Relaxing Bertrand Competition: Capacity Commitment Beats Quality Differentiation

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Abstract

Both product differentiation through quality and capacity commitment have been shown to relax price competition. However, they have not been considered simultaneously. To this end we consider a three stage game where firms choose quality then commit to capacity and finally compete in price. We show that in equilibrium, firms differentiate their products less than if they were not able to commit to limited capacities. This is because they are able to enjoy Cournot profits at the stage where capacity are chosen. Furthermore if the cost of quality is low, capacity pre-commitment completely eliminates the incentives to differentiate.

JEL codes: L13

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1. Introduction

It is well-known since Gabszewicz & Thisse (1979)'s seminal contribution that quality differentiation offers a powerful way out of the Bertrand paradox. Many scholars have further elaborated on their pioneering work and today a robust "principle of differentiation" prevails in the literature studying vertically differentiated industries. As nicely summarized in Shaked & Sutton (1983), firms are indeed likely to "relax price competition through product differentiation". Interestingly enough, capacity commitment also has the virtue of relaxing price competition. The seminal contribution in this area is Kreps & Scheinkman (1983) who showed how capacity commitment may be instrumental in sustaining Cournot outcomes in pricing games. The strategic value of capacities has then been widely studied though almost exclusively in markets for non-differentiated goods.

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Casual observation suggests that many industries exhibit both product differentiation through quality and limited capacities in the short run. It is hard to see however which of the two aspects governs firms' behavior at the price competition stage. In other words, we ignore if firms' incentives with respect to quality choices are dependent on the possibility to commit to capacities or the reverse. If either is true one may wonder whether these instruments are complements or substitutes in relaxing price competition. Our aim in the present paper is to address this issue which does not seem to have been previously studied neither theoretically nor empirically.

Our main result summarizes as follows: *within the standard model of vertical differentiation, capacity commitment is more effective than quality differentiation as a mean of relaxing price competition.* To show this, we consider a three stage game where firms choose their quality level, then their level of production capacities and finally compete in prices. In our model, the possibility to commit in capacities in the second stage tends to destroy much of the incentives to choose different qualities in the first stage. In particular, when quality costs are low, *firms end up selling homogeneous products in equilibrium.*

This result may seem surprising at first sight, in particular because it runs against the well-established "principle of differentiation". In fact our finding is quite intuitive. Eaton & Harrald (1992) or Ireland (1987) have already shown that under quantity competition, firms are not inclined to differentiate in quality unless this allows to reduce sunk costs. In particular, under quantity competition, when there are no costs to quality upgrading, choosing the best available quality is a dominant strategy for all firms. When quality is costly, differentiation arises in equilibrium because it is more profitable to select a lower quality in order to incur lower sunk costs if the other firm has already chosen the highest quality.

In our model, the main effect of capacity commitment is precisely to transform the initial pricing game into a quantity game. More precisely, the reduced form of each firm's payoff at the quality stage are exactly equivalent to the Cournot payoffs. Therefore, the no-differentiation outcome naturally follows when quality costs are low. When quality improvement is costly, the possibility to commit in capacities systematically induces less differentiation in equilibrium as compared to the no-commitment case. Note that this result should not be viewed as invalidating the idea of vertical differentiation. It underlines however that the principle of quality (vertical) differentiation, as opposed to variety (horizontal) differentiation, is crucially rooted in asymmetries of costs rather than on a willingness to relax competition. In this last respect indeed, quality differentiation is clearly supplemented by capacity commitment.

Incidentally, the previous finding suggests that the standard Cournot outcomes (i.e. for homogeneous goods) can be sustained as subgame perfect equilibrium outcomes, thereby replicating Kreps & Scheinkman (1983)'s result within an enlarged game. We will show that this is only partially true. Cournot outcomes will indeed obtain as subgame perfect equilibrium outcomes but many other outcomes, including the fully collusive ones will be sustainable as well.

The paper is organized as follows. In section 2 we present the model and recall of the equilibrium of a quality-price game when production capacities are infinite. We then introduce the capacity commitment stage in section 3 and solve for a subgame perfect equilibrium. We establish at this step that capacity commitment induces a marked tendency towards no differentiation. This leads to analyze the behavior of our model at the no-differentiation limit (homogeneous goods) in section 4. Finally section 5 concludes.

2. The Setup

Consumers' preferences are set according to the simplified framework of Mussa & Rosen (1978) as popularized by Tirole (1988). Consumers are characterized by a "taste for quality" x which is uniformly distributed in the $[0;1]$ interval. Furthermore consumers have unit demand for the good and make their choice according to the indirect utility function $u(i,x) = xs_i - p_i$ for $i = 1, 2$. Not consuming yields a utility normalized to 0.

We consider a three-stage game. In stage 1, firms $i = 1, 2$ choose quality levels $s_i \in [0, 1]$ at a cost $\frac{s_i^2}{F}$.[1] Observe that when F is large the cost of choosing a positive quality becomes negligible. The incentives to differentiate are then exclusively related to the price competition mechanism. In stage 2, firms have the opportunity to commit to capacities before competing in price in the last stage. The capacity cost is small but positive. We retain at this step the framework proposed by Dixit (1980) within a quantity competition model and recently used by Maggi (1996) for price competition. The installed capacity k_i allows firm $i = 1, 2$ to produce up to k_i at constant marginal cost c whereas producing beyond capacity is possible at a constant unit cost $c + \theta$. Formally, the relevant marginal cost at the price competition stage is given by

$$mc_i(q) = \begin{cases} c & \text{if } q \leq k_i \\ c + \theta & \text{if } q > k_i \end{cases} \quad \text{for } i = 1, 2$$

We assume w.l.o.g. that c is zero and for simplicity that $\theta > 1$ to guarantee that it is never profitable to produce beyond capacity. Given costs, firms produce to satisfy demand, i.e. we assume that firms cannot turn consumers away once they have named their prices. We follow in this respect the definition of Bertrand competition suggested by Vives (1989) and endorsed by Bulow, Geanakoplos & Klemperer (1985), Vives (1990), Kuhn (1994), Dastidar (1995, 1997) and Maggi (1996). This assumption of *no rationing* automatically turns price competition into quantity competition and therefore considerably eases the formal analysis of the capacity game.

[1] Setting a finite common upper bound to qualities and consumers reservation price is a potential limitation of our model. We show in the next section that it is not a severe restriction.

Proposition 1, Lemma 2 and 3 of the appendix prove this claim for vertical differentiation, homogeneous good and horizontal differentiation respectively. We shall discuss at more length this hypothesis of no rationing in section 5.

