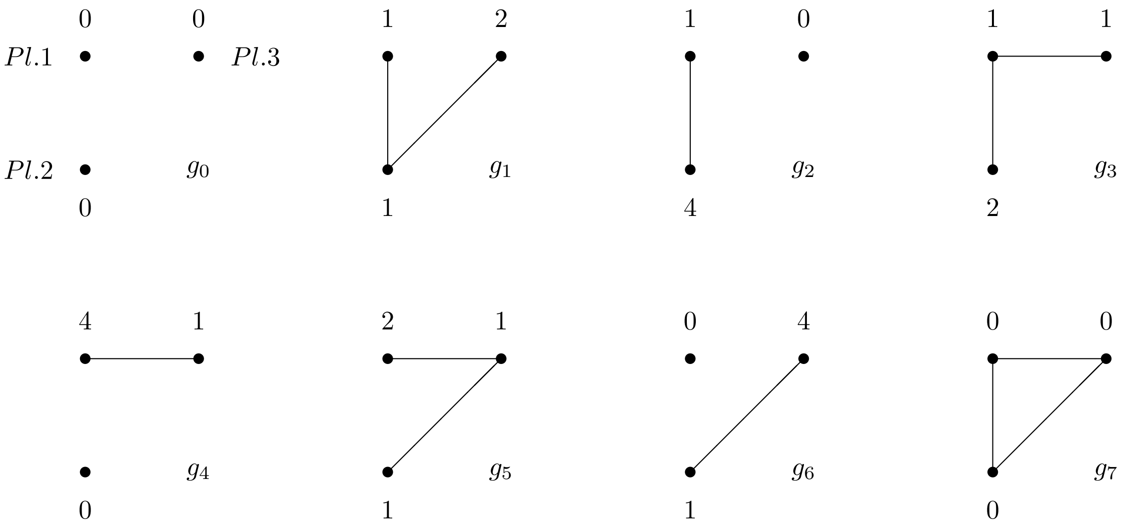

Figure 4: Non-existence of a vNM farsightedly stable set.

Concept	Example 1	Example 2	Example 3	Example 4
F	g_4, g_5, g_6	g_5, g_6, g_7	g_7	g_1, g_3, g_5
$F_\#$	g_4, g_5, g_6	g_7	g_7	g_1, g_3, g_5
A	g_4, g_5, g_6	g_4, g_5, g_6, g_7	$g_1, g_2, g_3, g_4, g_5, g_6, g_7$	g_1, g_3, g_5
$A_\#$	g_4, g_5, g_6	g_5, g_6, g_7	g_7	g_1, g_3, g_5
A_+	g_4, g_5, g_6	g_5, g_6	g_7	g_1, g_3, g_5

Table 1: Minimally farsighted unstable networks and maximally farsighted absorbing networks.

5 Relationship with Set Concepts

A set of networks G is a vNM farsighted stable set (FSS) if the following two conditions hold. Internal stability (**IS**): for any two networks g and g' in the farsighted stable set G there is no farsighted improving path from g to g' (and vice versa). External stability (**ES**): from any network outside the set G there is a farsighted improving path to some network within the set G.[10] vNM farsighted stable sets do not always exist.

Definition 10. A set of networks $G \subseteq \mathcal{G}$ is a vNM farsighted stable set if: **(IS)** for every $g, g' \in G$, it holds that $g' \notin \phi(g)$; and **(ES)** for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$.

Proposition 7. *If $\{g\}$ is a vNM farsighted stable set then g is minimally farsighted unstable ($g \in F$) and maximally farsighted absorbing ($g \in A$).*

Proof. If $\{g\}$ is a vNM farsighted stable set then $\{g\}$ satisfies **ES** and so, for every $g' \in \mathcal{G}$,

[10] Herings, Mauleon and Vannetelbosch (2020) and Luo, Mauleon and Vannetelbosch (2021) define the notion of myopic-farsighted stable set that extends the notion of vNM farsighted stable set to a mixed population of myopic and farsighted players.

$g' \neq g$, $\phi(g') \cap \{g\} \neq \emptyset$. Hence, from Corollary 1 and Proposition 5 we have that $g \in F$ and $g \in A$. $\square$

Remark that if $\{g\}$ is a vNM farsighted stable set and there is no $g' \neq g$ such that $\{g'\}$ is a vNM farsighted stable set, then g is also a #-maximally farsighted absorbing network ($g \in A_{\#}$).

Herings, Mauleon and Vannetelbosch (2009) introduce the pairwise farsightedly stable set (PFSS). It is obtained by requiring the deterrence of external deviations (**DED**), external stability (**ES**), and minimality (**MIN**). A set of networks G is pairwise farsightedly stable if (**DED**) all deviations from any network $g \in G$ to a network outside G are deterred by a credible threat of ending worse off or equally well off, (**ES**) there exists a farsighted improving path from any network outside the set leading to some network in the set, and (**MIN**) there is no proper subset of G satisfying (**DED**) and (**ES**).

Definition 11. A set of networks $G \subseteq \mathcal{G}$ is pairwise farsightedly stable if: (**DED**) $\forall ij \notin g$ such that $g + ij \notin G$, $\exists\, g' \in \phi(g + ij) \cap G$ such that $(u_i(g'), u_j(g')) = (u_i(g), u_j(g))$ or $u_i(g') < u_i(g)$ or $u_j(g') < u_j(g)$, and $\forall ij \in g$ such that $g - ij \notin G$, $\exists\, g', g'' \in \phi(g - ij) \cap G$ such that $u_i(g') \leq u_i(g)$ and $u_j(g'') \leq u_j(g)$; (**ES**) $\forall g' \in \mathcal{G} \setminus G$, $\phi(g') \cap G \neq \emptyset$; (**MIN**) $\nexists$ $G' \subsetneq G$ such that G' satisfies (**DED**) and (**ES**).

