

# R&D Networks among Unionized Firms\*

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## Abstract

We develop a model of strategic networks in order to analyze how trade unions will affect the stability and efficiency of R&D collaboration networks in an oligopolistic industry with three firms. Whenever firms settle wages, the complete network is always pairwise stable and the partially connected network is stable if and only if spillovers are large enough. If spillovers are small, the complete network is the efficient network; otherwise, the efficient network is the partially connected network. Thus, a conflict between stability and efficiency may occur: efficient networks are pairwise stable, but the reverse is not true. Strong stability even reinforces this conflict. However, once unions settle wages such conflict disappears: the complete network is the unique pairwise and strongly stable network and is the efficient network whatever the spillovers.

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# 1 Introduction

Traditionally, the theoretical literature has emphasized the role of trade unions in distorting relative prices and the empirical studies have concentrated on the determinants of union membership and on the effects of unions on wages and profitability. More recently, economists have shifted their attention to the long-term effects of trade unions, that is, on investment, technology and productivity growth. Menezes-Filho and Van Reenen (2003) have provided a survey of the economic literature on the impact of trade unions on innovation and R&D. The effects of unions on innovation are generally ambiguous both in theory and in empirical practice. There does, however, seem to be some emerging consensus that there is a negative association between unions and R&D in North America (see Acs and Audretsch (1988), Betts, Odgers and Wilson (2001)). This is not the case for Europe where no such relationship is found (see Schnabel and Wagner (1992), Menezes-Filho, Ulph and Van Reenen (1998)). Despite this evidence, there has not been to date a study of the impact of trade unions on research collaborations between firms in the theoretical literature on R&D in industries with market power.

Many markets are characterized by inter-firm collaboration in R&D activity.<sup>1</sup> Goyal and Moraga-González (2001) have analyzed the incentives for R&D collaboration between horizontally related firms that are not unionized. In a three-firm market for a homogeneous good, they have basically shown that a conflict between the incentives of firms to collaborate and social welfare is likely to occur.<sup>2</sup> The purpose of this paper is to go beyond their analysis by making endogenous the wage formation.

In this paper we address the following questions:

- (i) When the industry is unionized, what are the incentives of firms to collaborate and what is the architecture of "stable" networks of collaboration?
- (ii) Do unions reconcile individual incentives to collaborate and social welfare?

To answer these questions we develop a four-stage game. In the first stage, firms form pairwise collaboration links. The purpose of these collaboration links is to share R&D knowledge about a cost-reducing technology. The collection of pairwise links between the firms defines a network of collaboration. In the second stage, each firm chooses independently and simultaneously a level of effort in R&D. In the third stage, wages are settled at the firm-level. By tractability, we consider two extreme cases of wage formation: (i) each firm chooses its own wage (or there is no union), which is our benchmark; (ii) each union

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<sup>1</sup>See Hagedoorn (2002) who has provided a survey of empirical work on R&D collaboration among firms.

<sup>2</sup>Beside the asymmetric situation among three firms, Goyal and Moraga-González (2001) have analyzed symmetric networks, i.e. networks in which all  $n$  firms maintain the same number of collaborative ties.

chooses the wage, which is the monopoly-union model. The wages and the R&D efforts, along with the network of collaboration, define the costs of the firms. In the fourth stage, firms compete in the oligopolistic market, taking as given the costs of production.

R&D effort of a firm decreases its marginal cost of production. It has also positive spillovers on the costs of firms that are linked to the firm that undertakes R&D effort. We distinguish between direct and indirect R&D collaborations. For instance, suppose firms 1 and 2 collaborate in R&D, firms 2 and 3 collaborate in R&D, while firms 1 and 3 do not collaborate. Then, we say that firms 1 and 2 (2 and 3) have a direct R&D collaboration, while firms 1 and 3 have an indirect R&D collaboration. Knowledge spillovers from direct R&D collaborations are partially absorbed. Spillovers from indirect collaborations are not excluded but are smaller than those obtained from direct R&D collaborations. Moreover, the spillover from indirect collaborations deteriorates in the distance of the relationship. Goyal and Moraga-González (2001) do not assume that spillovers across collaborating firms are related to the distance between firms in the collaborating network. They assume that the research knowledge of a direct collaboration is fully absorbed, while the research knowledge of a no direct collaboration (indirect collaboration or no collaboration at all) is partially absorbed (public spillovers).

