Network Formation with Myopic and Farsighted Players

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June 2, 2018

Abstract

We study the formation of networks where myopic and farsighted individuals decide with whom they want to form a link, according to a distance-based utility function that weighs the costs and benefits of each connection. We propose the notion of myopic-farsighted stable set to determine the networks that emerge when some individuals are myopic while others are farsighted. A myopic-farsighted stable set is the set of networks satisfying internal and external stability with respect to the notion of myopic-farsighted improving path. In the case of a homogeneous population (either all myopic or all farsighted), a conflict between stability and efficiency is likely to arise. But, once the population becomes mixed, the conflict vanishes if there are enough farsighted individuals. In addition, we characterize the myopic-farsighted stable set for any utility function when all individuals are myopic.

Key words: networks; stable sets; myopic and farsighted players; distance-based utility.

JEL Classification: A14, C70, D20.

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1 Introduction

The organization of individuals into networks plays an important role in the determination of the outcome of many social and economic interactions. For instance, a communication or friendship network in which individuals have very few acquaintances with whom they share information will result in different employment patterns than one in which individuals have many such acquaintances. A central question is predicting the networks that individuals will form. Up to now, it has been assumed that all individuals are either myopic or farsighted when they decide with whom they want to link. Jackson and Wolinsky (1996) propose the notion of pairwise stability to predict the networks that one might expect to emerge in the long run. A network is pairwise stable if no individual benefits from deleting a link and no two individuals benefit from adding a link between them. Pairwise stability presumes that individuals are myopic: they do not anticipate that other individuals may react to their changes. Farsighted individuals may not add a link that appears valuable to them as this can induce the formation of other links, ultimately lowering their payoffs.[1] However, recent experiments provide evidence in favor of a mixed population consisting of both myopic and farsighted individuals (see Kirchsteiger, Mantovani, Mauleon and Vannetelbosch, 2016).

We reconsider Bloch and Jackson (2007) model of network formation where individuals decide with whom they want to form a link, according to a distance-based utility function that weighs the costs and benefits of each connection. Benefits of a connection decrease with distance in the network, while the cost of a link represents the time an individual must spend with another individual for maintaining a direct link. Adding a link requires the consent of both individuals, while deleting a link can be done unilaterally. We now allow the population of individuals to include not only myopic individuals but also farsighted ones. Farsighted individuals are able to anticipate that once they add or delete some links, other individuals could add or delete links afterwards.

We propose the notion of myopic-farsighted stable set to determine the networks that emerge when some individuals are myopic while others are farsighted. A myopic-farsighted stable set is the set of networks satisfying internal and external stability with respect to the notion of myopic-farsighted improving path. When all individuals are farsighted, the definition of a myopic-farsighted stable set boils down to the farsighted stable set.[2]

We focus on the range of costs and benefits such that a star network is the unique strongly efficient network.[3] When all individuals are myopic, Jackson (2008) shows that

[1]Mauleon and Vannetelbosch (2016) provide a comprehensive overview of the (myopic and farsighted) solution concepts for solving network formation games.

[2]See Chwe (1994), Herings, Mauleon and Vannetelbosch (2009), Mauleon, Vannetelbosch and Vergote (2011), Ray and Vohra (2015, 2017), Roketskiy (2018) for definitions of the farsighted stable set. Alternative notions of farsightedness are suggested by Dutta, Ghosal and Ray (2005), Dutta and Vohra (2017), Herings, Mauleon and Vannetelbosch (2004, 2018), Page, Wooders and Kamat (2005), Page and Wooders (2009) among others.