Condition (**DED**) captures that adding a link ij to a network $g \in G$ that leads to a network outside of G, is deterred by the threat of ending in g'. Here g' is such that there is a farsighted improving path from $g + ij$ to g'. Moreover, g' belongs to G, which makes g' a credible threat. There is a similar requirement, but then for the case where a link is severed. Since the set $\mathcal{G}$ (trivially) satisfies (**DED**) and (**ES**), a minimality condition (**MIN**) is required.

Proposition 8. *If $\{g\}$ is a pairwise farsightedly stable set then g is minimally farsighted unstable ($g \in F$) and maximally farsighted absorbing ($g \in A$).*

The proof of Proposition 8 is similar to the one of Proposition 7. Again, if $\{g\}$ is a pairwise farsightedly stable set and there is no $g' \neq g$ such that $\{g'\}$ is a pairwise farsightedly stable, then g is also a #-maximally farsighted absorbing network ($g \in A_{\#}$).

Herings, Mauleon and Vannetelbosch (2009) show that a set $G \subseteq \mathcal{G}$ is the unique pairwise farsightedly stable set if and only if $G = \{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ and for any $g' \in \mathcal{G} \setminus G$, $\phi(g') \cap G \neq \emptyset$.

Proposition 9. *If $G \subseteq \mathcal{G}$ is the unique pairwise farsightedly stable set, then $F = F_{\#} = G$.*

Proof. Take G such that $G = \{g \in \mathcal{G} \mid \phi(g) = \emptyset\}$ and for any $g' \in \mathcal{G} \setminus G$, $\phi(g') \cap G \neq \emptyset$. It follows that $\phi(g) \subsetneq \phi(g'))$ for all g, g' such that $g \in G, g' \notin G$. Hence, $F = F_{\#} = G$. $\square$

Chwe (1994) introduce the notion of largest consistent set for general social environments. By considering a network as a social environment, and by allowing only pairwise deviations, we obtain the definition of the largest pairwise consistent set (Page, Wooders and Kamat, 2005; Herings, Mauleon and Vannetelbosch, 2009). The pairwise largest consistent set (LPCS) contains the vNM farsightedly stable set.

Definition 12. A set of networks $G \subseteq \mathcal{G}$ is a pairwise consistent set if $\forall\ g \in G$: (**DD**) $\forall\ ij \notin g$, $\exists\, g' \in G$, where $g' = g + ij$ or $g' \in \phi(g + ij) \cap G$, such that $u_i(g') < u_i(g)$ or $u_j(g') < u_j(g)$ or $(u_i(g'), u_j(g')) = (u_i(g), u_j(g))$; $\forall\ ij \in g$, $\exists\, g', g'' \in G$, where $g' = g - ij$ or $g' \in \phi(g - ij) \cap G$, and $g'' = g - ij$ or $g'' \in \phi(g - ij) \cap G$, such that $u_i(g') \leq u_i(g)$ and $u_j(g'') \leq u_j(g)$. The largest pairwise consistent set is the pairwise consistent set that contains any pairwise consistent set.

The set G is a pairwise consistent set if both external and internal deviations are deterred (i.e. condition (**DD**)). The largest pairwise consistent set is the set that contains any pairwise consistent set. It follows from the results in Chwe (1994) that the largest pairwise consistent set exists, is non-empty, and satisfies (**ES**).

Concept	**Example 1**	**Example 2**	**Example 3**	**Example 4**
FSS	$\{g_4\}, \{g_5\}, \{g_6\}$	$\{g_5\}, \{g_6\}, \{g_4, g_7\}$	$\{g_1, g_2, g_3, g_7\}$	None
PFSS	$\{g_4\}, \{g_5\}, \{g_6\}$	$\{g_5\}, \{g_6\}$, $\{g_4, g_7\}$, $\{g_1, g_2, g_3, g_7\}$	$\{g_4, g_5, g_7\}, \{g_4, g_6, g_7\}$, $\{g_5, g_6, g_7\}$, $\{g_1, g_2, g_3, g_7\}$, $\{g_3, g_4, g_7\}, \{g_1, g_6, g_7\}$, $\{g_2, g_5, g_7\}$	$\{g_1, g_2, g_3\}$, $\{g_3, g_4, g_5\}$, $\{g_1, g_5, g_6\}$
LPCS	$\{g_4, g_5, g_6\}$	$\{g_1, g_2, g_3, g_4, g_5, g_6, , g_7\}$	$\{g_1, g_2, g_3, g_7\}$	$\{g_1, g_2, g_3, g_4, g_5, g_6\}$

Table 2: The vNM farsightedly stable sets (FSS), the pairwise farsightedly stable sets (PFSS), and the largest pairwise consistent set (LPCS).

In Table 2 we give the vNM farsightedly stable sets (FSS), the pairwise farsightedly stable sets (PFSS) and the largest pairwise consistent set (LPCS) found in the four examples. We observe that FSS, PFSS and LPCS often support more networks than F, $F_{\#}$, A, $A_{\#}$ or A_{+}.