3. The Benchmark without capacity commitment

Having defined our game completely, we now review the standard quality-price game (i.e. we neglect for the moment capacity commitment). This will provide a suitable benchmark for the analysis of the full game. Consider the price stage where we denote by s_h and s_l with $s_h > s_l$ the qualities chosen by the firms. Let us first define firms' demand as they result from consumers' choices given prices. Standard computations yield

$$D_l(p_l, p_h) = \begin{cases} \frac{p_h s_l - p_l s_h}{s_l(s_h - s_l)} & \text{if } p_l \leq p_h \frac{s_l}{s_h} \\ 0 & \text{if } p_l \geq p_h \frac{s_l}{s_h} \end{cases} \tag{1}$$

$$D_h(p_l, p_h) = \begin{cases} 1 - \frac{p_h - p_l}{s_h - s_l} & \text{if } p_l \leq p_h \frac{s_l}{s_h} \\ 1 - \frac{p_h}{s_h} & \text{if } p_l \geq p_h \frac{s_l}{s_h} \end{cases} \tag{2}$$

Note that for demands to be well-defined, we need $s_h > s_l$, i.e. products cannot be homogeneous. Whenever $p_l \geq p_h \frac{s_l}{s_h}$, firm l has an incentive to reduce its price to obtain a positive demand. Hence only the first segment of D_l and D_h are relevant. As a consequence we focus exclusively on this case to identify firms' best replies. The best reply functions in this benchmark pricing game are $\psi^h(p^l) = \frac{s_h - s_l + p_l}{2}$ and $\psi^l(p^h) = p_h \frac{s_l}{2s_h}$. They intersect at $\left(p_l^* = \frac{s_l(s_h - s_l)}{4s_h - s_l}, p_h^* = \frac{2s_h(s_h - s_l)}{4s_h - s_l}\right)$ which is the unique pure strategies price equilibrium. Demands addressed to the firms at these prices are $D_l^* = \frac{s_h}{4s_h - s_l}$ and $D_h^* = \frac{2s_h}{4s_h - s_l}$. It then remains to consider the first stage of the game where qualities are chosen. In this no-commitment case, the payoffs are

$$\Pi_i(s_i, s_j) = \begin{cases} \Pi_l^{NC}(s_j, s_i) & \text{if } s_i < s_j \\ \Pi_h^{NC}(s_i, s_j) & \text{if } s_i > s_j \end{cases}$$

where $\Pi_h^{NC}(s_h, s_l) \equiv \frac{s_h^2 4(s_h - s_l)}{(4s_h - s_l)^2} - \frac{s_h^2}{F}$ and $\Pi_l^{NC}(s_h, s_l) \equiv \frac{s_l s_h (s_h - s_l)}{(4s_h - s_l)^2} - \frac{s_l^2}{F}$

If quality is costless then F is infinite and simple computations show that there exist two subgame perfect equilibria. They involve one firm choosing the best available quality and the other one optimally differentiating to a lower quality. Formally when $F = +\infty$, $(s_1, s_2) = \left(1, \frac{4}{7}\right)$ and $(s_1, s_2) = \left(\frac{4}{7}, 1\right)$ are the only subgame perfect equilibria in pure strategies.[2] Similar qualitative results are obtained when quality costs are taken into account ($F < +\infty$). One firm then chooses a high quality whose level does not depend on the other's choice but solely on

[2]Lutz (1997) provides a detailed derivation of this equilibrium.

costs (numerically : $s_h(F) \simeq F/8$). The other firm's quality $s_l(F)$ is increasing, concave and converges very slowly towards the limit $\frac{4}{7}$ as F tends to zero. Given our assumption on the range of admissible qualities,[3] the previous results are depicted on Figure 1.

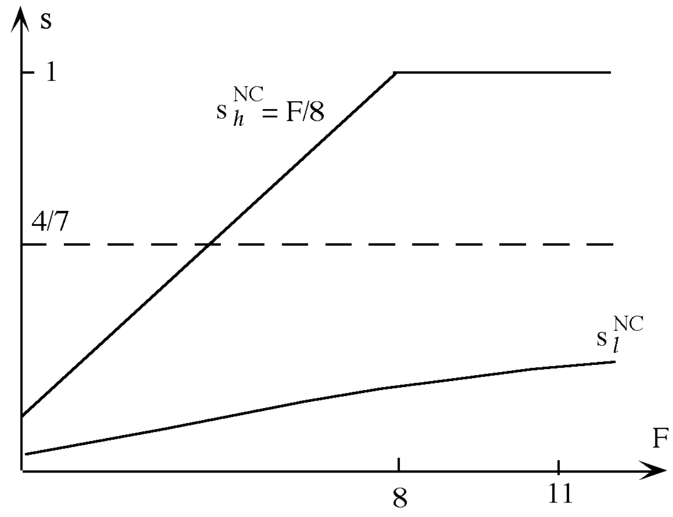

Figure 1

Let us summarize this section. The quality-price game studied here is similar to the battle of the sexes where one player chooses his most preferred action (a high quality) while the other accommodates with a lower quality. This illustrates the so-called "principle of differentiation".

4. The Game with Capacity Commitment

We consider now the full game where firms are allowed to commit to capacities before price competition takes place. We solve the corresponding three stage game by backward induction. We start by analyzing pricing games where firms have committed to qualities s_h and s_l with $s_h > s_l$ and then to capacities k_h and k_l. Note first that the assumption of no rationing and $\theta > 1$ imply that a firm will not find it profitable to name a price such that given the other's price, it sells beyond capacity. Thus whenever $D_i\left(\psi^i(p_j), p_j\right) > k_i$, firm i prefers to stick to its capacity by naming the price which solves $D_i(., p_j) = k_i$. The best reply functions are thus

$$\phi^h(p^l) = \begin{cases} \frac{s_h - s_l + p_l}{2} & \text{if } p^l \le (2k_h - 1)(s_h - s_l) \\ p_l + (1 - k_h)(s_h - s_l) & \text{if } p^l \ge (2k_h - 1)(s_h - s_l) \end{cases} \quad (3)$$

$$\phi^l(p^h) = \begin{cases} p_h \frac{s_l}{2s_h} & \text{if } p^h \le 2k_l(s_h - s_l) \\ p_h - k_l(s_h - s_l) & \text{if } p^h \ge 2k_l(s_h - s_l) \end{cases} \quad (4)$$

[3] The apparent arbitrariness of setting a finite upper bound to qualities is now easy to justify: if cost matters ($F < 8$) no firm wishes to choose top quality. It is only for the limit $F = +\infty$ that there is a problematic tendency to adopt an infinite quality.

Existence of a pure strategy equilibrium follows from the fact that firms always produce to satisfy demand. In this case indeed, no rationing can occur so that the typical non-concavities associated with Bertrand-Edgeworth competition are ruled out from the outset. More precisely, observe that D_h is linear decreasing for low p_l's and then steeper, thus Π_h is concave. The average over the distribution of p_l is also concave, hence firm h plays a pure strategy. Given this fact, D_l needs to be analyzed only on the domain where it is positive so that Π_l is also concave and firm l is playing a pure strategy. Uniqueness of the equilibrium follows then from the presence of product differentiation.

According to equations (3) and (4) there are four possible candidate equilibria involving either no firm, one firm, or two firms, selling at their full installed capacity. These equilibria as well as the parameter constellations in which they apply are given hereafter.