[3]In cases of intermediate link costs relative to benefits, individuals obtain their highest possible payoff

a conflict between stability and efficiency is likely to occur. In addition, starting from the empty network, a random process where pairs of players meet to add or to delete links becomes unlikely to reach a star network as the number of players increases (see Watts, 2001; Jackson, 2008). When the population consists of both myopic and farsighted individuals, we show that the conflict between stability and efficiency vanishes if there are enough farsighted individuals. Indeed, the set consisting of all star networks where the center of the star is a myopic individual is the unique myopic-farsighted stable set. However, once all individuals become farsighted, every set consisting of a star network encompassing all players is a myopic-farsighted stable set, but there may be other myopic-farsighted stable sets. For instance, the set of circles among four farsighted players can be a myopic-farsighted stable set.

One can then conclude that diversity guarantees the emergence in the long run of the efficient outcomes. When all individuals are myopic or all individuals are farsighted, a conflict between stability and efficiency can occur. However, if the population is mixed, then the conflict disappears. Farsighted individuals try to avoid ending up in the central position of the star, and so, if all of them are farsighted, this can lead to a worse inefficient outcome. But, if some individuals are myopic, farsighted individuals are able to manipulate myopic individuals by placing them in positions where they have myopic incentives to move towards some star network where one of the myopic individual ends up being the center of the star. However, if there are too many myopic individuals with respect to farsighted ones, farsighted individuals may fail to engage a path from some inefficient network towards a star network.

In addition, we provide a general characterization (i.e. for any utility function) of the myopic-farsighted stable set when all individuals are myopic: a set of networks is a stable set if and only if it consists of all pairwise stable networks and one network from each closed cycle.

Another strand of the literature that was initiated by Bala and Goyal (2000) studies the formation of two-way flow networks where individuals unilaterally form costly links in order to access the benefits generated by other individuals. Benefits flow in both directions, irrespective of who pays the cost of the link.[4]

The paper is organized as follows. Section 2 introduces networks, myopic-farsighted improving paths, myopic-farsighted stable sets, and distance-based utility functions. Section 2 also characterizes the stable set when all individuals are myopic. Section 3 provides a characterization of the myopic-farsighted stable sets when the population consists of a mixture of myopic and farsighted individuals. Section 4 concludes and discusses directions

when they are the peripherals in a star network. The center of the star is worse off compared to the peripherals.

[4]In Galeotti, Goyal and Kamphorst (2006), individuals are heterogeneous with respect to benefits and costs of forming links. In Bloch and Dutta (2009), individuals choose how much to invest in each link. See also Hojman and Szeidl (2008) and Feri (2007) among others.

for future research.

2 Network formation

2.1 Modelling networks

We study networks where players form links with each other in order to exchange information. The population consists of both myopic and farsighted players. The set of players is denoted by $N = M \cup F$, where M is the set of myopic players and F is the set of farsighted players. Let n be the total number of players and $m \geq 0$ ($n - m \geq 0$) be the number of myopic (farsighted) players. A network g is a list of which pairs of players are linked to each other and $ij \in g$ indicates that i and j are linked under g. The complete network on the set of players $S \subseteq N$ is denoted by g^S and is equal to the set of all subsets of S of size 2. It follows in particular that the empty network is denoted by $g^{\emptyset}$. The set of all possible networks on N is denoted by $\mathcal{G}$ and consists of all subsets of g^N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \text{there is } j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g. Let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbors of player i in g. A star network is a network such that there exists some player i (the center) who is linked to every other player $j \neq i$ (the peripherals) and that contains no other links (i.e. g is such that $N_i(g) = N \setminus \{i\}$ and $N_j(g) = \{i\}$ for all $j \in N \setminus \{i\}$). A path in a network g between i and j is a sequence of players $i_1, \ldots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$. A network g is connected if for all $i \in N$ and $j \in N \setminus \{i\}$, there exists a path in g connecting i and j. A nonempty subnetwork $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$.[5]

A network utility function (or payoff function) is a mapping $U_i : \mathcal{G} \to \mathbb{R}$ that assigns to each network g a utility $U_i(g)$ for each player $i \in N$. A network $g \in \mathcal{G}$ is strongly efficient if $\sum_{i \in N} U_i(g) \geq \sum_{i \in N} U_i(g')$ for all $g' \in \mathcal{G}$.

