Who matters in coordination problems on networks: myopic or farsighted agents?

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Abstract

This paper studies a model of social interaction in a fixed network where agents play a coordination game - a game where it is optimal for a player to choose an action like most of her friends. The different actions correspond to different projects the player can invest into. A project is successful once a certain amount of players have chosen it. All players have a certain type: A player can be either an extremist for one of the projects or she can be a moderate. Extremist players only obtain utility from one project, while moderate players are ex ante indifferent between the projects. In addition, the players may also differ in their level of farsightedness: Some players cannot foresee the reactions that their actions cause while others anticipate all induced changes.

We propose a solution concept to study the strategy profiles that are stable when myopic and farsighted players interact with each other. In such profile no player has any incentive to deviate to another action. We show how the set of stable strategy profiles changes if a myopic player becomes farsighted, a moderate player into an extremist or vice versa. Furthermore, we characterize the set of stable strategy profiles for common network structures.

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1   Introduction

Coordination problems play a huge role in our everyday life. We coordinate with our partners and family what to eat, how to spend the evening or where to go on vacation. With co-authors we have to coordinate time and date of meetings and the splitting of tasks. In economics, coordination games have been studied under different aspects. In this paper we analyse a coordination game that is played between players who are connected by a network. The players can choose between different projects and while some players may be ex ante indifferent between all projects, other players are very strongly in favor of a certain project. Each player wants to invest into a project she likes, but at the same time she wants to coordinate with her neighbors in the network. The projects the players invest to can either be successful or not. For a project to be successful enough players have to choose the project such that the threshold is reached. Only then the players get a positive payoff. Those thresholds can differ between projects. We assume that players can only choose a single project or they can decide to abstain. Players are heterogeneous in their taste for the different projects and, in addition, heterogeneous in their degree of farsightedness. A player can either be myopic or completely farsighted. Myopic players do not anticipate that individual deviations can yield to subsequent deviations while farsighted players forecast how others might react to their actions.

In this paper we characterize the set of stable strategy profiles. In a stable profile all players have chosen a project (or abstention) and no player has any incentive to deviate. A player deviates from a profile if her utility in the resulting profile is higher. A myopic players only compares the current profile to the profile in which only she deviates, while a farsighted player anticipates that other players might adjust their actions as well.

We show that the set of stable profiles is the set of Nash equilibria if all players are myopic. Modifying the degree of farsightedness of a player such that this player becomes farsighted reduces the size of the set and works as an equilibrium refinement. We prove that even if all players are farsighted the set of stable profiles is non-empty, which implies that for all combinations of myopic and farsighted players the set is non-empty.

Furthermore, we analyse the impact of extremism. A player is an extremist for a project if her taste between the projects is such that she only likes one project. An extremist will only invest into that project, or abstain. We state results how the set of stable profiles changes when the taste of a player is modified. In addition, we show how extremists can divide networks into different parts which can be analysed separately.

Besides the farsightedness and the extremism, the thresholds of the projects influence the set of stable profiles. We characterize the change in the set of stable profiles if we increase the threshold of a project. In that case some profiles may be not stable anymore, but other profiles may become stable.

We find many different examples in the literature and applications in our everyday life. *Example 1:* The classic example for coordination games is the decision of a couple

about their evening activity. In this battle of the sexes the woman prefers going to the opera with the man over watching a football match, while the man's preferences are the opposite way. He prefers football over going to the opera. No player gets any utility from spending the evening apart. We can adjust this example to work in our model: Let the network be a social network where links imply that the linked players are friends. All players have to make a decision about their evening activity. They can either stay at home (abstention) or choose one of several projects. They can go to the cinema, play darts, bowling, football or go for drinks and they enjoy the activity the most if all their friends are there as well. Clearly it is possible to go to the cinema or for drinks alone, but for playing darts or bowling you need at least two players and for football even more. Some players may spend each evening at the bar, they are extremists for that project. A player that decides to go and play football only and finds that no one else is there receives a negative utility. She would have preferred to stay at home over wasting her time to go to the pitch.

Example 2: Another example is the cooperation between researchers (or R&D firms). Clearly, the researchers build a network, where the links can be interpreted as previous cooperation. The players can do research into different problems. While some problems might be solved by a single player, other problems need the cooperation of several players to be solved. If there are not enough players to solve the problem, they do not get any result and have a negative utility. In this example the cooperation with the neighbors might not give higher payoff to the researcher directly, but lowers the effort she has to use for her research. In established research cooperations, the players know how the others work and how to explain things. While some researchers are interested in several problems, other researchers are extremists for a certain problem and only want to solve this one problem.

Example 3: Our model can be applied to social networks like Twitter as well. The users can choose between different hashtags for their posts and want to be part of a viral campaign, i.e. a hashtag that is used by a large group of users. If they use a hashtag that is not used often their post does not reach many people. If they are part of a viral campaign, each user benefits from followers that use the same hashtag as they might retweet the others posts.

Example 4: The choice of different problems can also be seen as the adoption of new technologies. Cities may choose one of several ways to make public transport more eco-friendly and benefit from exchange with neighboring cities. If one of these new types of transport is not supported by enough cities the development might be stopped. In that case the cities are not able to further improve their public transport.
Similarly the choice of mobile phone operation systems work. Users benefit from friends that can help them with problems, but if there are not enough users the developing company might stop further development as it happened with Windows Phone.

Coordination games have been analyzed intensively in the literature. The battle of sexes already appeared in Luce and Raiffa (1957) and is used as an example also in

Fudenberg and Tirole (1991).

Closest to our work, but without the network aspect is the work of Sákovics and Steiner (2012). They focus on a continuum of players where each player can either decide to invest into a project or to abstain from investment. The project has an investment threshold and is only successful if enough players invest. The players receive a noisy signal about this threshold. The authors show that depending on the noise in the signal there can be miscoordination, while if the noise is small the players coordinate their actions.

Jackson and Watts (2002a) model coordination games on networks, but focus on repeated interaction. In each period a player can adjust her action. With a small probability a player makes a mistake which might lead to changes of other players. All players are myopic and have the same taste between the different options. The authors use a different stability concept (stochastically stable) and in addition to analyzing the game, Jackson and Watts study the network formation when links are costly.

Goyal and Vega-Redondo (2005) study a coordination game which is repeatedly played on networks where players can choose between two different options. Players can revise their strategy in each period with a certain probability. In their model the links between players are paid only by one player. The authors show that depending on the costs for a link the players either coordinate on the risk-dominant action or on the efficient action.

Leister et al. (2018) analyze a global game played on a network. The players receive a noisy signal about the value of a technology and can choose whether to adopt the technology or not. Similar to our model, the value is increasing in the amount of neighbors who adopt the technology as well. The authors show under which conditions all players (or groups of players) coordinate their actions.

In our examples players have incentives to coordinate with their neighbors, but in other applications the opposite might be the case. The model of Bramoullé (2007) focuses on these anti-coordination games. The author defines frustration as a function of neighbors who play the same action and neighbors who choose the different action. This frustration is the highest in complete networks and lowest in bipartite networks. Bramoullé et al. (2004) analyze a similar model with network formation.

More specific models with network effects similar to coordination games are for example for public goods Allouch (2015) and Bramoullé and Kranton (2007) or the model of spill-overs by Ballester et al. (2006).

In our model we will often observe communities that choose the same action. Recent work on communities in networks has been done by Jackson and Storms (2018). They study a game in which a player can either adopt the behavior of her neighbors or not. If a certain amount of neighbors of a player choose to adopt, it is beneficial for the player to adopt as well. Several equilibria might exist and the authors define a community as a set of players that coordinates in each equilibrium.

Our model uses the idea of improving paths as in Jackson and Watts (2002b). Farsightedness has been studied in many different models and applications, see for example Ray and Vohra (2015) or Herings et al. (2009).

The structure of our paper is as follows: In Section 2 we describe the model. Section 3 defines the stability concept and characterises the set of stable strategy profiles when all players are myopic. In Section 4 we analyze the impact of farsightedness, extremism and the thresholds on the strategy profiles. We prove in Section 5 that this set of stable profiles is never empty. Results for specific networks are given in Section 6. In Section 7 we discuss the incentives and possibilities of a social planner. Section 8 concludes. All proofs are relegated to the appendix.

2 Model

Let $N = \{1, ..., n\}$ be a finite set of players, which are arranged in an undirected network g. A link between two players i and j is denoted by $ij \in g$. By $N_i(g) = \{j \in N : ij \in g\}$ we denote the set of neighbors of player i in the network g. A player who only has one neighbor is called a loose end. The complete network is the network in which each player i is linked to all other players, i.e. $N_i(g) = N \setminus i$. In the star network there is one player i in the center of the star and all other players j are only linked to the center, i.e. $N_i(g) = N \setminus i$ and $N_j(g) = \{i\}$. The line network is the network in which each player $i \in \{2, \ldots n-1\}$ is linked to the players $i-1$ and $i+1$ and players 1 and n are loose-ends. Player 1 is only linked to player 2 and players n only link is to player $n-1$.