Lemma 1. *The Nash equilibrium of the price game is*

[**A**] $p_l^A = \frac{s_l(s_h - s_l)}{4s_h - s_l}, p_h^A = \frac{2s_h(s_h - s_l)}{4s_h - s_l}$ *if* $k_l \geq \frac{s_h}{4s_h - s_l}$ and $k_h \geq \frac{2s_h}{4s_h - s_l}$

[**B**] $p_l^B = (1 - k_h)\frac{s_l(s_h - s_l)}{2s_h - s_l}, p_h^B = (1 - k_h)\frac{2s_h(s_h - s_l)}{2s_h - s_l}$ *if* $k_l \geq \frac{(1-k_h)s_h}{2s_h - s_l}$ and $k_h \leq \frac{2s_h}{4s_h - s_l}$

[**C**] $p_l^C = (1 - 2k_l)\frac{s_l(s_h - s_l)}{2s_h - s_l}, p_h^C = (s_h - k_l s_l)\frac{(s_h - s_l)}{2s_h - s_l}$ *if* $k_l \leq \frac{s_h}{4s_h - s_l}$ and $k_h \geq \frac{s_h - k_l s_l}{2s_h - s_l}$

[**D**] $p_l^D = (1 - k_h - k_l)s_l, p_h^D = (1 - k_h)s_h - k_l s_l$ *if* $k_l \leq \frac{(1-k_h)s_h}{2s_h - s_l}$ and $k_h \leq \frac{s_h - k_l s_l}{2s_h - s_l}$

In region $[A]$ installed capacities are large enough to sustain the standard Nash equilibrium in prices identified in the previous section (i.e., without capacity constraints). When k_h decreases we enter area $[B]$ while if k_l decreases we enter area $[C]$; in both cases the low capacity firm sticks to its capacity while the other keep playing along its standard best reply ψ^h. Finally, in region $[D]$ both firms sell their capacity at the highest possible price: they virtually mimic the behavior of the Walrasian auctioneer. Given the capacities that have been installed, the Nash equilibrium is given by the pair of prices which "clear the market" i.e., for which demands equal capacities. It is in this sense that price competition without rationing is similar to Cournot competition.

Lemma 1 shows that any configuration of parameters (k_h, k_l, s_h, s_l) defines a unique Nash equilibrium in the corresponding pricing game. We can go backward in the game tree to consider the game of capacity choices. We prove in Proposition 1 that it possesses a unique equilibrium that enables us to easily study how qualities are chosen in the first stage.

Proposition 1. *There exists a unique pure strategy equilibrium in the capacity game replicating Cournot outcomes under quality differentiation.*

Proof On Figure 2 below the frontiers of the four areas A, B, C and D are the thin plain lines. Let us consider first the best reply of firm l against k_h. The payoffs in region A and B do not depend on capacity levels. Thus the presence of an arbitrarily small cost to capacity installation induces firm l to move to the frontier with region C and D as seen on Figure 2 below (recall that payoffs are continuous throughout regions). In region C , the payoff to firm l is $\pi_l^C(k_l, k_h) = k_l(1-2k_l)\frac{s_l(s_h-s_l)}{2s_h-s_l}$ so that the best reply against k_h is $\frac{1}{4}$. In region D, the payoff is $\pi_l^D(k_l, k_h) = k_l(1-k_h-k_l)s_l$, leading to the best reply $\frac{1-k_h}{2}$. The last step is to compare the respective merits of those two best reply candidates; letting $\overline{k_h}$ solve $\pi_l^D\left(\frac{1-k_h}{2}, k_h\right) = \pi_l^C\left(\frac{1}{4}, k_h\right)$ we obtain

$$\kappa^l(k_h) = \begin{cases} \frac{1-k_h}{2} & \text{if } k_h \leq \overline{k_h} \\ \frac{1}{4} & \text{if } k_h \geq \overline{k_h} \end{cases} \tag{4.1}$$

A similar analysis shows that firm h's best reply is defined in regions D and B as

$$\kappa^h(k_l) = \begin{cases} \frac{s_h-k_l s_l}{2} & \text{if } k_l \leq \overline{k_l} \\ \frac{1}{2} & \text{if } k_l \geq \overline{k_l} \end{cases} \tag{4.2}$$

where $\overline{k_l}$ solves $\pi_h^D\left(\frac{s_h-k_l s_l}{2}, k_l\right) = \pi_h^B\left(\frac{1}{2}, k_l\right)$. The best reply functions (displayed in bold on Figure 2) are discontinuous but this does not prevent the existence of a unique pure strategy equilibrium in the capacity game. The solution of system $(5-6)$ is $k_l^* = \frac{s_h}{4s_h-s_l}$ and $k_h^* = \frac{2s_h-s_l}{4s_h-s_l}$. This equilibrium lies in the interior of region D as $k_l^* < \overline{k_l}$ and $k_h^* < \overline{k_h}$. Figure 2 below summarizes our findings.

In order to establish the equivalence of this equilibrium with Cournot outcomes, observe that the demand system defined by equations (1) and (2) is invertible and yield the system characterizing the price equilibrium of region D i.e., p_l^D and p_h^D as functions of quantity variables k_l and k_h. Solving for a Nash equilibrium of this new quantity game we obtain (k_l^*, k_h^*). ■

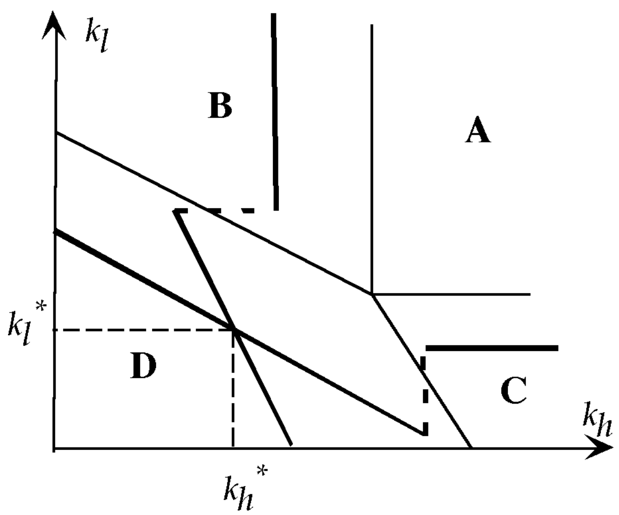

Figure 2

This proposition has therefore established that *under Bertrand competition and vertical differentiation , capacity commitment yields Cournot outcomes.* This result is reminiscent of Kreps & Scheinkman (1983).

It then remains to consider the first stage of the game where qualities are chosen.[4] The payoff arising from the capacity equilibrium are $\Pi_h^C(s_h, s_l) = \frac{s_h(2s_h - s_l)^2}{(4s_h - s_l)^2}$ for the high quality firm and $\Pi_l^C(s_h, s_l) = \frac{s_l s_h^2}{(4s_h - s_l)^2}$ for the low quality one. When quality is not costly (F large), it is straightforward to see that the firm exhibiting the high quality will choose the highest possible quality because $\frac{\partial \Pi_h^C}{\partial s_h} > 0$. This is exactly what happened when no capacity commitment was available.