2.2 Myopic-farsighted improving paths and stable sets

We propose the notion of myopic-farsighted stable set to determine the networks that emerge in the long run when some players are myopic while others are farsighted. A set of networks is a myopic-farsighted stable set if (internal stability) there is no myopic-farsighted improving path between networks within the set and (external stability) there is a myopic-farsighted improving path from any network outside the set to some network

[5] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, # will refer to the notion of cardinality.

within the set.[6]

A myopic-farsighted improving path is a sequence of networks that can emerge when farsighted players form or delete links based on the improvement the end network offers relative to the current network while myopic players form or delete links based on the improvement the resulting network offers relative to the current network. Since we only allow for pairwise deviations, each network in the sequence differs from the previous one in that either a new link is formed between two players or an existing link is deleted. If a link is deleted, then it must be that either a myopic player prefers the resulting network to the current network or a farsighted player prefers the end network to the current network. If a link is added between some myopic player i and some farsighted player j, then the myopic player i must prefer the resulting network to the current network and the farsighted player j must prefer the end network to the current network.[7]

Definition 1. A myopic-farsighted improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, \ldots, K-1\}$ either

(i) $g_{k+1} = g_k - ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $i \in M$ or $U_j(g_K) > U_j(g_k)$ and $j \in F$; or

(ii) $g_{k+1} = g_k + ij$ for some ij such that $U_i(g_{k+1}) > U_i(g_k)$ and $U_j(g_{k+1}) \geq U_j(g_k)$ if $i, j \in M$, or $U_i(g_K) > U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ if $i, j \in F$, or $U_i(g_{k+1}) \geq U_i(g_k)$ and $U_j(g_K) \geq U_j(g_k)$ (with one inequality holding strictly) if $i \in M, j \in F$.

If there exists a myopic-farsighted improving path from a network g to a network g', then we write $g \to g'$. The set of all networks that can be reached from a network $g \in \mathcal{G}$ by a myopic-farsighted improving path is denoted by $\phi(g)$, $\phi(g) = \{g' \in \mathcal{G} \mid g \to g'\}$. A set of networks G is a myopic-farsighted stable set if the following two conditions hold. Internal stability: for any two networks g and g' in the myopic-farsighted stable set G there is no myopic-farsighted improving path from g to g' (and vice versa). External stability: for every network g outside the myopic-farsighted stable set G there is a myopic-farsighted improving path leading to some network g' in the myopic-farsighted stable set G (i.e. there is $g' \in G$ such that $g \to g'$).

Definition 2. A set of networks $G \subseteq \mathcal{G}$ is a myopic-farsighted stable set if: **(IS)** for every $g, g' \in G$, it holds that $g' \notin \phi(g)$; and **(ES)** for every $g \in \mathcal{G} \setminus G$, it holds that $\phi(g) \cap G \neq \emptyset$.

When all players are farsighted, our notion of myopic-farsighted improving path reverts to Herings, Mauleon and Vannetelbosch (2009) notion of farsighted improving path, and the myopic-farsighted stable set is simply the farsighted stable set as defined in Herings, Mauleon and Vannetelbosch (2009) or Ray and Vohra (2015).

[6] Herings, Mauleon and Vannetelbosch (2017) define the myopic-farsighted stable set for two-sided matching problems.

[7] Along a myopic-farsighted improving path, myopic players do not care whether other players are myopic or farsighted, while farsighted players know exactly who is farsighted and who is myopic.