6 Experimental Data

Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016) test whether players are either myopic or farsighted when forming links between them. We now use their experimental data to test the relevance of the new solution concepts. In Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016), participants are grouped by four to play a sequential network formation game. Players are initially unconnected (i.e. they start from the empty network). At each stage, a potential link between two players is randomly selected. If the link exists in the current network, then both players choose simultaneously whether to keep the link or to delete the link. If one of the players decides to delete the link, then the link is cut. If the link is not yet formed in the current network, then both players choose simultaneously whether or not to add this link. Only if both players decide to add the link, then the link is formed. The four players are informed about the linking choices of the players involved in the selected link. All players know perfectly the resulting network and its payoffs. Before moving to the next stage where another potential link is randomly selected, players declare if they want to do further changes to the network (satisfaction choice). The game ends either when all players do not want to modify the current network or when it is randomly stopped after stage 26.[11] The game is repeated three times and groups are kept the same through repetitions. A vector of payoffs that allocates a number of points to each player is associated to every network. Players receive points depending only on the final network of each repetition. Participants are informed about the payoffs associated to every possible network.

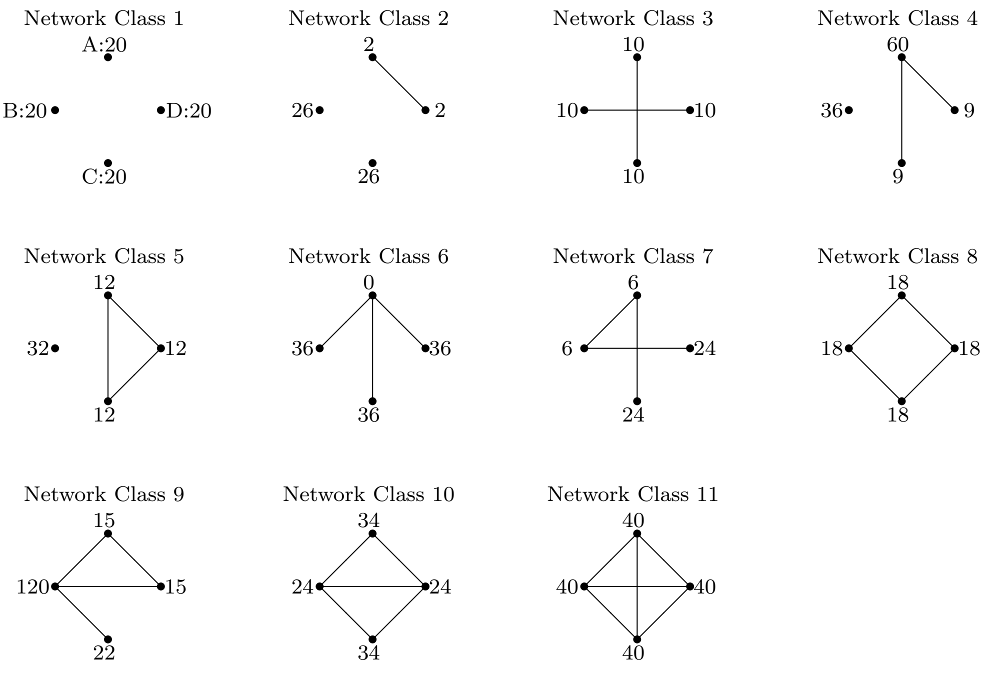

Figure 5: Payoffs in treatment T1.

Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016) run three treatments (T1, T2, T3) where the payoffs are modified in some networks to get vNM farsighted stable sets with different properties. Figure 5, Figure 6 and Figure 7 display, respectively, the payoffs

[11]The design of the termination rule allows each individual to decide unilaterally to continue playing (at least for the first 26 stages).

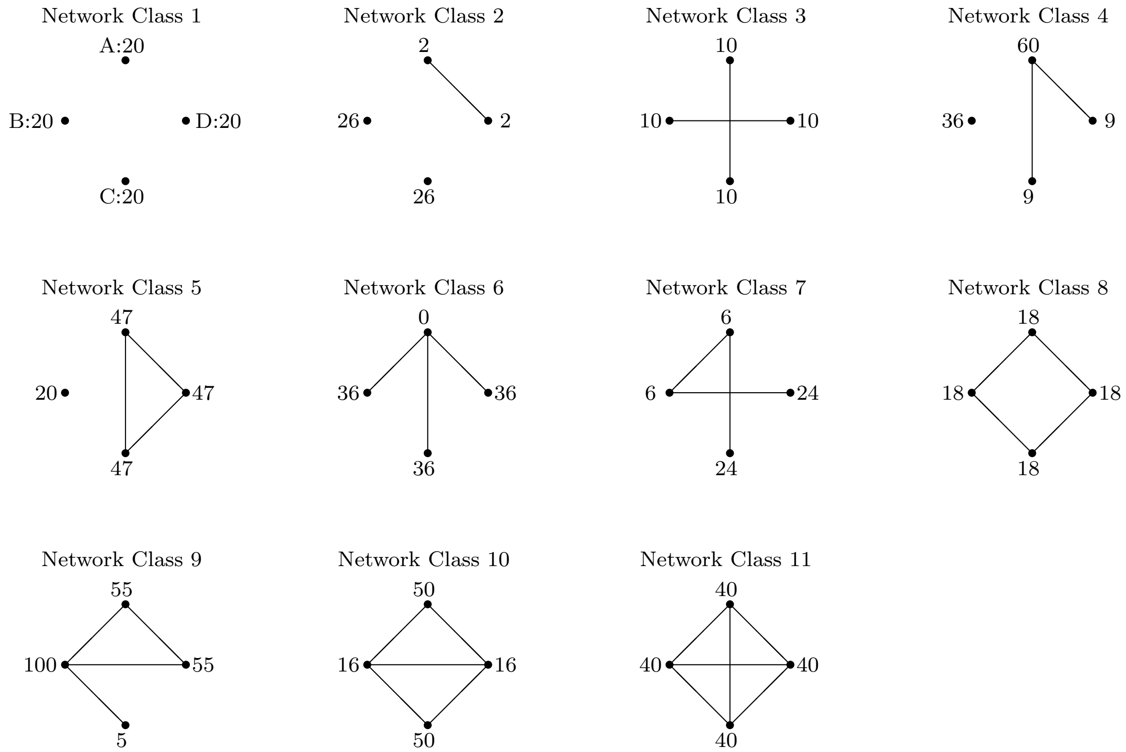

Figure 6: Payoffs in treatment T2.