We may then focus on the best reply of firm l against $s_h = 1$. Recall that in the absence of capacity commitment, it is well known that the low quality firm optimally differentiates to $s_l = \frac{4}{7}$. On the contrary, we show hereafter in proposition 2 that the ability to commit to a given capacity is so powerful as a mean of limiting price competition that there is no need to differentiate anymore. Formally, when cost for quality is low we obtain $\frac{\partial \Pi_l^C}{\partial s_l} > 0$: the low quality firm imitates the high quality one. When quality is more costly, the low quality firm differentiates but less than in the no-commitment game. Proposition 2 states this result in the general case of positive and convex cost to quality.

Proposition 2. *The Nash equilibrium of the quality game with Commitment and quality cost factor F is asymmetric: One firm chooses a high quality $s_h^C(F)$ almost identical to the monopoly choice while the other differentiates to $s_l^C(F) > s_l^{NC}(F)$.*

Proof The profit function of firm i in the quality game (with commitment) is

$$\Pi^C(s_i, s_j) = \begin{cases} \Pi_h^C(s_i, s_j) - \frac{s_i^2}{F} & \text{if } s_i > s_j \\ \Pi_l^C(s_j, s_i) - \frac{s_i^2}{F} & \text{if } s_i \le s_j \end{cases}$$

Observe that for the high quality firm $\frac{\partial^2 \Pi_h^C}{\partial s_h^2} = -\frac{8s_l^2(s_h - s_l)}{(4s_h - s_l)^4} < 0$ and $\frac{\partial^2 \Pi_h^C}{\partial s_l \partial s_h} = \frac{8s_h s_l(s_h - s_l)}{(4s_h - s_l)^4} > 0$ thus the solution $\gamma_h(s_l)$ of $\frac{\partial \Pi_h^C}{\partial s_h} = 2\frac{s_h}{F}$ is a maximum of Π_h^C and is increasing with s_l. Furthermore $\frac{\partial \Pi_h^C}{\partial s_h} = (2s_h - s_l)\frac{2s_h(s_h - s_l) + 6s_h^2 + s_l^2}{(4s_h - s_l)^3} > \frac{1}{4}$ implies that over the domain $\{s_h \ge s_l\}$, the high quality firm choose a quality above the monopoly one $F/8$.[5]

On the other hand we have for the low quality firm: $\frac{\partial^2 \Pi_l^C}{\partial s_l^2} = \frac{2s_h^2(8s_h + s_l)}{(4s_h - s_l)^4} > 0$ and $\frac{\partial^2 \Pi_l^C}{\partial s_l \partial s_h} = -\frac{2s_h s_l(s_l + 8s_h)}{(4s_h - s_l)^4} < 0$. If F is so large that $\frac{\partial \Pi_l^C}{\partial s_l} > 2\frac{s_l}{F}$ the low quality firm tries to imitate the high quality one over the domain $\{s_h \le s_l\}$ because Π_l^C is convex. For a higher cost of quality (lower F), $\gamma_l(s_h)$ the solution of $\frac{\partial \Pi_l^C}{\partial s_l} = 2\frac{s_l}{F}$ lies between 0 and 1 and is a decreasing function of s_h.

[4]Recall that our analysis is valid only when qualities are strictly different.

[5]Indeed the monopoly price is $\frac{1}{2}s$ yielding a payoff of $\frac{1}{4}s$. Hence the FOC for optimal quality choice is $\frac{1}{4} = \frac{2}{F}s$ and leads to $s^M = \frac{1}{8}F$.

Comparing the payoff $\Pi_h^C(\gamma_h(s_j), s_j)$ and $\Pi_l^C(s_j, \gamma_l(s_j))$ we are able to determine the point α at which the best reply of firm i jumps down. The intuition is easy to understand: as long as s_j is low firm i is better off leading the game by choosing a large quality that is even greater than the monopoly choice $\frac{F}{8}$. When s_j is large a similarly high quality leads to losses because of the fierce quantity competition, thus firm i optimally differentiates to a low level. Figure 3a below displays γ_h over $[0; \alpha]$ and γ_l over $[\alpha; 1]$ for $F = 5$.

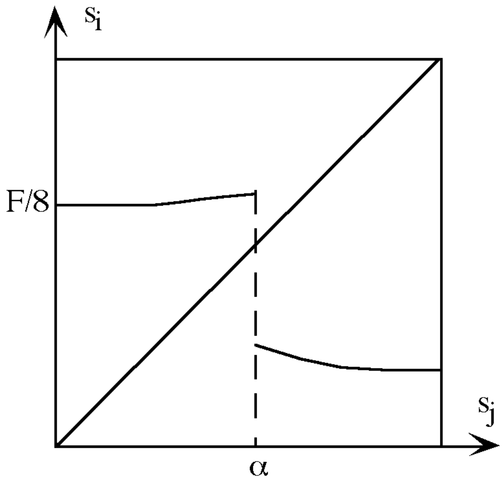

Figure 3a

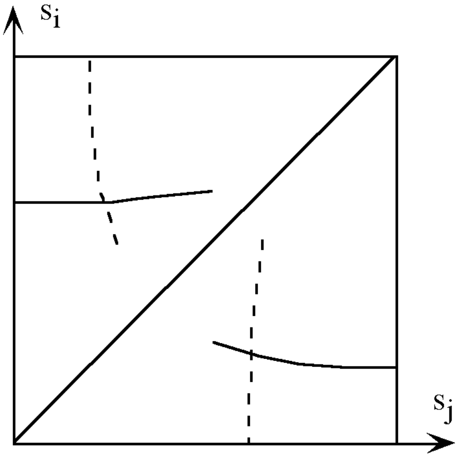

Figure 3b

Displaying both best reply functions on Figure 3b above we see that $\alpha < \frac{F}{8}$ (we check numerically that it holds true whatever F) thus there exists two asymmetric pure strategy equilibria in the first stage where qualities are chosen.

We now study the equilibrium as a function of the cost parameter F. Although $\frac{\partial \Pi_l^C}{\partial s_l} = 2\frac{s_l}{F}$ is a 4^{th} degree polynomial equation, it can be solved analytically. Let us denote $\psi^C(s_h, F)$ the unique root among the four possible ones that lies in $[0; 1]$. The subgame perfect Nash equilibrium of the whole game is the solution of $\frac{\partial \Pi_h^C(s_h, \psi^C(s_h,F))}{\partial s_h} = 2\frac{s_h}{F}$. This equation is obviously not solvable analytically but can be solved numerically to yield a unique equilibrium. The choice of the high quality firm is $s_h^C(F) \simeq F/8$. We then derive $s_l^C(F) = \psi^C\left(s_h^C(F), F\right)$ which is increasing, convex and reaches the top level at $F = 11$. ■

Figure 4 summarizes our findings and compare them to the no-commitment case. s_l^{NC} denotes the low quality equilibrium choice in the no-commitment case whereas s_l^C applies for the commitment case.