We study a coordination game that is played by all players. There exist r different projects. The set of projects is denoted by $P = \{P^1, \ldots, P^r\}$. A project $Q \in P$ is successful if at least E^Q players choose this project. We call $E^Q \in \{1, \ldots, n\}$ the investment threshold of project Q. Let $E = (E^1, ..., E^r)$ denote the vector of investment thresholds. All players in the network choose a project or abstain. The choice of player i is captured in p_i, i.e. $p_i \in \bar{P} = P \cup \emptyset = \{\emptyset, P^1, \ldots, P^r\}$, where $p_i = \emptyset$ means that player i abstains. We denote a strategy profile by the vector[1] $p = (p_1, \ldots, p_n)$. The set of all possible strategy profiles is $\mathcal{P} = \bar{P}^n = \{\emptyset, P^1, \ldots, P^r\}^n$.

Players have a taste for the different projects, where $t_i^Q \in \{0, 1/r, 1\}$ denotes the taste of player i for project Q with $\sum_{Q=1}^r t_i^Q = 1$. We assume that a player can either be an extremist for one project Q, i.e. $t_i^Q = 1$ or be moderate, i.e. $t_i^Q = 1/r$ for all $Q \in P$. The set of extremists for project Q is denoted by $X_Q = \{i \in N : t_i^Q = 1\}$ and the set of moderate players by $Y = \{i \in N : t_i^Q = 1/r \ \forall Q \in P\}$. Let $t = ((t_1^Q)_{Q=1}^r, ..., (t_n^Q)_{Q=1}^r)$ denote the tastes of the players for the different projects.

In addition to their different tastes, players can vary in their degree of farsightedness. Let $s_i = \{m, f\}$ denote the degree of farsightedness of player i. If $s_i = m$, player i is myopic, while player i is farsighted if $s_i = f$. We denote by M the set of myopic players and by F the set of farsighted players. Obviously, we have $M \cup F = N$. Either set (M or F) is allowed to be empty. The taste of player i for the different projects $(t_i^Q)_{Q=1}^r$ and her

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[1]For a $n \times 1$ vector a we denote by a_{-i} the vector of size $n-1$ with the i-th entry removed and analogue we use $a_{-i,j}$ to denote the vector in which the i-th and the j-th elements have been deleted.

degree of farsightedness s_i determines the type of player i, $\theta_i = ((t_1^Q)_{Q=1}^r, s_i)$. We denote by $\theta = (\theta_1, ..., \theta_n)$ the vector of types of the different players.

The utility function u_i of player i, given (g, θ, E), is assumed to be as follows:

$$u_i(p) = \begin{cases} 0 & \text{if } p_i = \emptyset, & (1a) \\ < 0 & \text{if } p_i \neq \emptyset \text{ and either } |\{j \in N \mid p_i = p_j\}| < E^{p_i} \text{ or } t_i^{p_i} = 0, & (1b) \\ > 0 & \text{if } p_i \neq \emptyset \text{ and } |\{j \in N \mid p_i = p_j\}| \geq E^{p_i} \text{ and } t_i^{p_i} > 0. & (1c) \end{cases}$$

Properties of u:

(P1) $u_i(p_i, p_{-i}) = u_i(p_i, p'_{-i})$ for any p_{-i} and p'_{-i} if $u_i(p_i, p_{-i})$ and $u_i(p_i, p'_{-i})$ as in (1a) or (1b).

(P2) $u_i(p_i, p_j, p_{-i,j}) > u_i(p_i, p'_j, p_{-i,j})$ if $p'_j \neq p_j = p_i$ and u_i as in (1c).

(P3) $u_i(p_i, p_j, p_{-i,j}) = u_i(p_i, p'_j, p_{-i,j})$ if $p_j \neq p_i \neq p'_j$.

(P4) u_i is independent of s_i and s_{-i}.

We normalize the utility of abstention to 0 (1a). An investment of player i into an unsuccessful project gives the player a negative utility (1b), while an investment into a successful project Q yields the player a positive utility as long as the taste of player i for this project Q is positive (1c). Furthermore, in that case, the utility of player i is increasing in the amount of neighbors in the network g who choose the same project (P2). In the other cases (1a and 1b) the utility of player i does not change if her neighbor, player j changes her action as long as the utility of player i stays non-positive (P1). Similarly, the utility of player i who invests in a successful project p_i does not change if her neighbor j invests in project $p_j \neq p_i$ or $p'_j \neq p_i$ (P3). We can also see that the utility of a player depends on her taste and project choice, but not on her degree of farsightedness. Furthermore, it follows directly from the utility function that an extremist for project Q will never choose project $R \neq Q$.

Example 1. One example for this class of utility function is given by:

$$u_i(p) = \begin{cases} 0 & \text{if } p_i = \emptyset, & (2a) \\ -c_i & \text{if } |\{j \in N : p_i = p_j\}| < E^{p_i}, & (2b) \\ t_i^{p_i}\left[\alpha_i + (1-\alpha_i)\frac{|\{j \in N_i : p_i = p_j\}|}{|N_i|}\right] - c_i & \text{if } |\{j \in N : p_i = p_j\}| \geq E^{p_i} & (2c) \end{cases}$$

Players have different interests in cooperating with their neighbors. This parameter of cooperation is captured by $\alpha_i \in (0, 1)$. The investment into a project is costly for each player and the cost for player i is denoted by c_i. In case of a successful investment the return to the player is always higher than her investment cost, i.e. $\alpha_i > r \cdot c_i$ holds.

(2c) corresponds to the case of a successful project p_i. In that case the player's utility depends on her taste ($t_i^{p_i}$) for the project p_i, her parameter for cooperation α_i and the amount of neighbors who also invested into project p_i. From this payoff the cost of investment c_i has to be subtracted. In case of an unsuccessful project p_i (see 2b), there is no benefit from the project, but only the cost c_i. (2a) represents the case of abstention.

From now on we only consider projects that can be successful, i.e. $\forall Q \in P : |\{i \in N : t_i^Q > 0\}| \geq E^Q$. This ensures that a project Q can be successful and that there are not too many extremists for all the other projects such that the investment threshold of project Q cannot be reached even if all extremists for project Q and all moderate players invest into project Q.
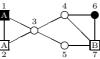

Figure 1: Network example

In Figure 1 we illustrate the notation we use for figures. Players with a round node are moderate players, while a player with a square node is an extremist for the project written in the square. If the node is filled the player is farsighted otherwise she is myopic. So in this example there are two extremists for project A, namely player 1 and 2. Player 7 is an extremist for project B and the players 1 and 6 are farsighted.

3 Myopic-farsighted stability

For normal form games, the stability concept studied in the literature is the concept of Nash equilibrium.

Definition 1. Consider (g, θ, E) as given. A strategy profile $p^* \in \mathcal{P}$ is a Nash Equilibrium if it satisfies

$$\forall i \in N, \forall p_i \in \bar{P} : u_i(p_i^*, p_{-i}^*) \geq u_i(p_i, p_{-i}^*)$$

Let $\mathcal{N}(g, \theta, E)$ denote the set of all Nash Equilibria.

The notion of Nash equilibrium considers that players are myopic in the sense that they do not forecast how others might react to their actions. This concept is based on the direct dominance relation and neglects the destabilizing effect of indirect dominance relations as introduced by Harsanyi (1974) and Chwe (1994). Indirect dominance captures the idea that farsighted players can anticipate the actions of other players and consider the end strategy profile that their deviations may lead to. Based on the concept of indirect dominance, Chwe (1994) has proposed the largest consistent set to predict which of the several pure strategy Nash equilibria will be played if players are farsighted. Up to now, no solution concept has been proposed in order to allow for heterogeneity in the degree of farsightedness among players. In the following, we propose a solution concept to study the strategy profiles that are stable when myopic and farsighted players interact with each other: the set of myopic-farsighted stable strategy profiles.

A myopic-farsighted improving path is a sequence of strategy profiles that can emerge when farsighted players choose an action based on the improvement the end strategy

profile offers them relative to the current strategy profile while myopic players choose an action based on the improvement the next strategy profile in the sequence offers them relative to the current one. Each strategy profile in the sequence differs from the previous one in that only one action of one of the players has been modified.