that were used in the three treatments for each class of networks C_k $(k = 1, ..., 11)$. Each class C_k regroups networks that share the same architecture but differ in the position occupied by each player. To identify F, $F_{\#}$, A, $A_{\#}$ and A_+ we need to compute $\phi(g)$ and $\phi^{-1}(g)$ for every g. The next proposition gives us the characterization of those concepts for the three treatments. The proof can be found in the appendix.

Proposition 10. *Consider a set of four players. Then,*

(*i*) *In T1 we have that* $F = F_{\#} = A = A_{\#} = A_+ = \{g^N\}$,

(*ii*) *In T2 we have that* $F_{\#} = \{g^{\emptyset}\}$, $F = A = A_{\#} = \{g^{\emptyset}\} \cup \{g \mid g \in C_5\} \cup \{g \mid g \in C_9\}$ *and* $A_+ = \{g^{\emptyset}\} \cup \{g \mid g \in C_5\}$.

(*iii*) *In T3 we have that* $F_{\#} = \{g^{\emptyset}\}$, $F = A_{\#} = \{g^{\emptyset}\} \cup \{g \mid g \in C_7\}$, $A = \{g^{\emptyset}\} \cup \{g \mid g \in C_7\} \cup \{g \mid g \in C_{10}\}$ *and* $A_+ = \{g \mid g \in C_7\}$.

In T1, since $\phi(g^N) = \emptyset$ and $g^N \in \phi(g)$ for all $g \neq g^N$ (so $\phi^{-1}(g^N) = \{g \mid g \neq g^N\}$) it follows directly that all concepts single out the complete network g^N. Hence, data from T1 cannot be used to discriminate between these concepts. In T2 and T3, the empty

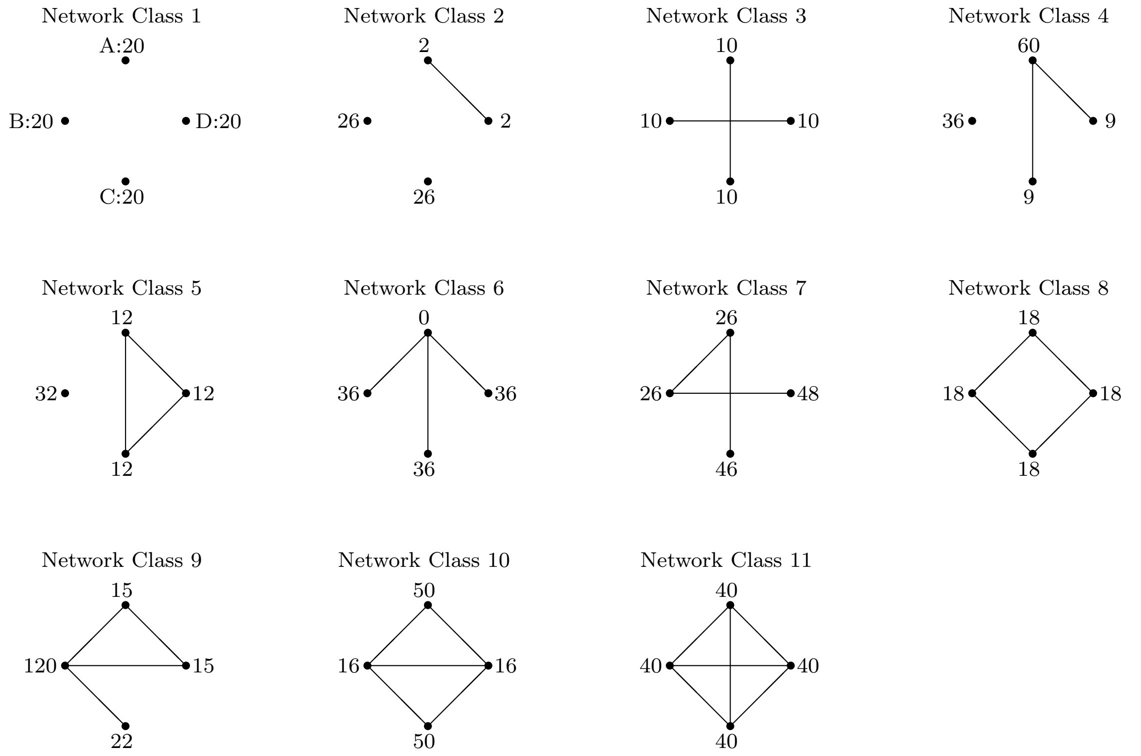

Figure 7: Payoffs in treatment T3.

network is the unique #-minimally farsighted unstable network. The set of minimally farsighted unstable networks and the set of #-maximally farsighted absorbing networks coincide in both T2 and T3 (i.e. $F = A_{\#}$ in T2, T3). Hence, data from the experiment cannot be used to discriminate between F and $A_{\#}$. In both T2 and T3 we get $F_{\#} \subsetneq F$ and $A_{+} \subsetneq A_{\#}$. Finally, we have $A_{\#} = A$ in T2 but $A_{\#} \subsetneq A$ in T3. Predictions are summarized in Table 3 included the vNM farsighted stable sets.[12]

In general, the relationships between the concepts are as follows: $F_{\#} \subseteq F$ (Proposition 1) and $A_{+} \subseteq A_{\#} \subseteq A$ (Proposition 4). From T2 and T3, we have $F_{\#} \subsetneq F = A_{\#} \subseteq A$ and $A_{+} \subsetneq F = A_{\#} \subseteq A$. Hence, we will test the following three hypothesis:

H1: $F_{\#} \subsetneq F$, H2: $A_{+} \subsetneq A_{\#}$, and H3: $A_{\#} \subsetneq A$.