2.3 Only myopic players: a characterization

When all players are myopic, our notion of myopic-farsighted improving path reverts to Jackson and Watts (2002) notion of (myopic) improving path. Using it they define the notions of cycle and closed cycle. A set of networks C, form a cycle if for any $g \in C$ and $g' \in C$ there exists a (myopic) improving path connecting g to g'. A cycle C is a closed cycle if no network in C lies on a (myopic) improving path leading to a network that is not in C. A network $g \in \mathcal{G}$ is pairwise stable if (i) for all $ij \in g$, $U_i(g) \geq U_i(g - ij)$ and $U_j(g) \geq U_j(g - ij)$, (ii) for all $ij \notin g$, if $U_i(g) < U_i(g + ij)$ then $U_j(g) > U_j(g + ij)$. Let P be the set of pairwise stable networks. Lemma 1 in Jackson and Watts (2002) shows there always exists at least one pairwise stable network or closed cycle of networks. Starting from any network, either it is pairwise stable (and no improving path leaves it) or it lies on an improving path to another network. Either the network reached is pairwise stable or the improving path can be continued forever and ends up running into a closed cycle. Using Lemma 1 of Jackson and Watts (2002) we provide a general characterization of the (myopic-farsighted) stable set when all players are myopic. A set of networks is a (myopic-farsighted) stable set if and only if it consists of all pairwise stable networks and one network from each closed cycle.

Theorem 1. *Suppose that all players are myopic, $\#M = n$. Let $C^1, ..., C^r$ be the set of closed cycles. A set of networks $G \subseteq \mathcal{G}$ is a myopic-farsighted stable set if and only if $G = P \cup \{g^1, ..., g^r\}$ with $g^k \in C^k$ for $k = 1, ..., r$.*

Proof. We first show that any $G = P \cup \{g^1, ..., g^r\}$ with $g^k \in C^k$ for $k = 1, ..., r$ satisfies (**IS**) and (**ES**). Since all players are myopic, the set G satisfies (**IS**) by definition of a pairwise stable network and of a closed cycle; i.e. for every $g, g' \in G$ we have that $g \notin \phi(g')$. From Lemma 1 in Jackson and Watts (2002) we have that, for every $g \notin G$, $\phi(g) \cap G \neq \emptyset$, and so G satisfies (**ES**).

Suppose now that G is a (myopic-farsighted) stable set. First, $P \subseteq G$, otherwise, G would violate (**ES**). Second, $C^k \cap G \neq \emptyset$ for $k = 1, ..., r$, otherwise, G would violate (**ES**). Third, take any G, G' such that $G \supsetneq G' = P \cup \{g^1, ..., g^r\}$ with $g^k \in C^k$ for $k = 1, ..., r$. Then, from Lemma 1 in Jackson and Watts (2002) we have that there is $g, g' \in G$ such that $g \in \phi(g')$ and G violates (**IS**). □

2.4 Distance-based utility

Players directly communicate with the players to whom they are linked. They benefit not only from direct communication but also from indirect communication from the players to whom their neighbors are linked. But, the benefit obtained from indirect communication decreases with the distance. As in Bloch and Jackson (2007) or Jackson (2008), if player i is connected to player j by a path of t links, then player i receives a benefit of $b(t)$ from her indirect connection with player j. It is assumed that $b(t) \geq b(t + 1) > 0$ for any t, and so information that travels a long distance becomes diluted and is less valuable than information obtained from a closer neighbor. Each direct link $ij \in g$ results in a benefit $b(1)$ and a cost c to both i and j. This cost can be interpreted as the time a player must spend with another player in order to maintain a direct link. Player i's distance-based utility or payoff from a network g is given by

$$U_i(g) = \sum_{j \neq i} b(t(ij)) - \#N_i(g) \cdot c,$$

where $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j), $c \geq 0$ is a cost per link, and b is a nonincreasing function. The symmetric connections model ($b(t) = \delta^t$) and the truncated connections model of Jackson and Wolinsky (1996) are special cases of distance-based payoffs.[8]

Proposition 4 in Bloch and Jackson (2007) tells us that the unique strongly efficient network is (i) the complete network g^N if $c < b(1) - b(2)$, (ii) a star encompassing everyone if $b(1) - b(2) < c < b(1) + ((n-2)/2)b(2)$, and (iii) the empty network $g^{\emptyset}$ if $b(1) + ((n-2)/2)b(2) < c$. Are the strongly efficient networks likely to arise when all players are myopic?