A number of theoretical arguments as well as some empirical findings suggest that knowledge spillovers are concentrated in spatial proximity from their respective source. Empirical evidence that knowledge spillovers are concentrated in spatial proximity to the respective source is provided in Acs, Audretsch and Feldman (1992), Audretsch and Feldman (1996), Anselin, Varga and Acs (1997) and Jaffe, Trajtenberg and Henderson (1993). The theoretical explanation is based on the notion that in most cases face-to-face-contacts are necessary for transferring *tacit* knowledge. Fritsch and Franke (2004) have analyzed the impact of spillovers on innovation activities in a German region and examine the significance of R&D cooperation for these knowledge spillovers. They demonstrate that significant differences between regions exist with regard to the productivity of R&D activities. These interregional differences can be explained by R&D spillovers from other R&D activities by actors located in the same region. They also find that R&D cooperation plays only a minor role as a medium for knowledge spillovers. Apparently, cooperative relationships, as such, do not lead to those kinds of knowledge spillovers that are important for the efficiency of innovation activities. In this sense, it seems that spillovers from direct R&D collaborations could not be perfect and that spillovers from indirect collaborations are smaller than those obtained from direct R&D collaborations. Moreover, spillovers from indirect collaborations deteriorates in the distance of the relationship.<sup>3</sup>

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<sup>3</sup>As in the connections model studied by Bala and Goyal (2000) and Jackson and Wolinsky (1996).

A simple way to analyze the networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that agents not benefit from altering the structure of the network. A weak version of such condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no agent benefits from severing one of their links and no other two agents benefit from adding a link between them, with one benefiting strictly and the other at least weakly. But, pairwise stability considers only deviations by at most a pair of agents at a time. It might be that some group of agents could all be made better off by some complicated reorganization of their links, which is not accounted for under pairwise stability. The definition of strong stable networks allows for larger coalitions than just pairs of agents to deviate, and is due to Jackson and van den Nouweland (2004). A strongly stable network is a network which is stable against changes in links by any coalition of agents.<sup>4</sup>

In a three-firm market for a homogeneous good, there are four possible network architectures: the complete network, the star network, the partially connected network, and the empty network. In the complete network every pair of firms is linked. The star network is a network in which there is a "hub" firm directly linked to every other firm, while none of the other firms have a direct link with each other. The partially connected network refers to a configuration in which two firms are linked while the third firm is isolated. In the empty network there are no collaboration links. We find that, whenever firms settle wages, the complete network is always pairwise stable while the partially connected network is stable if and only if spillovers are large enough. Indeed, smaller spillovers destabilize the partially connected network rapidly. The intuition behind this is that the stability of the partially connected network relies on the great cost asymmetry existing between the linked firms and the isolated firm. It is this asymmetry that discourages a linked firm from forming a link with the isolated firm, for large spillovers. As spillovers decrease, this asymmetry reduces, and that destabilizes the partially connected network. However, the complete network is the efficient network if spillovers are small, while the partially connected network is the efficient network if spillovers are large. Thus, a conflict between stability and efficiency may occur: efficient networks are pairwise stable, but the reverse is not true. Moreover, the concept of strong stability even reinforces this conflict: efficient networks are not always strongly stable.