The experiment took place at the University of Milan-Bicocca in 2010 for T1 and 2012 for T2 and T3. Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2016) run 16 sessions for a total of 288 participants and 72 groups (38 for T1, 18 for T2, 18 for T3).

[12]Since the empty network is pairwise stable in all treatments, myopia predicts the empty network in T1, T2 and T3.

Concept	T1	T2	T3
F	g^N	$g^{\emptyset}, g \in C_5, C_9$	$g^{\emptyset}, g \in C_7$
$F_{\#}$	g^N	$g^{\emptyset}$	$g^{\emptyset}$
A	g^N	$g^{\emptyset}, g \in C_5, C_9$	$g^{\emptyset}, g \in C_7, C_{10}$
$A_{\#}$	g^N	$g^{\emptyset}, g \in C_5, C_9$	$g^{\emptyset}, g \in C_7$
A_+	g^N	$g^{\emptyset}, g \in C_5$	$g \in C_7$
FSS	$\{g^N\}$	$\{g \in \mathcal{G} \mid g \in C_5\}$	$\{g, g' \in \mathcal{G} \mid g, g' \in C_7, \#N_i(g) = \#N_i(g') \forall i\}$

Table 3: Predictions in the three treatments of the experiment.

Network	$\#C$	T1	T2	T3	T1	T2	T3
$g \in C_1$	1 (1)	21	12	26	19.4	22.2	48.1
$g \in C_2$	6 (6)	11/6	2/6	7/6	1.7	0.6	2.2
$g \in C_3$	3 (1)	0	0	0	0.0	0.0	0.0
$g \in C_4$	12 (12)	4/12	0	6/12	0.3	0.0	0.9
$g \in C_5$	4 (4)	1/4	29/4	1/4	0.2	13.4	0.5
$g \in C_6$	4 (4)	0	0	0	0.0	0.0	0.0
$g \in C_7$	12 (6)	1/12	2/12	10/12	0.1	0.3	1.5
$g \in C_8$	3 (1)	0	0	0	0.0	0.0	0.0
$g \in C_9$	12 (12)	0	8/12	0	0.0	1.2	0.0
$g \in C_{10}$	6 (6)	5/6	0	2/6	0.7	0.0	0.6
$g \in C_{11}$	1 (1)	65	1	2	60.2	1.9	3.7
		Average number			Average percentage		

Table 4: Number of groups ending on average in each network of each class of networks in the experiment. Column $\#C$: in parenthesis the number of payoff relevant networks in each class.

Each group played three consecutive rounds. Table 4 gives us the number of groups ending on average in each network of each class of networks in the experiment. The column $\#C$ gives us the number of different networks within each class.[13] Table 4 can be interpreted as follows. In T1, 19.4% of the groups end up in the empty network while 60.2% end up in the complete network. In T2, 22.2% of the groups end up in the empty network while 13.4% end up in a network where player i is isolated and players j, k, l form a complete network between them.[14] In T3, half of the groups end up in the empty network.

Using a proportion test, it turns out that H1 and H2 are consistent with the data: $F_{\#}$ is significantly different of F (P-value < 0.001) and A_+ is significantly different of $A_{\#}$

[13] From the experimental data, we only have the number of groups ending in each class in the experiment. The average number of groups ending in any given network is then equal to the number of groups ending in each class divided by the number of different networks in the class.

[14] There are four different networks in class C_5. In total 53.6% of the groups end up in class C_5.

(P-value < 0.001). Only H3 is rejected (P-value of 0.83). It is not surprising since in T2 both $A_{\#}$ and A lead to the same predictions. In T3, A predicts more networks but networks in which nearly no groups were ending up. Hence, $A_{\#}$ is even not significantly different from A in T3 (P-value of 0.68).

Result 1. *In the experiment:*

(*i*) *The set of #-minimally farsighted unstable networks is a significant refinement of the set of minimally farsighted unstable networks.*

(*ii*) *The set of +-maximally farsighted absorbing networks is a significant refinement of the set of #-maximally farsighted absorbing networks.*

(*iii*) *The set of #-maximally farsighted absorbing networks is not significantly different from the set of maximally farsighted absorbing networks.*

Concept	**T1**	**T2**	**T3**	**T1**	**T2**	**T3**	**T3***
F	60.2	90.7	66.7	75.6	100	92.8	94.7
$F_{\#}$	60.2	22.2	48.1	75.6	29.3	92.8	68.9
A	60.2	90.7	70.4	75.6	100	92.8	94.7
$A_{\#}$	60.2	90.7	66.7	75.6	100	92.8	94.7
A_{+}	60.2	75.9	18.5	75.6	100	0	25.8
FSS	60.2	53.7	18.5	75.6	70.7	0	25.8
x		0			2.5		2.5

Table 5: Percentage of networks predicted by each concept among the networks in which at least x% of the groups were ending up. In the column T3* the percentage is computed considering only the number of payoff relevant networks in each class.