Definition 2. Consider (g, θ, E) as given. A myopic-farsighted improving path of length L from a strategy profile $p \in \mathcal{P}$ to a strategy profile $p' \in \mathcal{P}$ is a finite sequence of strategy profiles $p^0, \ldots, p^L \in \mathcal{P}$ with $p^0 = p$, $p^L = p'$ and $p^j \neq p^k$ for all $j, k \in \{1, \ldots, L-1\}$ such that for every $\ell \in \{0, \ldots, L-1\}$ there is a unique player i such that $p_i^{\ell+1} \neq p_i^{\ell}$ and

$$
\begin{cases}
u_i(p^{\ell+1}) > u_i(p^{\ell}), & \text{if } i \in M \\
u_i(p^L) > u_i(p^{\ell}), & \text{if } i \in F
\end{cases}.
$$

If there exists a myopic-farsighted improving path from the strategy profile p to p' we write $p \to p'$. The set of all strategy profiles that can be reached from p by a myopic-farsighted improving path is denoted by $h(p) = \{p' \in \mathcal{P} \mid p \to p'\}$.
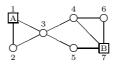
Example 2.

Figure 2: Network example

For this example we consider the network shown in Figure 2. All players are myopic. Players 1 and 7 are extremists while all other players are moderate. There are only two projects A and B and both have low thresholds, i.e. $E^A = E^B = 1$. Consider the profile $p^0 = \{A, B, B, A, A, A, B\}$. The players 1, 4, 5 and 6 chose project A and the other players selected B. One example for an improving path out of this profile is the following:

Player 3 has only one neighbor (player 2) with the same choice as her and three neighbors who chose project A. So it is an improvement for her to select A as well. From this profile ($p^1 = \{A, B, A, A, A, A, B\}$) there is the second step to $p^2 = \{A, A, A, A, A, A, B\}$. Caused by the change of player 3, player 2 is not coordinating with any neighbor. She improves if she changes her choice to project A.

In this example all players are myopic and only consider the immediate improvement they get from changing their action. This is different if we consider farsighted players. Let us take once again the example from Figure 2, but assume that player 6 is farsighted.

In the profile $p^0 = \{A, A, A, A, A, A, B\}$ player 6 is coordinating with one of her two neighbors. From this profile there is an improving path initiated by player 6. First, player 6 will change to project B, which yields to the profile $p^1 = \{A, A, A, A, A, B, B\}$. In this profile player 4 has an incentive to deviate to project B as well, because two of her three neighbors have selected project B. This yields to the profile $p^2 = \{A, A, A, B, A, B, B\}$.

We see that in p^2 player 6 is coordinating with both of her neighbors. Since she is farsighted, she had the incentive to start the improving path, taking into account that player 4 will follow her example and deviate from project A to B. This makes player 6 in profile p^2 better off than in p^0, even though she did not immediately (in p^1) benefit from her change.

Only farsighted players anticipate the changes of the other players and compare their current utility with the utility they would get at the end of the improving path.

Nash equilibrium requires strategy profiles to be immune to immediate deviations and does not capture that farsighted players anticipate the actions of other players and consider the end strategy profile that their deviations may lead to. In order to be stable, a strategy profile should be immune to deviations of both myopic and farsighted players. These considerations lead to the following definition of myopic-farsighted stable strategy profiles.

Definition 3. Consider (g, θ, E) as given. A strategy profile p is myopic-farsighted stable if it satisfies $h(p) = \emptyset$. The set of myopic-farsighted stable strategy profiles $Z(g, \theta, E)$ is defined as follows: $Z(g, \theta, E) = \{p \in \mathcal{P} : h(p) = \emptyset\}$.

According to Definition 3, a strategy profile p is myopic-farsighted stable if there is no myopic-farsighted improving path leaving p. Let us first analyze some properties of the set of myopic-farsighted stable strategy profiles.

The next proposition establishes that there cannot be a myopic-farsighted improving path that starts at a strategy profile p and ends at the same strategy profile p. The result holds when both myopic and farsighted players deviate along the path.

Proposition 1. *For any strategy profile $p \in \mathcal{P}$, it holds that $p \notin h(p)$.*

Notice that if $p^* \in \mathcal{N}(g, \theta, E)$ is a Nash equilibrium, then there is no profitable individual deviation from p^*. Thus, $h(p^*) = \emptyset$, when all players are myopic. As a corollary of Proposition 1, we have that there always exist a myopic-improving path from any strategy profile $p \notin \mathcal{N}(g, \theta, E)$ to some Nash equilibrium strategy profile $p^* \in \mathcal{N}(g, \theta, E)$.

Corollary 1. *Assume all players are myopic. Then, for any strategy profile $p \notin \mathcal{N}(g, \theta, E)$, there exists a Nash equilibrium strategy profile $p^* \in \mathcal{N}(g, \theta, E)$ such that $p^* \in h(p)$.*

From Proposition 1 and Corollary 1, we obtain the next proposition.

Proposition 2. *If all players are myopic, a set of strategy profiles is a set of myopic-farsighted stable strategy profiles if and only if it is equal to the set of Nash equilibrium strategy profiles.*

Proposition 2 states an important result as it shows the connection between the set of myopic-farsighted stable strategy profiles and the set of Nash equilibria. As long as all players are myopic the sets are the same.

4 Farsightedness, Extremism and Thresholds

In the previous section we have characterized the set of stable strategy profiles for the case that all players are myopic. Besides the network, the myopic-farsighted stable set $Z(g, \theta, E)$ depends on three different sets of parameters: It plays an important role whether a player is myopic or farsighted and if she is moderate or an extremist. In addition, the thresholds of the projects have a big impact on the myopic-farsighted stable set. In this section we characterize the impact of farsightedness, extremism and the thresholds and answer our question "Who matters in coordination games?".

We first state some remarks that simplify our analysis.

Remarks 1.

- In all stable profiles with at least one successful project, all moderate players will choose a successful project and will never abstain.
- Assume player i is an extremist for project Q. In all stable strategy profiles where project Q is successful, player i will choose project Q.
- Assume player i is moderate and a loose-end and player j is the neighbor of player i. In all stable strategy profiles where j does not abstain, player i will choose the same project as player j.
- If all thresholds are low, i.e. $E^Q = 1$ for all $Q \in P$ no player will abstain in a stable profile.
- If the thresholds are such that only one project can be successful (i.e. $E^Q + E^R > n$ for all $Q, R \in P$), then there exist only two different types of stable profiles: Either one project Q is successful and all extremists for Q and all moderate players choose Q, while the other extremists abstain or all players abstain.

4.1 Impact of Farsightedness

What happens with the set of myopic-farsighted stable strategy profiles once some players become farsighted? Our next proposition shows that $Z(g, \theta, E)$ weakly decreases (increase) when a myopic (farsighted) player i becomes farsighted (myopic) while keeping the types of the other players (θ_{-i}) constant.

Let $Z(g, ((t_i^Q)_{Q=1}^r, m), \theta_{-i}, E)$ be the set of stable myopic-farsighted strategy profiles when player i is myopic and let $Z(g, ((t_i^Q)_{Q=1}^r, f), \theta_{-i}, E)$ be the set of stable myopic-farsighted strategy profiles when player i is farsighted. The type of the rest of the players does not change.

Proposition 3.
$Z(g, ((t_i^Q)_{Q=1}^r, f), \theta_{-i}, E) \subseteq Z(g, ((t_i^Q)_{Q=1}^r, m), \theta_{-i}, E).$

Proposition 3 implies that the set of stable profiles weakly decreases when a myopic player turns farsighted. In Section 5 we show that even though the set weakly decreases it will never become empty. One immediate conclusion from Proposition 3 is that, if players

i and j become farsighted, the order in which they become farsighted does not matter for the set of stable strategy profiles as illustrated in Figure 3.

$$Z(g, ((t_i^Q)_{Q=1}^r, m), ((t_j^Q)_{Q=1}^r, m), \theta_{-i,j}, E)$$

$$Z(g, ((t_i^Q)_{Q=1}^r, f), ((t_j^Q)_{Q=1}^r, m), \theta_{-i,j}, E) \quad Z(g, ((t_i^Q)_{Q=1}^r, m), ((t_j^Q)_{Q=1}^r, f), \theta_{-i,j}, E)$$

$$Z(g, ((t_i^Q)_{Q=1}^r, f), ((t_j^Q)_{Q=1}^r, f), \theta_{-i,j}, E)$$

Figure 3: Impact of farsightedness

From Proposition 3 we obtain the following corollary.

Corollary 2. *The set of myopic-farsighted stable strategy profiles when there are both myopic and farsighted players $Z(g, \theta, E)$ is a subset of the set of Nash equilibrium strategy profiles $\mathcal{N}(g, \theta, E)$: $Z(g, \theta, E) \subseteq \mathcal{N}(g, \theta, E)$.*

Thus, in the type of coordination games studied in the paper, the set of myopic-farsighted stable strategy profiles when players are both myopic and farsighted is a refinement of the set of Nash equilibrium strategy profiles.
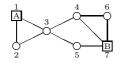
Example 3.