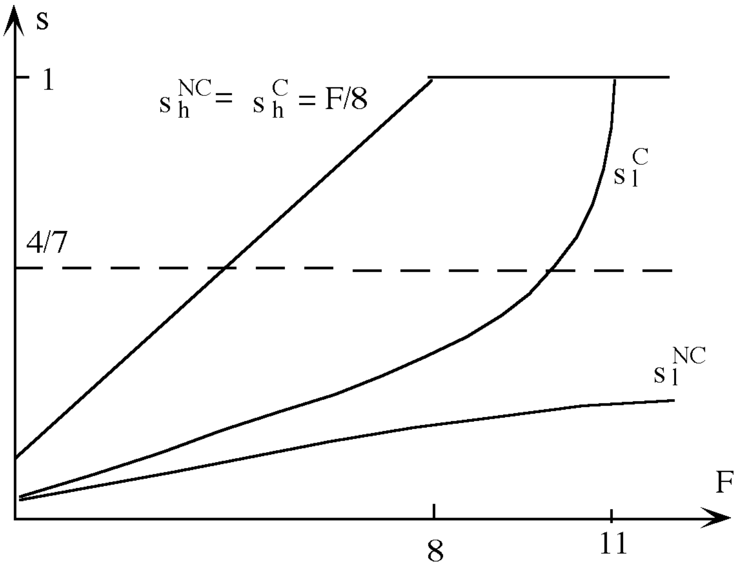

Figure 4

As a direct consequence of Proposition 2 we may thus state our central result: *Capacity commitment and Bertrand competition systematically induce less product differentiation than without the power to commit to a given capacity.* Further, if the cost of quality is below some critical level then *the ability to commit to capacities before Bertrand competition leaves no room for quality differentiation as a way of relaxing competition.*

Proposition 3. *As the cost of quality decreases, the degree of quality differentiation slowly decreases towards a positive level in the* no-commitment *case whereas in the* commitment *case it quickly decreases towards zero.*

Proposition 1 told us that if firms can commit to capacities and play Bertrand competition afterwards, we are literally back to a standard Cournot game at the quality stage. It is then obvious that the incentives to differentiate that are left for are those prevailing under quantity competition. Under Cournot competition, choosing a lower quality essentially amounts to enjoy a lower residual demand against the other's quantity, which cannot be profitable under the assumption that quality is not too costly.[6] In the case of negligible costs for quality ($F > 11$), this is exactly what happens in our model. Given this marked tendency towards identical quality choices in equilibrium we have to study the behavior of our model in the limiting case of no-differentiation. In order to do this, a different analysis is called for since the analysis up to now is only valid for differing qualities. In the next section, we develop the formal analysis of the capacity-price game with Bertrand competition and homogeneous products.

[6]See Eaton & Harrald (1992) on this point when there is no cost to quality.

5. The Limit Model when Products Are Homogeneous

When identical qualities are chosen by firms in the first stage, our model simplifies to the linear demand $D(p) = s - p$. We shall show later that choosing the best available quality is the only "robust" equilibrium of the full game with negligible quality costs. We therefore set $s = 1$ to ease the exposition of the capacity-price competition. Firms choose capacities k_1 and k_2 and then compete in price. Recall that firms name prices and produce to satisfy demand. We assume that in case of a tie, demand is split equally between the two firms.[7]

Consider a subgame $G(k_i, k_j)$, the profit function for $i = 1, 2$ is

$$\Pi_i(p_i, p_j) = \begin{cases} p_i(1-p_i) & \text{if } 1-k_i \leq p_i < p_j \\ p_i k_i + (p_i - 1)(1 - k_i - p_i) & \text{if } p_i < p_j \text{ and } p_i < 1 - k_i \\ p_i \frac{1-p_i}{2} & \text{if } 1 - 2k_i \leq p_i = p_j \\ p_i k_i + (p_i - 1)(\frac{1-p_i}{2} - k_i) & \text{if } p_i = p_j < 1 - 2k_i \\ 0 & p_i > p_j \end{cases}$$

Notice that a firm's payoff is totally independent of the other's capacity in complete opposition to Bertrand-Edgeworth models. Introducing quantitative restrictions while preventing rationing has two direct effects. Because the "no-rationing" rule prevents the existence of demand spillovers, the kind of high price strategic deviation that generates price instability in Bertrand-Edgeworth models is not at work in the present model. On the other hand, undercutting the other's price may lead to huge losses if the capacity is low relative to the demand that has to be served. Quite naturally, the strategy that will emerge in equilibrium consists in the matching of the other's price in order to avoid being forced to fully serve market demand. As a consequence, there will exist a continuum of Nash equilibria in the pricing game (as in Dastidar (1997)) whose range will depend on firms' capacities. Cournot prices but also Collusive ones will belong to this continuum for a wide range of capacity levels, i.e. in many price subgames.

Regarding the incentives to capacity choices the intuition is then simple: a large capacity makes undercutting attractive in the pricing game for a wide range of prices. Therefore, the level of prices that can be sustained as equilibrium ones tend to be low when capacities are high. On the other hand, choosing a too low capacity level does not allow to take the full benefit of sustaining identical prices in equilibrium. Equilibrium capacity choices are thus located "in between". Cournot capacities can be part of a subgame perfect equilibrium but many other capacities including the collusive ones are also equilibrium choices. The following proposition is proved in Lemma 2 of the appendix.

Proposition 4. *Capacity commitment and Bertrand price competition for an homogeneous good yield*

[7] This (standard) convention simplifies the exposition without affecting the nature of our results.

► *A multiplicity of subgame perfect equilibria*

► *The Cournot equilibrium as a lower bound on equilibrium payoffs*

► *The collusive outcome if the Pareto selection is used at the price stage*

Studying the behavior of our model at the no-differentiation limit reveals how crucial is the assumption of *no-rationing* to the analysis. In differentiated markets, capacity commitment and price competition without rationing force firms to play quantity competition because the rules of the game leave them no other choice. Indeed, Proposition 1 proves this result for vertically differentiated products but the same is true under horizontal differentiation as formally shown in Lemma 3 of the appendix. The main problem with the no-rationing hypothesis is therefore its lack of continuity at the limit when goods become homogeneous as illustrated by Proposition 4.

Proposition 4 also contrasts with Kreps & Scheinkman (1983) and the bulk of the literature on capacity-price competition with rationing were uniqueness of equilibrium and Cournot outcome is the rule while here the Cournot outcome is only the lower bound in terms of payoffs of a continuum of equilibria.

We are now in a position to relate our analysis of the homogeneous case with Proposition 3. Since we are dealing with continuous games the notion of trembling hand perfection is not well defined meaning that we cannot formally prove the following claim.

Claim Under Bertrand competition and low costs of quality, capacity commitment yields no quality differentiation and collusion.