Since $A_{\#}$ and F are not significantly different of A, we only focus on F, $F_{\#}$ and A_{+} and we compare those concepts with respect to the following two criteria. A first criteria is the percentage of networks predicted by each concept. A higher percentage is better. Table 5 gives us the percentage of networks predicted by each concept among the networks in which at least x% of the groups were ending up.[15] A second criteria is the percentage of networks predicted by each concept among the networks in which less than 2.5% of the groups were ending up. A lower percentage is better. Table 6 gives us the percentage of networks predicted by each concept among the networks in which less than 2.5% of the groups were ending up. We observe that, with respect to the first criteria, A_{+} is (weakly) dominated by F, and with respect to the second criteria, A_{+} is (weakly) dominated by

[15]In the column T3* the percentage is computed considering the number of payoff relevant networks in each class. For instance, the number of groups ending on average in a representative network of class C_7 becomes 3% (instead of 1.5% in Table 4) since there are only 6 different payoff relevant networks in this class.

$F_{\#}$. With respect to the first criteria, $F_{\#}$ is (weakly) dominated by F, but with respect to the second criteria, $F_{\#}$ is not dominated by one of the other concepts. Conversely, with respect to the first criteria, F is not dominated by one of the other concepts, while with respect to the second criteria, F is (weakly) dominated by $F_{\#}$.[16] Based on these two criteria, we cannot discriminate between F (and $A_{\#}$) and $F_{\#}$. However, both perform better than A_{+} and FSS for fitting the data. F (and $A_{\#}$) predicts most networks that occur with positive probability while $F_{\#}$ is good at selecting the network with the highest percentage among those predicted by F. This observation is confirmed by Table 7 that gives us the mean percentage of a network predicted by a given concept. Thus, F (and $A_{\#}$) predicts the most relevant networks[17] while $F_{\#}$ provides narrower but more robust predictions.

Concept	**T1**	**T2**	**T3**	**T3***
F	0	59.5	37.3	0
$F_{\#}$	0	0	0	0
A	0	59.5	44.8	11.9
$A_{\#}$	0	59.5	37.3	0
A_{+}	0	0	37.3	0
FSS	0	0	37.3	0

Table 6: Percentage of networks predicted by each concept among the networks in which less than 2.5% of the groups were ending.

Treatment	F	$F_{\#}$	A	$A_{\#}$	A_{+}	**FSS**
T2	5.31	22.20	5.31	5.31	15.16	13.40
T3	5.09	48.10	3.67	5.09	1.50	1.50
T2+T3	5.20	35.15	4.49	5.20	8.33	7.45

Table 7: Mean percentage of a network predicted by a given concept.

The analysis suggests that the frequency of a network g depends negatively on $\#\phi(g)$ and positively on $\#\phi^{-1}(g)$. Table 8 shows the naive regression of the percentage per representative network on $\#\phi(g)$, $\#\phi^{-1}(g)$ and $\#\phi^{-1}(g) - \#\phi(g)$, and it confirms this intuition. Moreover, $\#\phi(g)$ impacts more the frequency of a network than $\#\phi^{-1}(g)$. Thus, both the stability requirement and the reachability requirement seem to matter for predicting the most frequent networks with a higher importance of the former.

[16]With respect to the first criteria, FSS is (weakly) dominated by A_{+} and F, and with respect to the second criteria, FSS is (weakly) dominated by A_{+} and $F_{\#}$.

[17]When we separate the networks between those with a frequency higher than 1% and the others, F (and $A_{\#}$) has the highest correlation with this binary criteria.

Percentage per network

	T1	**T2**	**T3**	**All**	**All**	**All**	**All**
$\#\phi(g)$	-1.602	-0.383	-1.119	-0.640+	-1.059**		
	(0.211)	(0.294)	(0.166)	(0.063)	(0.001)		
$\#\phi^{-1}(g)$	0.143	0.124	0.191	0.219+		0.347**	
	(0.655)	(0.355)	(0.315)	(0.050)		(0.000)	
$\#\phi^{-1}(g) - \#\phi(g)$							0.304**
							(0.000)
constant	24.45	8.309	24.63	12.02	23.42**	-1.358	4.650*
	(0.290)	(0.329)	(0.182)	(0.113)	(0.000)	(0.609)	(0.021)
Num. obs.	11	11	11	33	33	33	33
adj. R-sq	0.433	0.449	0.227	0.369	0.305	0.314	0.368

P-values in parentheses, $^{+}$ $P < 0.10$, * $P < 0.05$, ** $P < 0.001$

Table 8: Regression of the percentage per representative network on $\#\phi(g)$, $\#\phi^{-1}(g)$ and $\#\phi^{-1}(g) - \#\phi(g)$.

7 Conclusion

We have proposed the notion of minimal farsighted instability to predict the networks that are more likely to be observed in the long run when agents are farsighted.[18] A network is said to be minimally farsighted unstable if there is no other network which is more farsightedly stable, where the comparison between two networks is made by comparing (in the set inclusion or cardinal sense) their sets of farsighted defeating networks. But, selecting a network that is more farsightedly stable than another one might be a more robust prediction if at the same time the former is more likely to be reached than the latter one. Hence, we have also proposed to compare networks in terms of their reachability. A network is said to be maximally farsighted absorbing if there is no other network which is more farsightedly absorbing, where the comparison between two networks is now made by comparing both their sets of farsighted defeating networks (i.e. in terms of their stability) and their sets of farsighted defeated networks (i.e. in terms of their reachability).