Figure 4: Network example

All players are myopic, two project (A and B) with $E^A = E^B = 1$. Stable strategy profiles:
1. $\{A, A, A, A, A, A, B\}$, 2. $\{A, A, A, A, B, A, B\}$,
3. $\{A, A, A, B, A, B, B\}$, 4. $\{A, A, A, B, B, B, B\}$,
5. $\{A, A, B, B, B, B, B\}$, 6. $\{A, B, B, B, B, B, B\}$.

Let us consider the network we have already used in previous examples and which is shown in Figure 4. We consider only two project A and B and thresholds $E^A = E^B = 1$. If all players are myopic the profiles shown on the right are stable. Hence, the set of Nash equilibria contains exactly those six profiles.

Turning any of the players 1 to 5 into farsighted players, does not change the stable set at all. On the other hand, if player 6 becomes farsighted, the first two profiles are not stable any more. In those profiles player 6 deviates from project A to project B and takes into account that player 4 then will change to B as well (as explained in Example 2).

Even when all players are farsighted the profiles 3 to 6 remain stable.

4.2 Impact of Extremism

In the previous results we have assumed that the taste of the players is fixed. In the examples, there was one extremist for project A, one extremist for project B and the

remaining players were moderate. We can clearly observe that the set of myopic-farsighted stable strategy profiles does not only depend on the farsightedness of the players, but also on their taste.

Let $Z(g, ((t_i^Q = 1/r)_{Q=1}^r, s_i), \theta_{-i}, E)$ be the set of myopic-farsighted stable strategy profiles when player i is moderate, and $Z(g, ((t_i^Q = 1, (t_i^R = 0)_{R \neq Q}), s_i), \theta_{-i}, E)$ be the set of myopic-farsighted stable strategy profiles when player i is an extremist for project Q.

Proposition 4.

$$Z(g, ((t_i^Q = 1/r)_{Q=1}^r, s_i), \theta_{-i}, E) \subseteq \bigcup_{Q=1}^m Z(g, ((t_i^Q = 1, (t_i^R = 0)_{R \neq Q}), s_i), \theta_{-i}, E)$$

Proposition 4 can be interpreted as follows. If choosing a project Q is part of a stable strategy profile p when player i is moderate, then the strategy profile p is still stable once player i becomes an extremist for project Q, while keeping constant the type of the other players. If there exists no stable strategy profile in which player i, while being moderate, choose project Q, profiles become stable when player i is an extremist for project Q.

Example 4. Let us consider the same network structure as in the previous examples and let us assume that all players are myopic and thresholds are low, i.e. $E^A = E^B = 1$.

In the case that all players are moderates, there are 8 different stable strategy profiles: $p_A^1 = \{A, A, A, B, A, B, B\}$, $p_A^2 = \{A, A, A, B, B, B, B\}$, $p_A^3 = \{A, A, B, B, B, B, B\}$, $p_A^4 = \{A, A, A, A, A, A, A\}$ and $p_B^1, \dots, p_B^4$ which are the same profiles with the actions A and B exchanged.

Turning player 7 into an extremist for project B keeps only those profiles stable in which player 7 chooses project B, namely: p_A^1, p_A^2, p_A^3 and p_B^4. In addition there are new stable strategy profiles: $p^5 = \{A, A, A, A, A, A, B\}$, $p^6 = \{A, A, A, A, B, A, B\}$ and $p^7 = \{B, B, A, A, A, A, B\}$. We see that the profiles p^5 and p^7 are modifications from the profiles p_A^4 respectively p_B^3: The actions of the players 1 to 6 in profile p^5 (p^7) are the same as in p_A^4 (p_B^3) only the action of player 7 is different. In the strategy profile p^6 there is a deviation from at least two players compared to the profiles p_A^1 to p_A^4 and p_B^1 to p_B^4.

If, in addition, we make player 1 an A-extremist we get the same six stable strategy profiles as in Example 3: Profiles $p_A^1, p_A^2, p_A^3, p^5, p^6$ and $p^8 = \{A, B, B, B, B, B, B\}$. Again, turning a player into an extremist makes some strategy profiles unstable, while it stabilizes others.

On the other hand turning a player into an extremist does not always create new stable strategy profiles:

Example 5. We use again the same network and thresholds as in the previous examples. As we have seen if all players are myopic and players 2 to 6 are moderate, while player 1 (7) is an extremist for A (B), there exist six stable strategy profiles (see Example 3). If we turn player 3 into an A-extremist only those four profiles where she choose project A remain stable. The same happens when we turn her into an extremist for project B: There are only two stable strategy profiles: Those in which player 3 invests into project B.

Proposition 5.

Assume the thresholds of all projects are low, i.e. $E^Q = 1$ for all $Q \in P$. If on all paths between two players i and j there is an extremist player, the choice of player i and j are independent of each other.

If we are analyzing the game when the thresholds are not important we can simplify our analysis with the help of Proposition 5. If there are two (or more) groups of players who are only connected to the other group(s) through paths that lead through at least one extremist we can split the graph into two (or more) components and analyze them separately. One example is illustrated in Figure 5. Players 1 and 6 are only connected through extremists (players 4 and 5), so instead of analyzing the stable profiles in g we can separately analyze g_1 and g_6.
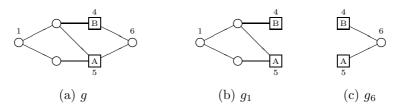

(a) g (b) g_1 (c) g_6

Figure 5: Illustration of Proposition 5.

4.3 Impact of the Thresholds

In addition to the characteristics of the players, the thresholds of the project have an impact on the stability of profiles. If we increase the threshold of project Q from E^Q to $E^Q + 1$ certain strategy profiles will not be stable, but other profiles can become stable. In profiles in which a project's threshold was exactly reached the increase in the threshold makes the project unsuccessful and the profile unstable.

Proposition 6.

$$Z(g, \theta, (E^Q + 1, (E^R)_{R \neq Q}))$$
$$\subseteq Z(g, \theta, E) \backslash \{p \in Z(g, \theta, E) : |\{i \in N : p_i = Q\}| = E^Q\}$$
$$\cup \{p : p_i = \emptyset \text{ if } t_i^Q = 1 \text{ and } p_i \in \bar{P} \backslash Q \text{ otherwise}\}$$

Proposition 6 shows that an increase of the threshold of project Q from E^Q to $E^Q + 1$ has the following effect: The new set of stable profiles consists of all those profiles that were stable before and where at least $E^Q + 1$ players chose project Q. In addition, the set might contain profiles in which no player chooses project Q and where the extremists for project Q abstain.

All profiles in $\{p \in Z(g, \theta, E) : |\{i \in N : p_i = Q\}| = E^Q\}$ can no longer be stable if the threshold is $E^Q + 1$. In those profiles the threshold for project Q was reached exactly, so an increase in the threshold makes project Q unsuccessful. In that case it is better for all players who invested into Q to choose another project or to abstain. These profiles

are described in the set $\{p : p_i = \emptyset$ if $t_i^Q = 1$ and $p_i \in \bar{P}\backslash Q$ otherwise$\}$. Notice that there might be some strategy profiles that are part of the first and second set.

Example 6. We use, once more, the network from Example 3 with only myopic players. In the case of low thresholds ($E^A = E^B = 1$) there are the six stable profiles as stated before:

$p^1 = \{A, A, A, A, A, A, B\}$, $p^2 = \{A, A, A, A, B, A, B\}$, $p^3 = \{A, A, A, B, A, B, B\}$,
$p^4 = \{A, A, A, B, B, B, B\}$, $p^5 = \{A, A, B, B, B, B, B\}$, and $p^6 = \{A, B, B, B, B, B, B\}$.

An increase of the threshold for project A to $E^A = 3$ (while keeping $E^B = 1$) makes all profiles unstable in which one or two players have selected project A. Only the profiles p^1 to p^4 remain stable. In p^5 player 2 will deviate to project B, while player 1 abstains. Player 1 will also change to abstention in profile p^6. This creates one new stable profile: $p^7 = \{\emptyset, B, B, B, B, B, B\}$.

An additional increase of E^B to $E^B = 2$, also makes profile p^1 unstable. Instead the profile $p^8 = \{A, A, A, A, A, A, \emptyset\}$ becomes stable. In addition there is another stable profile: The profile in which all players abstain: $p^9 = \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}$. This profile is stable, because all players are myopic. In this profile no player will deviate to a project, because the investment of a single player is not enough to make the project successful.

The previous example has shown that the profile in which all players abstain can be a stable strategy profile if the thresholds are high and there are no farsighted players. The following Proposition characterizes exactly when this profile is stable.