Jackson (2008) characterizes the pairwise stable networks. He shows that a conflict between pairwise stability and efficiency is likely to occur except if link costs are small. For $c < b(1) - b(2)$, the unique pairwise stable network is the complete network g^N. For $b(1) - b(2) < c < b(1)$, a star encompassing all players is pairwise stable, but not necessarily the unique pairwise stable network. For $b(1) < c$, any pairwise stable network which is nonempty is such that each player has at least two links and thus is inefficient. Only for $c < b(1) - b(2)$, there is no conflict between efficiency and pairwise stability. When $b(1) - b(2) < c < b(1)$, the efficient network is pairwise stable, but there are other pairwise stable networks that are not efficient. For $b(1) < c < b(1) + ((n-2)/2)b(2)$, the efficient network is never pairwise stable. And, finally, for $b(1) + ((n-2)/2)b(2) < c$, the efficient network is pairwise stable, but there could be other pairwise stable networks that are not efficient.

Hence, from Theorem 1, the concept of myopic-farsighted stable set confirms that, for a large range of parameter values, a conflict between stability and efficiency is likely to occur when all players are myopic.

3 Characterization of myopic-farsighted stable sets

We denote by g^{*i} the star network where player i is the center of the star.

[8] Johnson and Gilles (2000) extend the connection model by introducing a cost of creating a link that is proportional to the geographical distance between two individuals. In Jackson and Rogers (2005) or de Marti and Zenou (2017), individuals belong to two different communities, and the cost for creating links depends whether it is an intracommunity link or an intercommunity link.

Proposition 1. *Consider the distance-based utility model in the case* $b(1) - b(2) < c < b(1)$*. If* $n > \#F \geq 1 + b(2)/(b(2) - b(3))$ *then the set* $G^* = \{g^{*i} \mid i \in M\}$ *is the unique myopic-farsighted stable set.*

Proof. We first show that $G^* = \{g^{*i} \mid i \in M\}$ satisfies both internal stability (i.e. condition (**IS**) in Definition 2) and external stability (i.e. condition (**ES**) in Definition 2).

IS. Farsighted players are peripherals in all networks in G^* so that they always obtain the same payoff: $U_i(g) = b(1) + (n-2)b(2) - c$ for all $i \in F$, $g \in G^*$. Myopic players who are peripherals have no incentive to delete their single link ($b(1) + (n-2)b(2) - c > 0$) or to add a new link ($2b(1) + (n-3)b(2) - 2c < b(1) + (n-2)b(2) - c$ since $b(1) - b(2) < c$). The center who is myopic has no incentive to delete one link since $c < b(1)$. Hence, for every $g, g' \in G^*$, it holds that $g' \notin \phi(g)$.

ES. Take any network $g \notin G^*$. We build in steps a myopic-farsighted improving path from g to some $g^{*i} \in G^*$.

Step 1: Starting in g, farsighted players delete all their links successively looking forward to some $g^{*i} \in G^*$, where they obtain their highest possible payoff given $b(1) - b(2) < c$. Notice that if g is a star network where the center is a farsighted player, then the center starts by deleting all her links since only the center is better off in g^{*i} compared to g (and we go directly to Step 8). We reach a network g^1 where all farsighted players have no link and myopic players only keep the links to myopic players they had in g.

Step 2: From g^1, looking forward to $g^{*i} \in G^*$, farsighted players build a star network g^{*jF} restricted to farsighted players with player j being the center (i.e. g^{*jF} is such that $j \in F$, $N_j(g^{*jF}) = F \setminus \{j\}$ and $N_k(g^{*jF}) = \{j\}$ for all $k \in F \setminus \{j\}$), and we obtain $g^2 = g^1 \cup g^{*jF}$ where all farsighted players are still disconnected from the myopic ones.