We have provided some general results that are helpful for characterizing minimally farsighted unstable networks and maximally farsighted absorbing networks, and we have investigated their relationships with set-valued notions of farsightedness. In terms of computational complexity, minimal farsighted instability turns to be much less demanding than set-valued concepts of farsightedness. Indeed, in addition to the computation of the set of farsighted defeating networks for each network, most set-valued concepts require to consider all possible combinations of networks into a set. We have used experimental data

[18]An alternative approach for solving the lack of farsighted stability is to require the consent of partners or neighbours for adding or deleting links. See Caulier, Mauleon and Vannetelbosch (2013) and Caulier, Mauleon, Sempere-Monerris and Vannetelbosch (2013).

to show the relevance of the new solution concepts. It turns out that the set of minimally farsighted unstable networks (F) is not significantly different than the set of maximally farsighted absorbing networks (A). The set of cardinally$^+$ maximally farsighted absorbing networks (A_+) performs better than the vNM farsighted stable set (FSS). Finally, the set of minimally farsighted unstable networks (F) predicts most networks that occur with positive probability while the set of cardinally minimally farsighted unstable networks ($F_\#$) is good at selecting the network with the highest percentage among those predicted by F. Both F and $F_\#$ outperform on average A_+ and FSS.

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Appendix

Experimental Data: Proof of Proposition 10

Let g^i be a generic network in class C_i and $c_i \subseteq C_i$ a generic proper subset of the corresponding class. We will write $g^i \to g$ with $g \in C_j$, and $g^i \to g$ with $g \in c_j$, when the generic network g^i in class C_i reaches with a farsighted improving path all the networks in class C_j or only a proper subset c_j of C_j, respectively.

Minimally unstable and maximally absorbing networks in T1

In treatment 1 (T1) the set of networks that can be reached from g^i by some farsighted improving path are:

$\phi(g^\emptyset) = \{g \mid g \in C_{10} \cup C_{11}\}$

$\phi(g^2) = \{g \mid g \in C_1 \cup C_{10} \cup C_{11}\}$

$\phi(g^3) = \{g \mid g \in C_1 \cup c_2 \cup c_5 \cup C_{10} \cup C_{11}\}$

$\phi(g^4) = \{g \mid g \in C_1 \cup c_2 \cup c_5 \cup C_{10} \cup C_{11}\}$

$\phi(g^5) = \{g \mid g \in C_1 \cup c_2 \cup C_{10} \cup C_{11}\}$

$\phi(g^6) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup c_5 \cup C_{10} \cup C_{11}\}$

$\phi(g^7) = \{g \mid g \in C_1 \cup c_2 \cup c_3 \cup c_4 \cup C_5 \cup C_{10} \cup C_{11}\}$

$\phi(g^8) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup C_5 \cup c_7 \cup C_{10} \cup C_{11}\}$

$\phi(g^9) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup c_5 \cup c_6 \cup c_7 \cup C_{10} \cup C_{11}\}$

$\phi(g^{10}) = \{g \mid g \in c_2 \cup c_4 \cup c_5 \cup c_6 \cup C_{11}\}$

Network	T1		T2		T3	
	$\#\phi(g)$	$\#\phi^{-1}(g)$	$\#\phi(g)$	$\#\phi^{-1}(g)$	$\#\phi(g)$	$\#\phi^{-1}(g)$
$g \in C_1$	7	56	4	46	12	50
$g \in C_2$	8	29	11	19	14	21
$g \in C_3$	14	4	19	4	21	0
$g \in C_4$	11	7	13	7	19	5
$g \in C_5$	11	30	9	51	19	12
$g \in C_6$	17	6	23	0	28	6
$g \in C_7$	18	3	23	3	16	62
$g \in C_8$	24	0	29	2	23	2
$g \in C_9$	18	0	14	52	27	0
$g \in C_{10}$	11	57	28	1	29	49
$g \in C_{11}$	0	63	22	0	18	0

Table 9: Number of defeating networks ($\#\phi(g)$) and number of defeated networks ($\#\phi^{-1}(g)$) for each network in each class of networks of the experiment.

$\phi(g^N) = \emptyset$.

Remember that a network $g \in \mathcal{G}$ is minimally farsighted unstable if there is no $g' \neq g$ such that $g' \gtrsim^{\sigma(\subseteq)} g$; i.e., $\phi(g') \subsetneq \phi(g)$. It follows that g^N is the unique minimally farsighted unstable network. Notice that $g^{\emptyset}$ is more farsightedly stable than all networks $g \neq g^{10}, g^N$.

A network $g \in \mathcal{G}$ is maximally farsighted absorbing if there is no $g' \neq g$ such that $g' \gtrsim^{\alpha(\subseteq)} g$; i.e., $\phi(g') \subseteq \phi(g)$ and $\phi^{-1}(g) \subseteq \phi^{-1}(g')$ with one inclusion holding strictly. Since $\phi(g^N) = \emptyset$ and $g^N \in \phi(g^i)$ for all $g^i \neq g^N$, it also holds that g^N is the unique maximally farsighted absorbing network.

A network $g \in \mathcal{G}$ is #-minimally farsighted unstable if there is no $g' \neq g$ such that $g' \gtrsim^{\sigma(\#)} g$; i.e., $\#\phi(g') < \#\phi(g)$. Thus, g^N is the #-minimally farsighted unstable network.

A network $g \in \mathcal{G}$ is #-maximally farsighted absorbing if there is no $g' \neq g$ such that $g' \gtrsim^{\alpha(\#)} g$; i.e., $\#\phi(g') \leq \#\phi(g)$ and $\#\phi^{-1}(g) \leq \#\phi^{-1}(g')$ with one inequality holding strictly. Then, g^N is the #-maximally farsighted absorbing network.

A network $g \in \mathcal{G}$ is +-maximally farsighted absorbing if there is no $g' \neq g$ such that $g' \gtrsim^{\alpha(+)} g$; i.e., $\#\phi(g') - \#\phi^{-1}(g') < \#\phi(g) - \#\phi^{-1}(g)$. Again, g^N is the +-maximally farsighted absorbing network.