Proposition 7.
The strategy profile in which all players choose to abstain is stable if and only if for all project $Q \in P$ it holds:

$$|\{i \in F : t_i^Q > 0\}| + \min\left(1, |\{i \in M : t_i^Q > 0\}|\right) < E^Q$$

Proposition 7 gives a certain level of farsighted players that is required to ensure that the profile in which all players abstain is not stable. There is no improving path out of the profile in which all players abstain if there are not enough farsighted players in comparison to the thresholds. To have an improving path, there has to be at least one project Q for which there exist at least $E^Q - 1$ farsighted players that are not extremist for a different project. In addition, there either has to be one more player i (farsighted or myopic) that is not an extremist for a different project. Then, the improving path is as follows: First the $E^Q - 1$ farsighted players change their decision and choose project Q. Then, player i choose project Q as well and her decision makes the project successful. All these players receive a positive utility.

5 Existence of myopic-farsighted stable profiles

We have shown that the set of myopic-farsighted stable strategy profiles Z is the set of Nash equilibria if all players are myopic. Furthermore, we know from Proposition 3 that

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the cardinality of the set (weakly) decreases if we replace a myopic player by a farsighted player. This section shows that Z is never empty and we provide a method to find a myopic-farsighted stable strategy profile if all players are farsighted.

Starting from an arbitrary profile p, we can start writing the decision-making process of a player along an improving path as a tree. At each node one player decides whether she wants to stay with her current decision or wants to deviate to another action. As players can change their decision many times along the improving path, the decision order of the players can be neglected. The order only decides which profile we reach, but if there is an improving path from p when players decide in the order 1 to n, then there is also one for every other decision order. This is caused by the fact that players can repeatedly change their decisions. Still, it is not possible to illustrate an entire improving path as a tree, because there is a infinite number of nodes. Again, the reason is the possibility of repeated deviations from each player.

The following lemma allows us to write down the decision-making process as a finite tree if all players are farsighted.

Lemma 1. *Assume all players are farsighted. If there exists an improving path from profile p to profile p' then there also exists an improving path from p to p' in which each player changes her action at most once.*

Lemma 1 implies that instead of considering all improving paths, we can focus our attention on improving paths in which each player changes her action once or not at all. To find a myopic-farsighted stable strategy profile when all players are farsighted we state a way to formulate a game tree.

First, we need to fix a decision order, wlog. $(1, \ldots, n)$ and we select an arbitrary profile p^0. Then, we can start to create the decision tree. In the first node player 1 decides if she wants to stay with her action p_1^0 or if she wants to deviate to any other action. In case of indifference we assume she stays. If she deviates, we arrive in a new profile p^1 with $p_1^0 \neq p_1^1$ and $p_{-1}^0 = p_{-1}^1$. In that case we remove player 1 from the decision order and we start from the beginning of the updated order, in this case with player 2. Player 2 then faces the same choice, staying in p^1 or deviating. If she deviates we move to p^2, remove player 2 from the decision order and start from the beginning of the order. If player i does not deviate, but stays in profile p^x, we move to the next player in the decision order and letting her decide whether she wants to deviate from p^x. When player i does not change her decision, she stay in the decision order and can revisit her choice once another player has deviated. We reach a stable strategy profile either when we reach the end of the decision order and no player wanted to deviate or when there are no players left in the decision order. To illustrate this game tree, we introduce the following notation. Let o be the decision order of the players, e.g. $(1, \ldots, n)$ and $o \setminus i$ implies that player i is removed from the decision order, i.e. $o \setminus i = (1, \ldots, i-1, i+1, \ldots n)$. We generate the game tree for $\Gamma(p^0, o)$, i.e. starting with profile p^0 and order o as shown in Figure 6. $\Gamma(p^j, \emptyset)$ gives the utility $u_i(p^j)$ to player i.
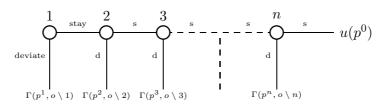
Figure 6: Game tree of $\Gamma(p^0, o)$

By construction, the finite game tree of $\Gamma(p^0, o)$ shows all the possible improving paths from p^0 in which each farsighted player deviates at most once. Together with Lemma 1 this yields the following result.

Corollary 3. *The subgame perfect Nash equilibrium of the game $\Gamma(p^0, o)$ as defined above is a myopic-farsighted stable profile.*

Corollary 3 implies that even if all players are farsighted there always exists at least one stable strategy profile. Starting from different profiles p^0 or using different orders o can result in different stable strategy profiles, but the main point of this result is to show that the set of myopic-farsighted stable profiles is non-empty independent of the players' types.

Example 7. Let us consider the network used in all previous examples and assume all players are farsighted. We can start from the profile in which all players choose project A, i.e. $p^o = \{A, A, A, A, A, A, A\}$. We use the decision order following the number of the players ($o = (1, \ldots, 7)$) and take thresholds $E^A = E^B = 1$. Starting from profile p^0 the players 1 to 6 have no incentive to deviate, but player 7 does. She is an extremist for project B and a profile in which she choose A can never be stable. She changes to project B and we enter game $\Gamma(\{A, A, A, A, A, A, B\}, (1, \ldots, 6))$. From this profile the players 1 to 5 again have no incentive to change their projects, but player 6 will deviate. She will select project B as well and even though this does not yield to an immediate improvement, she changes, because she takes into account that afterwards player 4 will also deviate to project B (in the game $\Gamma(\{A, A, A, A, A, B, B\}, (1, \ldots, 5))$). Those two changes bring us to the game $\Gamma(\{A, A, A, B, A, B, B\}, (1, 2, 3, 5))$. Neither of the players 1, 2, 3 or 5 have any incentive to change, so the profile is stable.

If we start from the profile $\{B, B, B, B, B, B, B\}$ only player 1 will deviate and the resulting profile is $\{A, B, B, B, B, B, B\}$, which is stable.

6 Results for specific networks

The stability of a strategy profile does not only depend on the thresholds and the type of the players, but also on the network structure. In this section we analyze the set of stable strategy profiles for the most common network structures: The complete network, the star network and line networks.

Complete network   In the case of the complete network, each player wants to coordinate with all other players. A moderate player wants to choose the project that gets chosen by the majority of the other players. This simplifies the analysis.

Proposition 8.

In all stable strategy profiles all moderate players choose the same project, i.e. for all $i, j \in Y : p_i = p_j$.

Proposition 8 confirms that the complete network structure leads to a complete coordination of the moderate players. There might be several stable profiles, but in all of them the moderate players choose the same action. It depends on the thresholds, the amount of extremists and farsighted players which profiles are contained in the set of stable profiles.

Star network   In a star network the center of the star plays an important role. Depending on the farsightedness and the taste of the player in the center, there can be different stable strategy profiles. Let player 1 be the center of the star and assume that project B is the project with most extremists. The following tables show the stable strategy profiles.

t_1	moderate				extremist for A (wlog)	
s_1	farsighted		myopic		myopic	
			$X_B > \frac{n-1}{2}$	$X_B \le \frac{n-1}{2}$		
	E^B low	E^B high	and E^B low	or E^B high		
p_1	B	R	B	R	A	$\emptyset$
$p_i, \, i \in Y$	B	R	B	R	A	$R \neq A$ or $\emptyset$
$p_i, \, t_i^Q = 1$	Q or $\emptyset$	Q or $\emptyset$	Q or $\emptyset$	Q or $\emptyset$	Q or $\emptyset$	Q or $\emptyset$

In addition to the profiles shown, in all cases the profile which all players abstain can be stable if the conditions from Proposition 7 holds. If there is a moderate, farsighted player in the center the choice of this player depends on the thresholds of the projects. She will select the project with the highest number of extremists that also has a sufficiently low threshold. If the project with most extremists (wlog project B) has a low threshold, she chooses this project and anticipates that all moderate players and all extremists for B will do the same. Through this choice she can coordinate with the highest possible amount of neighbors. If the threshold E^B is too high and project B is not successful player 1 chooses a different project R and the moderate periphery players do the same.

A myopic and moderate center behaves similarly under one condition: More than half of the periphery players have to be extremists for project B and the threshold of this project has to be sufficiently low. Then it is the best outcome for the center to choose project B. If at most half of the periphery players are extremists for B or if the threshold of project B is too high other stable strategy profiles may exist. In those the player in the center choose one project R and all other moderate players do the same. The myopic player in the center is not able to anticipate that the moderate players will always follow

her choice. If the center has selected project A and more of the periphery players chose A than B, the center has no incentive to deviate from her decision.

If the center is an extremist for project A (wlog), there can be only two possible types of myopic-farsighted stable strategy profiles. Either the center and all moderate players choose project A (while extremists for different projects abstain or invest into the project they are extremists for), or the center (and all other extremists for project A) abstains, all moderate players choose a project $R \in \bar{P} \backslash A$ and all extremists for project $Q \neq A$ either abstain or choose project Q. In the case where the center abstains, no periphery player can achieve any coordination. This implies that each periphery player i chooses a project R which she likes (i.e. $t_i^R > 0$) and that is successful. If no project is successful, all players abstain.