Step 3: From g^2, looking forward to $g^{*i} \in G^*$, the farsighted player j who is the center of g^{*jF} adds a link to some myopic player, say player 1. Player j is better off in g^{*i} compared to g^2, $b(1) + (n-2)b(2) - c > (n-m-1)(b(1) - c)$, while player 1 is better in $g^2 + j1$ since $b(1) > c$.

Step 4: From $g^2 + j1$, looking forward to $g^{*i} \in G^*$, the farsighted player j adds a link successively to the myopic players who are neighbors of player 1 (if any), say player 2. Player 2 who is myopic and linked to player 1 has an incentive to add the link $j2$ if and only if $b(2) + (n-m-1)b(3) < b(1) - c + (n-m-1)b(2)$. Thus, the necessary and sufficient condition for adding the link is

$$c < b(1) - b(2) + (n-m-1)(b(2) - b(3)). \tag{1}$$

Since $c < b(1)$, a sufficient condition is

$$b(1) \leq b(1) - b(2) + (n-m-1)(b(2) - b(3)) \text{ or } 1 + \frac{b(2)}{b(2) - b(3)} \leq n - m \tag{2}$$

where $n-m$ is the number of farsighted players ($\#F$). In $g^2 + j1 + \{jl \mid l \in N_1(g^2 + j1) \cap M\}$, player j is (directly) linked to all other farsighted players, player 1 and all neighbors of player 1.

Step 5: From $g^2 + j1 + \{jl \mid l \in N_1(g^2 + j1) \cap M\}$, the myopic players who are neighbors of player 1 and have just added a link to the farsighted player j delete their link successively with player 1. They have incentives to do so since $b(1) - b(2) < c < b(1)$ and we reach $g^2 + j1 + \{jl \mid l \in N_1(g^2 + j1) \cap M\} - \{1l \mid l \in N_1(g^2 + j1) \cap M\}$.

Step 6: Next, looking forward to $g^{*i} \in G^*$, the farsighted player j adds a link successively to the myopic players who are neighbors of some $l \in N_1(g^2 + j1) \cap M$ and we proceed as in Step 4 and Step 5. We repeat this process until we reach a network g^3 where there is no myopic player linked directly to the myopic neighbors of player j (i.e. $N_k(g^3) \cap M = \emptyset$ for all $k \in N_j(g^3) \cap M$).

Step 7: From g^3, player j adds a link to some myopic player belonging to another component (if any) as in Step 3 and we proceed as in Step 4 to Step 6. We repeat this process until we end up with a star network g^{*j} with player j (who is farsighted) in the center (i.e. $N_j(g^{*j}) = N \setminus \{j\}$ and $N_k(g^{*j}) = \{j\}$ for all $k \in N \setminus \{j\}$).

Step 8: From g^{*j}, looking forward to $g^{*i} \in G^*$, the farsighted player j deletes all her links successively to reach the empty network $g^{\emptyset}$. From $g^{\emptyset}$, myopic and farsighted players have both incentives (since $b(1) > c$) to add links successively to build the star network $g^{*i} \in G^*$ where some myopic player $i \in M$ is the center.

We now show that G^* is the unique myopic-farsighted stable set. Farsighted players who are peripherals in all networks in G^* obtain their highest possible payoff. Myopic players who are peripherals have no incentive to delete their single link or to add a new link. The center who is myopic has no incentive to delete one link. Hence, $\phi(g) = \emptyset$ for every $g \in G^*$. Suppose that $G \neq G^*$ is another myopic-farsighted stable set. (1) G does not include G^*: $G \not\supseteq G^*$. External stability would be violated since $\phi(g) = \emptyset$ for every $g \in G^*$. (2) G includes G^*: $G \supsetneq G^*$. Internal stability would be violated since for every $g \in \mathcal{G} \setminus G^*$, it holds that $\phi(g) \cap G^* \neq \emptyset$. □

In fact, the set G^* satisfies a stronger external stability requirement: for every $g \in \mathcal{G} \setminus G^*$, it holds that $\phi(g) \supseteq G^*$. The internal stability condition is satisfied for G^* even when $\#F < 1 + b(2)/(b(2) - b(3))$.[9]

If $b(1) - b(2) < c < b(1) - b(3)$ then the sufficient condition for having external stability becomes $b(1) - b(3) \leq b(1) - b(2) + (n - m - 1)(b(2) - b(3))$ or $2 \leq n - m$. Thus, once linking costs are intermediate but not so high, it suffices to have two farsighted players to guarantee that only efficient networks are going to emerge in the long run.