When $F < 11$, small perturbations from $s_i = F/8$ and $s_j = s_l^C(F)$ lead firms back to the unique SPE which is therefore robust. However when $F > 11$, both firms have a tendency to imitate each other in quality. Yet any $s > 8/9$ chosen by both firms along with collusive capacities of $s/8$ form an SPE of G since the best deviation that a firm can make is to choose $s = 1$ which yields the Cournot payoff $1/9$ in the unique continuation equilibrium of $G(s, 1)$. Furthermore no firm gets an equilibrium payoff in G lesser than $1/9$. Our claim is supported by limited rationality arguments and models of evolutionary game theory which tend to indicate that players are able to coordinate on the equilibrium whose payoffs are Pareto-dominating. It is therefore very likely that firms will increase quality and limit capacities if they anticipate that they will play the Pareto price equilibrium described in Proposition 4.

6. Conclusion

In this article, we have shown that quality differentiation *as a mean of relaxing price competition* was not a robust principle once capacity commitment is allowed. There exists a recent literature which sees the mode of competition in the market as the result of a richer game. For instance Maggi (1996) shows that the degree of competitiveness of equilibrium market outcomes can be viewed as the result of capacity choices with limited commitment value. Motta & Polo (1999) establish a similar result using product differentiation. They show that *"The extent to which firms can differentiate their products ... determines the toughness of price competition"*. The present contribution clearly belongs to the same vein.

We have shown indeed that considering a richer game where capacity commitment is possible sheds a new light on differentiation issues as well as on price competition. In our setting, capacity commitment relaxes price competition so effectively that differentiation becomes unprofitable (for large F). Two remarks are called for at this step. First, our result should not be viewed as disqualifying vertical differentiation. It emphasizes rather the fact that quality differentiation may rely more heavily on costs considerations than on a willingness to relax competition. Second, Bertrand competition (as opposed to Bertrand-Edgeworth) appears to be central in obtaining our minimum-differentiation principle so easily. Allowing for rationing severely complicates the picture because non existence of pure strategy equilibrium is endemic in the corresponding pricing games. Preliminary results obtained in a more simple setting (Boccard & Wauthy (1998)) suggest that our present findings could indeed generalize to Bertrand-Edgeworth games. At this step however, this remains an open conjecture.

From an empirical point of view our analysis suggests that in industries whose technology exhibits rigid production capacities, quality differentiation should basically reflect costs differentials so that if upgrading quality is not too costly, less product differentiation should be observed.

7. Appendix

Lemma 2. *Capacity commitment and Bertrand price competition yield*

- ▶ *a multiplicity of subgame perfect equilibria including the Cournot equilibrium*
- ▶ *the collusive outcome if the Pareto selection is used at the price stage*

Proof : The proof is in four steps

i) Firm i's best reply in $G(k_i, k_j)$.

Two strategy profiles are relevant: undercutting, or matching the other's price. Notice the novelty here: undercutting may be less profitable than matching because this may entail losses on units sold beyond capacities.

Observing that $p_i k_i + (p_i - 1)(1 - k_i - p_i) = k_i - (1 - p_i)^2$, hence undercutting a price lesser than $1 - \sqrt{k_i}$ yields negative profits. Likewise $pk_i + (p-1)(\frac{1-p}{2} - k_i) = k_i - \frac{1}{2}(1-p)^2$ implies that matching the other's price yields negative profits whenever $p_j \leq 1 - \sqrt{2k_i}$. For $p_j < 1 - \sqrt{2k_i}$, both undercutting and matching yield negative profits so that the best reply is any higher price. The price that leaves firm i indifferent between matching and not being "constrained" and undercutting while being constrained is the negative root of equation $\frac{(1-p)p}{2} = k_i - (1-p)^2$ which is $\rho(k_i) \equiv \frac{1}{2}\left(3 - \sqrt{1+8k_i}\right) > 1 - 2k_i$ so that firm i is indeed not constrained. When $1 - \sqrt{2k_i} < p_j < 1 - 2k_i$ (this is meaningful for $k_i < \frac{1}{2}$ only) matching leads to a constrained capacity but is still better than undercutting by continuity. Noticing finally that $\rho(k_i) > 1 - \sqrt{2k_i}$, the best reply function is

$$BR_i(p_j) = \begin{cases} p_j^+ & \text{if } p_j < Max\left\{0; 1 - \sqrt{2k_i}\right\} \\ p_j & \text{if } Max\left\{0; 1 - \sqrt{2k_i}\right\} < p_j < \rho(k_i) \\ p_j^- & \text{if } p_j \geq \rho(k_i) \end{cases} \quad \text{for } i = 1, 2$$

ii) Analysis of the symmetric price equilibria and of the Pareto correspondence..

For asymmetric capacities with $k_j \geq k_i$, a continuum of symmetric equilibria (p, p) exists over the segment

$$\begin{aligned} \Phi(k_1, k_2) &\equiv \left[Max\left\{0; 1 - \sqrt{2k_i}\right\}; \rho(k_i)\right] \cap \left[Max\left\{0; 1 - \sqrt{2k_j}\right\}; \rho(k_j)\right] \\ &= \left[Max\left\{0; 1 - \sqrt{2k_i}\right\}; \rho(k_j)\right] \end{aligned}$$

When capacities are not too dissimilar $\Phi(k_1, k_2)$ is non void; otherwise the equilibria are asymmetric. Firm i plays any p in $\left]\rho(k_j); 1 - \sqrt{2k_i}\right[$ and firm j plays $p - \epsilon$ for any small positive ϵ. Firm i obtains a zero profit in these equilibria.

The multiplicity of equilibria might be problematic for going backward in the game tree. We rely on bounded recall and limited rationality arguments like those of Aumann & Sorin (1989) to select the Pareto dominant equilibrium from the Nash correspondence. This equilibrium is either the purely collusive one or as close as possible to it.

If $p < 1 - 2k_j$ the equilibrium payoff of firm i and j are $k_i - \frac{1}{2}(1-p)^2$ and $k_j - \frac{1}{2}(1-p)^2$ thus coordinating to a higher price is Pareto dominating. If $p \geq 1 - 2k_i$, $\Pi_i = p\frac{1-p}{2}$ so that the Pareto dominant equilibrium is the price in $[Max\{0; 1 - 2k_i\}; \rho(k_j)]$ that is the nearest to the monopoly price 1/2. When $1 - 2k_j < p < 1 - 2k_i$ all prices larger than 1/2 lead to Pareto optimal equilibrium outcomes because firm i is paid $k_i - \frac{1}{2}(1-p)^2$ and wishes to increase p while firm j is paid $p\frac{1-p}{2}$ and wish to decrease p toward the monopoly price. This will not be a problem because the incentive for firm i will be to raise k_i.