Minimally unstable and maximally absorbing networks in T2

Let us denote B_g the networks that are adjacent to g,

$$B_g = \{g' \mid g' = g + ij \vee g - ij, \text{ for some } ij\},$$

and let $\overline{B}_g$ be its complement. In treatment 2 (T2) the set of networks that can be reached from g^i by some farsighted improving path are:

$\phi(g^\emptyset) = \{g \mid g \in C_5\}$
$\phi(g^2) = \{g \mid g \in C_1 \cup C_5 \cup c_9\}$
$\phi(g^3) = \{g \mid g \in C_1 \cup c_2 \cup C_5 \cup C_9\}$
$\phi(g^4) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup C_5 \cup c_9\}$
$\phi(g^5) = \left\{g \mid g \in C_9 \cap \overline{A}_{g^5}\right\}$
$\phi(g^6) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup C_5 \cup C_9\}$
$\phi(g^7) = \{g \mid g \in C_1 \cup c_2 \cup c_3 \cup c_4 \cup C_5 \cup C_9\}$
$\phi(g^8) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup C_5 \cup c_7 \cup C_9\}$
$\phi(g^9) = \{g \mid g \in c_4 \cup (C_5 \cap A_{g^9}) \cup (C_9 \setminus g^9)\}$
$\phi(g^{10}) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup C_5 \cup c_7 \cup c_8 \cup C_9\}$
$\phi(g^N) = \{g \mid g \in C_5 \cup C_9 \cup C_{10}\}$.

Note that $g^\emptyset$ is more farsightedly stable than all networks $g \neq g^5, g^9$. Moreover, neither g^5 nor g^9 are more farsightedly stable than $g^\emptyset$. Thus, $g^\emptyset$ is minimally farsighted unstable. Note also that it is not always true that $(C_9 \setminus g^9) \supseteq (C_9 \cap \overline{B}_{g^5})$ since g^9 could belong to $(C_9 \cap \overline{B}_{g^5})$. Then, also g^5, g^9 are minimally farsighted unstable.

The maximally farsighted absorbing networks are $g^\emptyset, g^5$ and g^9. Indeed:

$\phi^{-1}(g^\emptyset) = \{g \mid g \in C_2 \cup C_3 \cup C_4 \cup C_6 \cup C_7 \cup C_8 \cup C_{10}\}$;
$\phi^{-1}(g^5) = \{g \mid g \in C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_6 \cup C_7 \cup C_8 \cup c_9 \cup C_{10} \cup C_{11}\}$ and
$\phi^{-1}(g^9) = \{g \mid g \in C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup c_9 \cup C_{10} \cup C_{11}\}$.

From Table 9, we have that $g^\emptyset$ is the #-minimally farsighted unstable network. Notice that the network $g^\emptyset$ is #-more farsightedly absorbing than the network g^2, but not more than g^5 and g^9. Finally, $g^\emptyset$ and g^5 are the +-maximally farsighted absorbing networks.

Minimally unstable and maximally absorbing networks in T3

In treatment 3 the set of networks that can be reached from g^i by some farsighted improving path are:

$\phi(g^\emptyset) = \{g \mid g \in C_7\}$
$\phi(g^2) = \{g \mid g \in C_1 \cup C_7 \cup c_{10}\}$
$\phi(g^3) = \{g \mid g \in C_1 \cup c_2 \cup C_7 \cup C_{10}\}$
$\phi(g^4) = \{g \mid g \in C_1 \cup c_2 \cup c_5 \cup C_7 \cup c_{10}\}$
$\phi(g^5) = \{g \mid g \in C_1 \cup c_2 \cup C_7 \cup c_{10}\}$
$\phi(g^6) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup c_5 \cup C_7 \cup C_{10}\}$
$\phi(g^7) = \{g \mid g \in (C_7 \setminus g \text{ s.t. } d_i(g) = d_i(g^7) \text{ for all } i \in N) \cup C_{10}\}$
$\phi(g^8) = \{g \mid g \in C_1 \cup c_4 \cup C_7 \cup C_{10}\}$
$\phi(g^9) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup c_5 \cup c_6 \cup C_7 \cup C_{10}\}$
$\phi(g^{10}) = \{g \mid g \in C_1 \cup c_2 \cup c_4 \cup c_5 \cup c_6 \cup C_7 \cup c_8 \cup C_{10} \setminus g^{10}\}$
$\phi(g^{11}) = \{g \mid g \in C_7 \cup C_{10}\}$.

Note that $g^{\emptyset}$ is more farsightedly stable than all networks $g \neq g^7$. Moreover, g^7 is not more farsightedly stable than $g^{\emptyset}$. Thus, $g^{\emptyset}$ and each g^7 are minimally farsighted unstable networks. The maximally farsighted absorbing networks are also $g^{\emptyset}, g^7$ and g^{10}. Indeed:

$\phi^{-1}(g^{\emptyset}) = \{g \mid g \in C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_8 \cup C_9 \cup C_{10}\}$;

$\phi^{-1}(g^7) = \{g \mid g \in C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup c_7 \cup C_8 \cup c_9 \cup C_{10} \cup C_{11}\}$ and

$\phi^{-1}(g^{10}) = \{g \mid g \in c_2 \cup C_3 \cup c_4 \cup c_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \setminus g^{10} \cup C_{11}\}$.

From Table 9, we have that $g^{\emptyset}$ is the #-minimally farsighted unstable network. Remark that the network g^7 is #-more farsightedly absorbing than all other networks except $g^{\emptyset}$. Finally, g^7 is the +-maximally farsighted absorbing network.

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