Line networks In the line network the positions of players can be sorted into two categories: Loose-ends (players 1 and n) and not loose-ends, i.e. players with two neighbors. We already know that in stable profiles moderate loose-ends will choose the same project as their neighbor as long as the neighbor does not abstain (Remark 1). On the other hand, the in-between players can either coordinate with one or two of their neighbors or with none.

When all players are myopic there exist many different stable profiles. In case of low thresholds, i.e. $E^Q = 1$ for all $Q \in P$, the line can be fragmented into groups of size 2 or larger. Players in each fragment choose the same project. If we consider a fragment of two moderate and myopic players, just turning one of them farsighted will increase the coordination. The farsighted player will choose the same project as her other neighbor (from a different fragment) and the myopic player will follow. In a fragment of 3 players we need to turn at least two of them farsighted in order to make the players in the fragment coordinate with players from a neighboring fragment. Similarly, if the fragment consist of x players, at least $x - 1$ players have to become farsighted to achieve coordination with the neighboring fragments. The following Proposition provides conditions for full coordination.

Proposition 9.
Consider a line network with only moderate players. If there is at most one myopic player in $\{2, \ldots, n-1\}$ then, there are only r stable strategy profiles (in which all players choose the same action). The opposite direction of the result ("only if") holds if the thresholds for all projects are low enough, e.g. $E^Q = 1$ for all $Q \in P$.

7 Social Planner

We have seen in the previous sections that the stable strategy profiles can vary in the amount of successful projects, in the amount of abstaining players and in the coordination

between neighbors. From a social planner's point of view there might be interest to let the players coordinate as much as possible.

This section discusses two different types of social planner, shows how he can influence the players choices and yields to the result that the social planner cannot always use the same receipt for success.

We return to our motivating examples from the introduction. In the case of adoption of a new technology we can clearly see that a social planner has incentives to intervene. For example when cities choose a way to make their public transport more eco-friendly the social planner can have one of the following two interests:

1. He wants to maximize the inter-city transport, i.e. maximize the investment in one project. In that case electric buses from city 1 can go to city 2 and be charged there. This avoids pollution and additionally to the eco-friendly intra-city transport, this increases the benefits for the environment.

2. He wants all cities to invest, i.e. minimize the abstention. More cities will provide eco-friendly public transport and the pollution decreases.

We give the social planner three different instruments to achieve his goal: He can change the degree of farsightedness, the taste of a player or modify the threshold of a project.

Maximize investment in one project A social planner who wants to maximize the investment in a single projects tries to maximize the following function: $U(p) = \max_{Q \in P} |\{i \in N : p_i = Q\}|$.

We return to our analysis of Example 6 with the same types of players and thresholds $E^A = 3$ and $E^B = 2$. There are six different stable strategy profiles. For simplicity we rename them: $p^1 = \{A, A, A, A, A, A, \emptyset\}$, $p^2 = \{\emptyset, B, B, B, B, B, B\}$, $p^3 = \{A, A, A, A, B, A, B\}$, $p^4 = \{A, A, A, B, A, B, B\}$, $p^5 = \{A, A, A, B, B, B, B\}$ and $p^6 = \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}$. We get $U(p^1) = U(p^2) = 6$, $U(p^3) = 5$, $U(p^4) = U(p^5) = 4$ and $U(p^6) = 0$. We denote the profile in which all players select A (B) by p^A (p^B).

The profiles p^1 and p^2 are the best stable profiles for the social planner, because all moderate players coordinate on a project. To move to full coordination, i.e. to p^A or p^B the social planner has to turn one of the extremists into a moderate player. From profile p^3 the social planner can improve either by increasing the threshold of project B to $E^B = 3$ (moving to profile p^1) or by turning player 7 into a moderate player (yielding in profile p^A). The difficulty arises in p^4. Changing the degree of farsighted or taste of one player or increasing or decreasing the threshold of one project by 1 does not destabilize this profile. Either several changes are necessary or for example an increase of E^B to 4 (moving to profile p^1). Even turning player 7 moderate will not destabilize the profile. To move from p^5 to a preferred profile (p^2) the social planner can increase the threshold of project A to $E^A = 4$. As stated in Proposition 7 the profile p^6 is stable if there are too few farsighted players relative to the thresholds of the project. Either turning any player except player 1 farsighted or lowering E^B to 1 would make the players move to profile p^2.

We see that different instruments can be used to destabilize certain profiles. Moreover destabilizing one profile might make the destabilization of another profile more difficult. In order to have a set of stable strategy profiles that only consist of p^1 and p^2, the social planner has to use several instruments. For example, he could increase the thresholds of both project to $E^A = E^B = 4$ and turn three players farsighted. Full coordination can only be reached when there do not exist two players that are extremists for different projects.

Finally, notice that making players farsighted does not always benefit the social planner. When all players are myopic and the planner makes player 6 farsighted, the profile p^1 is destabilized and the players move to profile p^4.

Minimizing abstentions   If the social planner wants to minimize the amount of players who abstain, he tries to minimize: $V(p) = |\{i \in N : p_i = \emptyset\}|$. We use the same example as before and see that in the profiles p^3, p^4 and p^5 there is no abstention, which is the most preferred outcome of the planner, i.e. $V(p^3) = V(p^4) = V(p^5) = 0$. In p^1 and p^2 one player abstains ($V(p^1) = V(p^2) = 1$), while p^6 is the worst for the social planner with $V(p^6) = 6$.

To move from p^1 the social planner can either turn player 6 farsighted and so end up in p^4, or turn player 7 into a moderate which yields to p^A. On the other hand, from p^2 there is no way out besides turning player 1 moderate (or reducing E^A to 1). For p^6 the possibilities for this social planner are the same as for the other type of social planner.

This example shows that it is hard to avoid abstention as long as there is a small amount of extremists for different projects. Reducing the thresholds of all projects to 1 is a very effective instrument. Then no player has an incentive to abstain. Returning to our example of cities and eco-friendly transport, this is equivalent to subsidizing all projects. This might be too expensive for the social planner and in that case other interventions, as described above, are more suitable.

8   Discussion

We have proposed a model to generalize coordination games on networks. Players want to coordinate with as many neighbors in their network as possible. In addition, players not only differ in their taste between the different projects, they also can be either myopic or farsighted. To analyze this game, we introduced a new stability concept. The set of stable strategy profiles is equivalent to the set of Nash equilibria as long as all players are myopic. Introducing farsighted players, i.e. players who anticipate changes that follow because of their actions, works as a refinement of the set. We have shown that even if all players are farsighted there exists at least one stable strategy profile. Furthermore, we have characterized the changes in the set of stable strategy profiles that arise when the taste of a player is changed.

The different projects, that the players can choose between, also add a novelty to the setting: Each project has a threshold which needs to be reached to make it successful. Our model allows for ignoring that threshold, i.e. setting all thresholds to 1, but the majority of our results hold for any level of thresholds. We have shown and illustrated how the thresholds influence the set of stable strategy profiles.

An important part in our model is the network. Players like to coordinate with their neighbors in the network. We have analyzed classical network examples such as the complete network or the star network.

We have discussed the role of a social planner and analyzed two different incentives for him. A social planner who wants to maximize investment into one project will behave differently than a social planner who wants to minimize abstention. Changing the type of a player or the threshold of a project can be good for one type of social planner and bad for the other one.

Appendix

For the proof of Proposition 1 we require two lemmas:

Lemma A1. *Consider a strategy profile p with $p_i \neq p_j$ for some $i \neq j$. If at p, player i prefers p'_i to p_i (i.e., $u_i(p'_i, p_{-i}) > u_i(p_i, p_{-i})$) and at (p'_i, p_{-i}), player j prefers p'_j to p_j (i.e., $u_j(p'_i, p'_j, p_{-i,j}) > u_i(p'_i, p_{-i})$) with $p'_j \neq p'_i$, then it also holds that at p, player j prefers p'_j to p_j (i.e., $u_j(p'_j, p_{-j}) > u_j(p_j, p_{-j})$).*

Proof. We prove that $u_j(p'_j, p_{-j}) \geq u_j(p'_i, p'_j, p_{-i,j}) > u_j(p'_i, p_{-i}) \geq u_j(p_j, p_{-j})$ holds.
$u_j(p'_j, p_{-j}) \geq u_j(p'_i, p'_j, p_{-i,j})$ follows directly from the fact that $p'_i \neq p'_j$. If p_i is equal to p'_j, then the inequality is strict, otherwise both terms are the same, because player i's choices (p_i and p'_i) both differ from p'_j.
It remains to be shown that $u_j(p'_i, p_{-i}) \geq u_j(p_j, p_{-j})$ holds. We use the same arguments as before: p'_i might be equal to p_j, but p_i is never equal to p_j. $\square$

The following lemma follows almost immediately and is presented without proof.