Corollary 1. *Consider the distance-based utility model in the case* $b(1) - b(2) < c < b(1) - b(3)$. *If* $n > \#F \geq 2$ *then the set* $G^* = \{g^{*i} \mid i \in M\}$ *is the unique myopic-farsighted stable set.*

[9] In the symmetric connections model where $b(t) = \delta^t$, the lower bound on the number of farsighted players, $1 + b(2)/(b(2) - b(3))$, becomes $1 + 1/(1 - \delta)$. Hence, the number of farsighted players needed for guaranteeing the emergence of the efficient networks increases with δ.

What happens if $\#F < 1 + b(2)/(b(2) - b(3))$ and (1) is not satisfied? If a myopic-farsighted stable set exists then G^* should be included in it. Otherwise, external stability would be violated since $\phi(g) = \emptyset$ for all $g \in G^*$.

Proposition 2. *Consider the distance-based utility model in the case $b(1) - b(2) < c < b(1)$. Suppose that all players are farsighted, $\#F = n$. If g is a star network then $\{g\}$ is a myopic-farsighted stable set.*

Proof. Since each set is a singleton set, internal stability (**IS**) is satisfied. (**ES**) Take any network $g \neq g^{*i}$, we need to show that $\phi(g) \ni g^{*i}$. (i) Suppose $g \neq g^{*j}$ $(j \neq i)$. From g, looking forward to g^{*i} (where they obtain their highest possible payoff), farsighted players $(\neq i)$ delete all their links successively to reach the empty network. From $g^{\emptyset}$, farsighted players have incentives (since $b(1) > c$) to add links successively to build the star network g^{*i} with player i in the center. (ii) Suppose $g = g^{*j}$ $(j \neq i)$. From g, looking forward to g^{*i}, the farsighted player j deletes all her links successively to reach the empty network. From $g^{\emptyset}$, farsighted players have incentives (since $b(1) > c$) to add links successively to build the star network g^{*i} with player i in the center. □

Once all players become farsighted (i.e. $\#F = n$), for $b(1) - b(2) < c < b(1)$, every set consisting of a star network encompassing all players is a myopic-farsighted stable set, but they are not necessarily the unique myopic-farsighted stable sets. For instance, when $n = 4$, the set of circles among the four farsighted players can be a myopic-farsighted stable set.[10]

Example 1. Take $N = F = \{1, 2, 3, 4\}$ and $b(1) - b(2) < c < b(1) - b(3) < b(1)$ in the distance-based utility model. Let $G^{c,4} = \{\{12, 23, 34, 14\}, \{13, 12, 34, 24\}, \{13, 14, 23, 24\}\}$ be the set of circles among the four farsighted players. The set $G^{c,4}$ is a myopic-farsighted stable set. It satisfies (**IS**) since the four players obtain the same payoffs in all circle networks. We now show that (**ES**) is satisfied: for every $g \notin G^{c,4}$, it holds that $\phi(g) \cap G^{c,4} \neq \emptyset$. (i) Take any g such that there is $g' \in G^{c,4}$ and $g \subsetneq g'$. In g, looking forward to g', players have incentives to add links successively to form g' since $c < b(1) - b(3)$, and so $g' \in \phi(g)$. (ii) Take any g^S such that $\#S = 3$. Players belonging to S have two links and are better off in any circle network g' than in g^S: $2b(1) - 2c < 2b(1) - 2c + b(2)$. Hence, from g^S, looking forward to some circle network g', some player deletes one of her links and we reach a network belonging to case (i) from which players have incentives to add links successively to form some circle network g', and so $g' \in \phi(g^S)$. (iii) Take