Observe that $\rho(k) = 1/2$ for $k = 3/8$ and $\frac{3}{8} < k_i \leq k_j$ implies $\rho(k_j) > 1 - k_i - k_j$, thus the Pareto dominant price when capacities are large is the upper bound $\rho(k_j)$. Over the complementary domain where $k_i < \frac{3}{8}$, $1 - \sqrt{2k_i} < 1 - k_i - k_j$ so that the monopoly price $\frac{1}{2}$ is reachable if $\frac{1}{2} \geq 1 - k_i - k_j$. To sum up the Pareto dominant price selection is

$$\hat{p}(k_i,k_j) = \begin{cases} \left[\frac{1}{2}; 1-2k_i\right] & \text{if } 2k_j^2 < k_i \leq \frac{1}{4} \\ 1-2k_i & \text{if } k_i \leq \frac{1}{4} \text{ and } 2k_j^2 \geq k_i \\ \frac{1}{2} & \text{if } \frac{1}{4} < k_i < \frac{3}{8} \\ \rho(k_j) & \text{if } \frac{3}{8} < k_i \end{cases} \text{ and } k_i \leq k_j.$$

iii) The capacity equilibrium is collusive

Consider firm i's best reply against k_j. If $k_i \in \left[\frac{3}{8}; k_j\right]$, firm i is paid independently of its own capacity and should thus reduce it as soon as there is a infinitesimal but positive cost to capacity installation. If $k_i \in \left[\frac{1}{4}; \frac{3}{8}\right]$ then firm i is paid one half of the monopoly profit minus its capacity cost which should be optimally set at 1/4. If $2k_j^2 < k_i < \frac{1}{4}$ then for any $p \in \left[\frac{1}{2}; 1-2k_i\right]$ there is a symmetric equilibrium that pays $\Pi_i = k_i - \frac{1}{2}(1-p)^2 < k_i(1-2k_i)$, thus firm i can profitably deviate to 1/4 the argument maximizer of $k_i(1-2k_i)$.

iv) Multiplicity of subgame perfect equilibria

If we nevertheless insist on considering all price equilibria there will obviously exist a continuum of SPE. For instance the Cournot quantities $(1/3, 1/3)$ may be sustain as follow. On the equilibrium path, there exist symmetrical price equilibria in the range $\Phi(1/3, 1/3) = [.18; .54]$ which includes the Cournot price 1/3. If a firm deviates to a larger capacity then the upper bound of Φ increases thus we may select the price equilibrium at the lower bound $.1835$ to "punish" the deviant. If a firm deviates downward then the lower bound of Φ equal to $\underline{p} \equiv 1 - \sqrt{2k_i}$ increases. If $\underline{p} < 1/3$ then we may select the price equilibrium at the lower bound $\underline{p}$ to punish the deviant. Because $k_i < 1/3$ the demand addressed to firm i is $\sqrt{2k_i}/2 > k_i$ thus its equilibrium payoff is $\underline{p}k_i + (\underline{p}-1)(\frac{1-\underline{p}}{2} - k_i) < \underline{p}k_i < 1/9$ the Cournot payoff. If k_i is so small that $1 - \sqrt{2k_i} > 1/3$ then firm i nets zero profit in any ensuing price equilibrium. ■

In the following lemma we consider an horizontally differentiated market adapted from Maggi (96) whose demand function for firm i is $D_i(p_i, p_j) = a - b_1 p_i + b_2 p_j$ with $a > 0$ and $b_1 > b_2 > 0$. Observing that $D_i + D_j = 2a - (p_i + p_j)(b_1 - b_2)$, the analogy with the classical aggregate demand for homogeneous goods $D(p) = 1 - p$ leads us to set $a = b_1 - b_2 = 1/2$ so that there is no scope for varying the degree of differentiation. Instead we will consider an horizontally differentiated market with a substitutability parameter a defined to $\left.\frac{\partial D_i(p_i,p_j)}{\partial p_i}\right|_{p_i=p_j}$. To keep the exposition simple we take the demand addressed to firm i to be $D_i(p_i, p_j) = Min\left\{1; Max\left\{0; \frac{1-p_j}{2} - a(p_i - p_j)\right\}\right\}$ although a smooth function with $D_i(0, p_j) = 1$ and $\lim_{p_i \to 1} D_i(., p_j) = 0$ would be more realistic but less tractable.

The technology of firms are now described. The marginal cost of production below capacity is c. The unit cost of capacity installation is δ and the marginal cost of producing beyond capacity is $c+\delta+\theta$ where θ measures legal and technical costs associated to the production of units beyond capacity. The Cournot quantity is $k^* \equiv \frac{(1-\delta-c)(4a-1)}{2(6a-1)}$ and the Cournot price is $\theta^* \equiv \frac{(1-\delta-c)(2a-1)^2}{2a(6a-1)}$.

Lemma 3. *Capacity commitment and Bertrand price competition in an horizontally differentiated market yield Cournot competition if $\theta > \theta^*$ and equilibrium capacities $\frac{a(1-c-\delta-\theta)}{1+2a} > k^*$ otherwise. The Bertrand (competitive) outcome is reached at $\theta = 0$.*

Corollary 1. *As goods become homogeneous ($a \to +\infty$), θ^* and k^* increase toward the Cournot level $\frac{1-\delta-c}{3}$ that is characteristic of the homogeneous goods model while if $\theta < \theta^*$ the equilibrium quantity tends to $\frac{1-c-\delta-\theta}{2}$ which is the individual (purely) competitive quantity for $\theta = 0$.*

Proof The proof is similar to that of Lemma 2 but easier since differentiation smooth things out. We first solve the pricing game for any pair (k_i, k_j) and then analyze the capacity game.

Solving for $D_i(p_i, p_j) = k_i$ yields $p_i = \gamma(k_i, p_j) \equiv \frac{(2a-1)p_j+1-2k_i}{2a}$. The profit function in the pricing game is

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c - \delta - \theta)D_i(p_i, p_j) + (\delta+\theta)k_i & \text{if } p_i < \gamma(k_i, p_j) \\ (p_i - c)D_i(p_i, p_j) & \text{if } p_i \geq \gamma(k_i, p_j) \end{cases}$$

Notice that if $D_i(c, p_j) = 0$ if $p_j < \frac{2ac-1}{2a-1}$, thus the best reply is any larger price. The unconstrained maximizer of $(p_i - c)D_i(p_i, p_j)$ is $\frac{c+p_j}{2} + \frac{1-p_j}{4a}$ (the traditional Bertrand best-reply). For this price to be eligible two conditions must be satisfied: it must yield a positive demand and it must be larger than $\gamma(k_i, p_j)$. The first condition is true because $D_i(p_i, p_j) > 0 \Leftrightarrow p_i < \frac{1-p_j}{2a} + p_j$ while the second is correct if $p_j < \frac{4k_i+2ac-1}{2a-1}$. The unconstrained argmax of $\pi_i(p_i, p_j)$ for $p_i < \gamma(k_i, p_j)$ is $\frac{c+\delta+\theta+p_j}{2} + \frac{1-p_j}{4a}$ which guarantees a positive demand and is lesser than $\gamma(k_i, p_j)$ if $p_j > \frac{4k_i+2a(c+\delta+\theta)-1}{2a-1}$. For values of p_j in between the best reply is to stick to the capacity by playing $\gamma(k_i, p_j)$. Therefore the best reply function is

$$BR_i(p_j) = \begin{cases} p_j^+ & \text{if } p_j < \frac{2ac-1}{2a-1} < c \\ \frac{c+p_j}{2} + \frac{1-p_j}{4a} > c & \text{if } \frac{2ac-1}{2a-1} \leq p_j < \frac{4k_i+2ac-1}{2a-1} \\ \gamma(k_i, p_j) & \frac{4k_i+2ac-1}{2a-1} \leq p_j < \frac{4k_i+2a(c+\delta+\theta)-1}{2a-1} \\ \frac{c+\theta+p_j}{2} + \frac{1-p_j}{4a} > c+\delta+\theta & \frac{4k_i+2a(c+\delta+\theta)-1}{2a-1} \leq p_j \end{cases}$$