Lemma A2. *Consider a strategy profile p from which player i wants to deviate to p'_i and some other player(s) j, $i \neq j$, wants to deviate to p'_j, with $p'_j \neq p'_i$; i.e., $u_i(p'_i, p_{-i}) > u_i(p_i, p_{-i})$ and $u_j(p'_j, p_{-j}) > u_j(p_j, p_{-j})$. Once the deviations have taken place, if player i deviates again to p_i (because $u_i(p_i, p'_j, p_{-i,j}) > u_i(p'_i, p'_j, p_{-i,j})$), then player j will never deviate back to p_j (since $u_j(p_i, p'_j, p_{-i,j}) > u_j(p_i, p_j, p_{-i,j})$).*

Proof of Proposition 1. Suppose there exists a myopic-farsighted improving path $p^0, p^1, ..., p^L$ with $p = p^0$ and $p = p^L$. First, notice that no farsighted player will initiate such a path. Only myopic players could move away from p. Let i_1 be the myopic player that deviates from p^0 to p^1, with $u_{i_1}(p^1_{i_1}, p^0_{-i_1}) > u_{i_1}(p^0_{i_1}, p^0_{-i_1})$. Consider now the move of player i_2 from p^1 to p^2. If $p^2_{i_2} = p^1_{i_1}$, then player i_1 will not deviate back to $p^0_{i_1}$ since the

support for the project she prefers is now even larger. Thus, as long as, along the path, the successive deviating players continue supporting projet $p^1_{i_1}$, player i_1 will not deviate back to $p^0_{i_1}$ and then the end strategy profile of the improving path will never be equal to p.

Let l be the first time along the path such that some myopic player j deviates from p^l to p^{l+1}, with $p^{l+1}_j \neq p^1_{i_1} = ... = p^l_{i_l}$ and $u_j(p^{l+1}_j, p^1_{i_1}, ..., p^l_{i_l}, p^0_{-j,i_1,...,i_l}) > u_j(p^1_{i_1}, ..., p^l_{i_l}, p^0_{-i_1,...,i_l})$. Then by Lemma A1, we also have that player j prefers p^{l+1}_j to $p^0_j = p^1_j = ... = p^l_j$ before players $i_1, ..., i_l$ deviates from $p^0_{i_1}, ..., p^{l-1}_{i_l}$ to $p^1_{i_1} = ... = p^l_{i_l} \neq p^{l+1}_j$. It holds that:

$u_j(p^{l+1}_j, p^1_{i_1}, ..., p^{l-1}_{i_{i-1}}, p^l_{i_l}, p^0_{-j,i_1,...,i_l}) > u_j(p^1_{i_1}, ..., p^l_{i_l}, p^0_{-i_1,...,i_l})$,
$u_j(p^{l+1}_j, p^1_{i_1}, ..., p^{l-1}_{i_{l-1}}, p^0_{-j,i_1,...,i_{l-1}}) > u_j(p^1_{i_1}, ..., p^{l-1}_{i_{l-1}}, p^0_{-i_1,...,i_{l-1}})$,
...,
$u_j(p^{l+1}_j, p^0_{-j}) > u_j(p^0_j, p^0_{-j})$.

Notice that, in order for the path to end at p, players $i_1, ..., i_{l-1}, i_l$ should have incentives to deviate back from $p^1_{i_1} = ... = p^l_{i_l}$ to $p^0_{i_1}, ..., p^{l-1}_{i_l}$, respectively. Thus, assume that this is the case once player j (and possibly other players that deviate later on to the same project as player j) deviates from p^l to p^{l+1}, with $p^{l+1}_j \neq p^1_{i_1} = ... = p^l_{i_l}$. But then, by Lemma A2, we have that player j (and possibly some other player) will never deviate back to $p^0_j \neq p^{l+1}_j$. Hence, p will never be reached.

Suppose now that, at l, a farsighted player j deviates from p^l to p^{l+1}, with $p^{l+1}_j \neq p^1_{i_1} = ... = p^l_{i_l}$ and $u_j(p^L_j, p^L_{-j}) > u_j(p^1_{i_1}, ..., p^l_{i_l}, p^0_{-i_1,...,i_l})$. Then, in order to induce player i_1 to deviate back to $p^0_{i_1}$, two cases have to be considered. First, if $p^0_j = p^1_{i_1} = ... = p^l_{i_l} \neq \emptyset$, we have that

$$u_j(p^0_j, p^0_{-j}) \leq u_j(p^1_{i_1}, p^0_j, p^0_{-j,i_1}) \leq ... \leq u_j(p^1_{i_1}, ..., p^l_{i_l}, p^0_j, p^0_{-j,i_1,...,i_l}) < u_j(p^L_j, p^L_{-j}) = u_j(p^0_j, p^0_{-j}),$$

a contradiction.

Second, if $p^0_j = \emptyset$ or $p^0_{i_1}, ..., p^0_{i_l} \neq p^0_j \neq p^1_{i_1} = ... = p^l_{i_l}$ then we have that

$$u_j(p^0_j, p^0_{-j}) = u_j(p^1_{i_1}, p^0_j, p^0_{-j,i_1}) = ... = u_j(p^1_{i_1}, ..., p^l_{i_l}, p^0_j, p^0_{-j,i_1,...,i_l}) < u_j(p^L_j, p^L_{-j}) = u_j(p^0_j, p^0_{-j}),$$

a contradiction. Thus, no farsighted player along the path from p to $p^L = p$ would be the first player deviating from p^l to p^{l+1}, with $p^{l+1}_j \neq p^1_{i_1} = ... = p^l_{i_l}$. $\qquad \square$

Proof of Proposition 2. We start with the "if" part of the proof and show that the set of Nash equilibrium strategy profiles $\mathcal{N}(g, \theta, E)$ is a set of myopic-farsighted stable strategy profiles $Z(g, \theta, E)$.

Let some $p^* \in \mathcal{N}(g, \theta, E)$ be given. It holds by definition of the Nash equilibrium that $h(p^*) = \emptyset$. Then, $p^* \in \mathcal{N}(g, \theta, E)$ is myopic-farsighted stable.

We continue with the "only if" part of the proof and show that $\mathcal{N}(g, \theta, E)$ is the only set of myopic-farsighted strategy profiles $Z(g, \theta, E)$. Let some $p \notin \mathcal{N}(g, \theta, E)$ be given. Then, by Corollary 1, we have that there exists a Nash equilibrium strategy profile $p^* \in \mathcal{N}(g, \theta, E)$ such that $p^* \in h(p)$. Thus, a strategy profile that is no Nash equilibrium

cannot be myopic-farsighted stable. □

Proof of Proposition 3. Proof by contradiction:
Assume there is a strategy profile $z \in Z(g, ((t_i^Q)_{Q=1}^r, f), \theta_{-i}, E)$ and that $z \notin Z(g, ((t_i^Q)_{Q=1}^r, m), \theta_{-i}, E)$. This means that there is another profile $p = (p_i, p_{-i})$ with $p_{-i} = z_{-i}$ such that $u_i(p) > u_i(z)$. So that the deviation from z_i to p_i is an immediate improvement for player i. Then, there is a myopic-farsighted improving path of length 1 from z to p when player i is myopic, with $p \in h(z)$.

But then, the previous myopic-farsighted improving path of length 1 from z to p is also a myopic-farsighted improving path of length 1 from z to p when player i is farsighted. This implies that z is not stable if i is farsighted and contradicts our assumption. □

Proof of Proposition 4. Let us consider all the strategy profiles $p \in Z(g, ((t_i^Q = 1/r)_{Q=1}^r, s_i), \theta_{-i}, E)$ with $p_i = \emptyset$. The only reason why player i chooses to abstain is that no project is successful. These strategy profiles are still stable when player i becomes an extremist for any of the projects.