[10] Dutta and Vohra (2017) propose two related solution concepts: the rational expectations farsighted stable set (REFS) and the strong rational expectations farsighted stable set (SREFS) where they restrict coalitions (or pairs in our case) to hold common, history independent expectations that incorporate maximality regarding the continuation path. REFS and SREFS coincide with a farsighted stable set when the latter consists of networks with a single payoff (Theorem 1 of Dutta and Vohra, 2017). Since every set consisting of a star network encompassing all players is a myopic-farsighted stable set, it is also a REFS and SREFS. When $n = 4$, the same holds for the set of circles among the four farsighted player.

any g such that at least one player has three links. Any star network g^{*i} is one of such network. Players who have three links are better off in any circle network g' than in g: $3b(1) - 3c < 2b(1) - 2c + b(2)$ or $b(1) - b(2) < c$. Hence, from g, looking forward to some circle network g', players who have three links successively delete one of their links and we reach either a circle network or a network belonging to case (i) or case (ii) from which players have incentives to add links successively to form some circle network g', and so $g' \in \phi(g)$.

4 Conclusion

In the context of network formation with distance-based utilities, we have shown that, once the population of myopic and farsighted players is mixed, there is no conflict between stability and efficiency. On the contrary, when all players are farsighted (or all players are myopic), a conflict is likely to arise.

We have focused on the range of costs and benefits such that a star network is the unique strongly efficient network. In the case of small (large) link costs relative to benefits, there is no conflict between stability and efficiency. The set consisting of the complete (empty) network is the unique myopic-farsighted stable set whatever the mixture of myopic and farsighted individuals.

Suppose now that player i's distance-based utility from a network g is given by $U_i(g) = \sum_{j \neq i} b_i(t(ij)) - \#N_i(g)c_i$ where $c_i \geq 0$ and b_i is a nonincreasing function. Assume that $b_i(1) - b_i(2) < c_i < b_i(1)$ for all $i \in N$. If $n > \#F$ and $\#F \geq 1 + b_i(2)/(b_i(2) - b_i(3))$ for all $i \in M$, then the set $G^* = \{g^{*i} \mid i \in M\}$ is the unique myopic-farsighted stable set, and Proposition 1 still holds. However, such asymmetries in benefits and costs would imply that a conflict between stability and efficiency could again arise. For instance, the efficient network might even lie outside the set G^* if it is a star network with some farsighted player in the center. Transfers might then be a solution for avoiding any conflict.[11]

An interesting direction for future research is to look at network formation when a player's payoff from a link is a decreasing function of the number of links the other players maintains.[12] Morrill (2011) shows that, in general, the socially efficient and stable networks diverge, but they coincide when players are able to make transfers to their partners. Could we stabilize the socially efficient networks without transfers when the population is mixed?

[11] When all players are myopic, Bloch and Jackson (2007) show that peripheral players can subsidize the center of the star to keep their links formed. Any (efficient) star network is supportable as a pairwise equilibrium of the direct transfer game when $b(1) - b(2) < c < b(1) + b(2)(n-2)/2$.

[12] Möhlmeier, Rusinowska and Tanimura (2016) consider a utility function that incorporates both the effects of distance and of neighbors' degree.

Acknowledgements

Vincent Vannetelbosch and Ana Mauleon are Senior Research Associates of the National Fund for Scientific Research (FNRS). Financial support from the Spanish Ministry of Economy and Competition under the project ECO2015-64467-R, from the MSCA ITN Expectations and Social Influence Dynamics in Economics (ExSIDE) Grant No721846 (1/9/2017-31/8/2020), from the Belgian French speaking community ARC project 15/20-072 of Saint-Louis University - Brussels, and from the Fonds de la Recherche Scientifique - FNRS research grant T.0143.18 is gratefully acknowledged.

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