For a large θ, a price p_j greater than $\frac{4k_i+2a(c+\delta+\theta)-1}{2a-1}$ would yield a nil demand for firm j thus the last entry of BR_i will never be relevant. If $k_i < \frac{1-c}{4}$ then $\frac{4k_i+2ac-1}{2a-1} < c$, thus firm j will never play such a low price and $BR_i(p_j) = \gamma(k_i, p_j)$.

► A symmetric equilibrium may involves non binding capacities if those are large; it is the traditional Bertrand competition. The solution of $\frac{c+p_j}{2} + \frac{1-p_j}{4a} = p_j$ is $p = \frac{1+2ac}{1+2a}$. This price is eligible if $\frac{2ac-1}{2a-1} \leq \frac{1+2ac}{1+2a} < \frac{4k_i+2ac-1}{2a-1}$ for $i = 1, 2$. The left inequality is always true while the other one necessitates k_i and k_j larger than $\frac{a(1-c)}{1+2a}$. The equilibrium payoff is $\frac{a(1-c)^2}{(1+2a)^2} - \delta k_i$, hence both firms have an incentive to decrease their capacity down to $\frac{a(1-c)}{1+2a}$.

▶ At the opposite extreme the equilibrium may involve sales in excess of capacities if the latter are small. The solution of $\frac{c+\delta+\theta+p_j}{2} + \frac{1-p_j}{4a} = p_j$ is $\frac{1+2a(c+\delta+\theta)}{1+2a}$. This price is eligible if k_i and k_j are lesser than $\frac{a(1-c-\delta-\theta)}{1+2a}$. The equilibrium payoff is $\frac{a(1-c-\delta-\theta)^2}{(1+2a)^2} + (\delta+\theta)k_i$ thus both firms have an incentive to increase their capacity up to $\frac{a(1-c-\delta-\theta)}{1+2a}$.

▶ The last candidate symmetric equilibrium is the solution of $\begin{cases} \gamma(k_i, p_j) = p_i \\ \gamma(k_j, p_i) = p_j \end{cases} \Leftrightarrow \begin{cases} D_i(p_i, p_j) = k_i \\ D_j(p_i, p_j) = k_j \end{cases}$.

This is the Cournot system of prices $\begin{cases} p_i^*(k_i, k_j) = \frac{4a(1-k_i-k_j)-1+2k_j}{4a-1} \\ p_j^*(k_i, k_j) = \frac{4a(1-k_i-k_j)-1+2k_i}{4a-1} \end{cases}$; it holds when k_i and k_j are between $\frac{a(1-c-\delta-\theta)}{1+2a}$ and $\frac{a(1-c)}{1+2a}$. The first period payoff function that we deduce from this equilibrium is $\Pi_i(k_i, k_j) = (p_i^* - c - \delta)k_i$ and the best reply at the capacity stage is $k_i = \frac{4a(1-k_j-\delta-c)+\delta-1+2k_j}{8a}$. The fixed point of this best reply operator is $k^* \equiv \frac{(1-\delta-c)(4a-1)}{2(6a-1)}$.

From the analysis of the 3 preceding points we deduce that the Cournot quantity k^* is the unique symmetric SPE if it lies between $\frac{a(1-c-\delta-\theta)}{1+2a}$ and $\frac{a(1-c)}{1+2a}$. The first condition is always true while the second leads to $\theta > \theta^* \equiv \frac{(1-\delta-c)(2a-1)^2}{2a(6a-1)} = p_i^*(k^*, k^*)$ the Cournot price. If $\theta < \theta^*$ the unique symmetric SPE involves capacities equal to $\frac{a(1-c-\delta-\theta)}{1+2a} > k^*$. ■

References

[1] **Aumann R. and S. Sorin (1989),** *Cooperation and Bounded Recall,* Games and Economic Behavior, vol 1(1), p 5-39.

[2] **Boccard N. and X. Wauthy. (1998),** *Import Quotas foster minimal differentiation under vertical differentiation,* Revision of CORE DP 9818.

[3] **Bulow J., J. Geanakoplos and P. Klemperer (1985)** *Multiproduct Oligopoly: Strategic Substitutes and Complements* Journal of Political Economy, 93, p 488-511.

[4] **Dastidar K. (1995)** *On the existence of pure strategy Bertrand equilibria* Economic Theory, 5, p 19-32.

[5] **Dastidar K. (1997))** *Comparing Cournot and Bertrand Equilibria in Homogeneous Markets,* Journal of Economic Theory, 75, p 205-212

[6] **Dixit A. (1980)** *The role of investment in entry deterrence* Economic Journal, 90, p 95-106

[7] **Eaton C. and P. Harrald (1992)** *Price versus quantity competition in the Gabszewicz-Thisse model of vertical differentiation* in *Market Strategy and Structure* A. Gee and G. Norman (eds) Harvester Wheatsheaf.

[8] **Edgeworth F. (1925)**, *The theory of pure monopoly*, in Papers relating to political economy, vol. 1, MacMillan, London.

[9] **Kuhn K.-U. (1994)** *Labour Contracts, Product Market Oligopoly and Involuntary Unemployment* Oxford Economic Papers, 46, p 366-384.

[10] **Kreps D. and J. Scheinkman (1983)** *Quantity precommitment and Bertrand competition yields Cournot outcomes* Bell journal of Economics, 14, p 326-337.

[11] **Lutz S. (1997)**, *Vertical Product Differentiation and Entry Deterrence*, Journal of Economics (Zeitschrift für Nationalokönomie), Vol. 65, p 79-102.

[12] **Maggi G. (1996)** *Strategic Trade Policies with Endogenous Mode of Competition* American Economic Review, 86, p 237-258.

[13] **Motta M. and M. Polo (1999)** *Product differentiation and Endogenous Mode of Competition*, Mimeo.

[14] **Shaked A. and J. Sutton (1982)** *Relaxing price competition through product differentiation*, Review of Economic Studies, 49, p 3-13

[15] **Vives X. (1989)** *Cournot and the oligopoly problem* European Economic Review, 33,p.503-514 .

[16] **Vives X. (1990)** *Nash Equilibria with Strategic complementarity* Journal of Mathematical Economics, 19, p 305-321.

[17] **Tirole J. (1988)** *The theory of industrial organization* MIT Press.