In the remaining profiles of $Z(g, ((t_i^Q = 1/r)_{Q=1}^r, s_i), \theta_{-i}, E)$ player i does not abstain. Let us consider a strategy profile $p \in Z(g, ((t_i^Q = 1/r)_{Q=1}^r, s_i), \theta_{-i}, E)$ where $p_i = Q$. This strategy profile can only be stable if project Q is successful because, otherwise, player i would abstain. This implies $p \in Z(g, ((t_i^Q = 1, (t_i^R = 0)_{R \neq Q}), s_i), \theta_{-i}, E)$. Repeating the same arguments for any other strategy profile where the moderate player i chooses some of the different projects completes the proof. □

Proof of Proposition 5. Let ℓ be an extremist. Wlog assume that $i, j \in N_\ell$ and there exists no other path between i and j (otherwise repeat the same arguments). Clearly p_ℓ is independent of p_i and p_j, because ℓ is an extremist and always choose the same project. This implies that p_i and p_j are independent of each other, that $p_s \in N_i$ is independent of p_j, that $p_t \in N_j$ is independent of p_i and so on. □

Proof of Proposition 6. We show how an increase of the threshold E^Q to $E^Q + 1$ can influence the set of stable strategy profiles. We have to differ between three cases: First, we characterize the profiles that are stable when the threshold is E^Q and that are also stable for the threshold $E^Q + 1$. Second, to compare the two sets of stable profiles for the thresholds E^Q and $E^Q + 1$, we have to remove those profiles from $Z(g, \theta, (E^Q, (E^R)_{R \neq Q}))$ that are not stable after increasing the threshold. Third, we have to add new profiles that are stable under the new threshold $E^Q + 1$ but not stable when the threshold is E^Q.
1. and 2. Let us consider $\bar{p} \in Z(g, \theta, (E^Q, (E^R)_{R \neq Q}))$. If we increase the threshold of project Q by one, there are two possible results. Either the success of all projects (including Q) stays unchanged or project Q is no longer successful. In the first case, the profile $\bar{p}$ is still stable, because the investment thresholds do not influence the utility, only the success of the projects do. In the second case, there is not enough investment into project

Q. Then for all players who has chosen that project it is better to abstain or invest into project $R \neq Q$, so there exists an improving path and $\bar{p}$ is not stable. This can only happen if before the increase of the threshold exactly E^Q players have chosen project Q. From this we get $Z(g, \theta, E) \setminus \{p \in Z(g, \theta, E) : |\{i \in N : p_i = Q\}| = E^Q\}$.

3. We show that a profile that is stable after the increase of the threshold, but was not stable before the increase is an element of $\{p : p_i = \emptyset$ if $t_i^Q = 1$ and $p_i \in \bar{P} \setminus Q$ otherwise$\}$. It is impossible that project Q (with threshold $E^Q + 1$) is successful, because than $\bar{p} \in Z(g, \theta, (E^Q + 1, (E^R)_{R \neq Q}))$ implies $\bar{p} \in Z(g, \theta, (E^Q, (E^R)_{R \neq Q}))$ and $\bar{p}$ would be stable under the lower threshold E^Q as well. So the only possibility is that project Q is not successful and that no player will choose project Q. The extremists for project Q will abstain and all other players will either select a project different than Q or abstain as well. The set of these profiles is $\{p : p_i = \emptyset$ if $t_i^Q = 1$ and $p_i \in \bar{P} \setminus Q$ otherwise$\}$. $\square$

Proof of Proposition 7. **If:** Assume for all projects $Q \in P$ it holds that $|\{i \in F \mid t_i^Q > 0\}| + \min\left(1, |\{i \in M \mid t_i^Q > 0\}|\right) < E^Q$. This implies that for no project Q there are neither enough farsighted players (with positive taste for project Q) such that they can either reach the threshold E^Q nor are there enough farsighted players to reach the level $E^Q - 1$ and one myopic player that has positive taste for project Q. So the farsighted players will not start an improving path from $p = \{\emptyset, \emptyset, ..., \emptyset\}$. Clearly, the myopic players will not deviate either. This implies $h(\{\emptyset, \emptyset, ..., \emptyset\}) = \emptyset$ and $p = \{\emptyset, \emptyset, ..., \emptyset\}$ is stable.

Only if: Proof by contradiction: Assume that there is a project $R \in P$ with $|\{i \in F \mid t_i^R > 0\}| + \min\left(1, |\{i \in M \mid t_i^R > 0\}|\right) \geq E^R$. There exists the following improving path from $p = \{\emptyset, \emptyset, ..., \emptyset\}$ to p' with $p_i' = R$ for all players i with $t_i^R > 0$. First all farsighted players (with positive taste for project R) will deviate from $p_i = \emptyset$ to $p_i' = R$. By assumption there are at least $E^R - 1$ such players. Once $E^R - 1$ farsighted players have selected project R, another player with $t_i^R > 0$ will deviate to project R as well and make project R successful. This can either be a myopic player or a farsighted player. The existance of this improving path implies that $p = \{\emptyset, \emptyset, ..., \emptyset\}$ cannot be in this case. $\square$

Proof of Lemma 1. Follows from Lemma A1 and the fact that the utility of player i who picks project p_i is the same when her neighbor invests into project $p_j \neq p_i$ or $p_j' \neq p_i$. $\square$

Proof of Proposition 8. We show by contradiction that there cannot be some moderate players that choose project Q and some moderate players that choose project R. The proof is easily replicable to show the same result for three or more projects. Let us assume there exist moderate players $r_1, \ldots, r_z$ who choose project R and players $q_1, \ldots, q_y$ who choose project $Q \neq R$.

It is obvious that players deviate if the threshold of either project Q or R is not reached. For the remainder we assume that both projects are successful.

Wlog, assume that $\mathcal{Q} = |X_Q \cup \{q_1, \ldots, q_z\}| \geq |X_R \cup \{r_1, \ldots, r_y\}| = \mathcal{R}$. Notice that, in the

complete network, for all $i \in \{r_1, \ldots, r_y\}$: $|\{j \in N_i(g) \mid p_j = Q\}| > |\{j \in N_i(g) \mid p_j = R\}|$. This implies that $u_i(p_i = Q) > u_i(p_i = R)$. Since $\mathcal{Q} > \mathcal{R}-1$ it follows that a myopic player $i \in \{r_1, \ldots, r_y\}$ will deviate to project Q. Farsighted players anticipate that if they choose Q, the moderate players will follow them and so they know that $\mathcal{Q} = N-1-|X_R| > \mathcal{R}-1$ holds and they will change to project Q as well. $\qquad\square$

Proof of Proposition 9. **If:** Let us assume that player $i \in \{2, \ldots, n-1\}$ is myopic and players $j \neq i \in \{2, \ldots, n-1\}$ are farsighted. Obviously the strategy profiles in which all players choose the same action are stable. We show that no other strategy profile can be stable:

Case q: Let us assume that two farsighted neighbors j, k (with $k = j+1$) choose different projects and all $\ell < j$ choose project p_j and all $\ell > k$ choose p_k. Wlog we can assume that $i > k$, i.e. $p_i = p_k$. Then player j has an incentive to change her decision and to select project p_k. If player j changes her project, then player $\ell = j - 1$ changes her project, because she knows that her neighbor $m = j-2$ will change. Player $j - x$ changes, because the farsighted player 2 who is the neighbor of the loose end knows that if she changes her decision the loose end will follow.

Case 2: Let us (wlog) assume that player i chooses project Q and all players located to her left do the same, i.e. $\forall j \in N$ with $j < i : p_j = p_i = Q$ while all other players choose R. In this case the neighbor of player i who chooses a different project than i, namely player $i + 1$ (with $p_{i+1} = R$) has an incentive to deviate. Player $i + 1$ changes because all other players $i + 2$ to $n - 1$ will change. These players change, because they anticipate that player n (the loose-end) will change as well.

So under these assumptions there can be no other stable strategy profile.

Only if (for $E^Q = 1$ for all $Q \in P$): We assume there are two myopic players i, j in $\{2, \ldots, n-1\}$.

1. Case: Assume i and j are neighbors, i.e. $j = i + 1$. Then there exists a stable strategy profile in which all players $k \leq i$ choose project Q and all players $\ell \geq j = i + 1$ choose project $R \neq Q$. In that case the farsighted players 2 to $i - 1$ and player 1 (whose degree of farsightedness does not matter) chose Q and the farsighted players $j + 1$ to $n - 1$ and player n chose project R. This implies that neither of the players 1 to $i - 1$ and $j + 1$ to n has any incentive to deviate. The two myopic players have also no incentive to change, because their utility after changing would be the same as before.

Case 2: Assume i and j are not neighbors, wlog we assume $i < j$.

We show that the following strategy profile is stable: $\forall k < j : p_k = Q$ and $\forall k \geq j : p_k = R$. In this strategy profile there are some farsighted players between i and j who choose project Q. The myopic player j has no incentive to deviate to project Q, because one of her neighbors chooses Q and the other chooses R. The farsighted player $j - 1$ would choose project R only if $j - 2$ chooses project R as well. Player $j - 2$ would deviate to project R only if $j - 3$ deviates to R as well, continuing this arguments leads to the problem that $i + 1$ only chooses project R if i deviates to R. Since i is myopic she does

not deviate, because even if $i+1$ choose R, she still get the same utility from the projects Q and R. So the strategy profile described is stable. $\qquad\square$

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