But, once unions settle wages such conflict disappears: the complete network is the unique pairwise and strongly stable network and is the efficient network whatever the spillovers. When firms settle wages, the isolated firm in the partially connected network will tend to be pushed out of the market as spillovers become very large. However,

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<sup>4</sup>Jackson (2003, 2004) provides surveys of models of network formation.



when unions settle wages, a large share of the benefits of the linked firms thanks to cost reductions due to R&D collaborations goes to the unions which diminishes their competitive advantage with respect to the isolated firm. As a consequence, collaborating firms have less incentives to make R&D, meanwhile the isolated firm may even make more R&D effort in presence of unions. In fact unionization reduces considerably the asymmetry between the linked firms and the isolated firm. Thus, unionization destabilizes the partially connected network making the complete network the unique pairwise and strongly stable network. Moreover, social welfare is increasing with the number of collaborative links, and hence, the complete network is the efficient network.

For each network architecture (except the partially connected network), we find that unions reduce research outputs, profits and quantities. In case of the partially connected network, unions reduces research outputs, profits and quantities of collaborating firms. However, unions reduce research outputs, profits and quantities of the isolated firm only if spillovers are very weak. Thus, there is no linear relationship between unions and R&D effort. This relationship depends on the network architecture and on the spillovers.

Before presenting the model, it is worth to mention some related literature. Goyal and Joshi (2003) have studied networks of collaboration between oligopolistic firms that are not unionized.<sup>5</sup> They assume that a collaboration link between two firms involves a fixed cost and leads to an exogenously specified reduction in marginal cost of production. By contrast, in Goyal and Moraga-González (2001) and in our paper the costs of forming links are taken to be negligible, and firms decide independently on a level of R&D, which in turns determines the level of cost reduction endogenously. For general background on R&D cooperation in oligopoly the reader is directed to Amir (2000), d'Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992), Katz (1986) and Suzumura (1992). Finally, Yi and Shin (2000) have analyzed the endogenous formation of research coalitions where coalition formation is modelled in terms of a coalition structure, which is a partition of the set of firms. But the restriction to partitions is a strong one indeed if our interest is in research collaborations, since it rules out situations in which, for example, firms 1 and 2 have a bilateral research agreement and firms 2 and 3 have a similar agreement but there is no agreement between 1 and 3. When this occurs, it is not appropriate to view firms 1, 2 and 3 as one coalition, and we cannot think of 1 and 2 and 2 and 3 being two distinct coalitions, since this violates the mutual exclusiveness property of coalitions. The theory of networks provides a natural way to think of such issues, since it allows for such intransitive relationships.

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<sup>5</sup> Recently, Goyal, Konovalov and Moraga-González (2003) have developed a model of R&D competition and collaboration in which individual firms carry out independent in-house research and also undertake joint research projects with other firms.

The paper is organized as follows. The model is presented in Section 2. In Section 3 we analyze the stability and efficiency of R&D networks, and we comment the aggregate performance of networks. In Section 4 we conclude.

## 2 The model

We develop a four-stage game. In the first stage, firms form pairwise collaboration links. In the second stage, each firm chooses a level of effort in R&D. In the third stage, wages are settled at the firm-level. The wages and the R&D efforts, along with the network of collaboration, define the costs of the firms. In the fourth stage, firms compete in quantities in the oligopolistic market, taking as given the costs of production.

We consider a market for a homogeneous commodity produced by 3 identical profit-maximizing firms. We denote by  $N = \{1, 2, 3\}$  the set of firms which are connected in a network of R&D collaboration. Let  $q_i$  denote the quantities of the commodity produced by firm  $i \in N$ . Let  $P(Q) = a - Q$  be the market-clearing price when aggregate quantity on the market is  $Q \equiv \sum_{i \in N} q_i$ . More precisely,  $P(Q) = a - Q$  for  $Q < a$ , and  $P(Q) = 0$  otherwise, with  $a > 0$ . The firms can undertake R&D to look for cost reducing innovations. The innovation technology is produced under decreasing returns to scale with the sole input  $y$ :  $x_i = \sqrt{y}$ , where  $x_i$  is the research output or effort for firm  $i \in N$ . It follows that the cost function for technology is given by

$$C_i(\gamma, x_i) = \gamma \cdot (x_i)^2, \quad (1)$$

where  $\gamma$  is the price of input  $y$ . We set  $\gamma$  equal to 1. This assumption suffices to ensure nonnegativity of all variables. The production technology is modeled as a Leontief function:

$$q_i = \min \{L_i, \theta_i \cdot K_i\}, \quad (2)$$

where  $L_i$  is labour,  $K_i$  is capital, and  $\theta_i$  is the fixed proportion at which the two factors are combined,  $i \in N$ . This technology gives rise to the cost function for producing the quantity  $q_i$ ,

$$C_i(w_i, r, q_i) = \left(w_i + \frac{r}{\theta_i}\right) \cdot q_i \quad (3)$$

where  $w_i$  is the wage paid by firm  $i$  to its workers and  $r$  is the price of capital which is normalized to one,  $r = 1$ . Associated with each firm there is a risk-neutral union. The workforce for each firm is drawn from separate pools of labour, and the union objective is to maximize the economic rent,

$$U_i(w_i, \bar{w}, L_i) = L_i \cdot (w_i - \bar{w}), \quad (4)$$

where  $\bar{w}$  is the reservation wage. Without loss of generality, the reservation wage is set equal to zero,  $\bar{w} = 0$ .<sup>6</sup>

In a network, firms are the nodes and each link indicates a pairwise R&D collaboration. Thus, a network  $g$  is simply a list of which pair of firms are linked to each other. If we are considering a pair of firms  $i$  and  $j$ , then  $\{i, j\} \in g$  indicates that  $i$  and  $j$  are linked under the network  $g$  and that a R&D collaboration is established between firms  $i$  and  $j$ . For simplicity, write  $ij$  to represent the link  $\{i, j\}$ , so  $ij \in g$  indicates that  $i$  and  $j$  are linked under the network  $g$ . The network obtained by adding link  $ij$  to an existing network  $g$  is denoted  $g + ij$  and the network obtained by deleting link  $ij$  from an existing network  $g$  is denoted  $g - ij$ . For any network  $g$ , let  $N(g) = \{i \in N \mid \exists j \text{ such that } ij \in g\}$  be the set of firms which have at least one link in the network  $g$ . Two firms  $i$  and  $j$  are connected if and only if there exists a sequence of firms  $i_1, \dots, i_K$  such that  $i_k i_{k+1} \in g$  for each  $k \in \{1, \dots, K-1\}$  with  $i_1 = i$  and  $i_K = j$ . Let  $N_i(g)$  be the set of firms which are connected with  $i$ , and let  $M_i(g)$  be the set of firms which have a direct link with  $i$ . Let  $G$  be the set of all possible networks. In this three-firm market, there are four possible network architectures: (i) the complete network,  $g^c$ , in which every pair of firms is linked, (ii) the star network,  $g^s$ , in which there is one firm that is linked to the other two firms, (iii) the partially connected network,  $g^p$ , in which two firms have a link and the third firm is isolated, and (iv) the empty network,  $g^e$ , in which there are no collaboration links. In the star network, the firm which is linked to the other two firms is called the "hub" firm, while the other two firms are called the "spoke" firms.

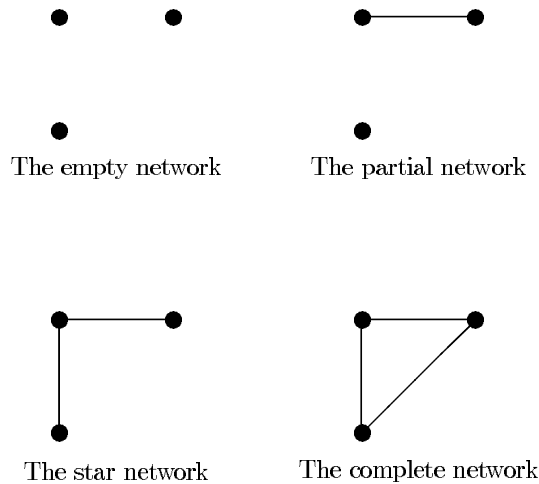


Figure 1: Four possible network architectures.

<sup>6</sup>It can be shown that all results are qualitatively robust to this assumption.

There is a function which relates the research output to the marginal cost of production.<sup>7</sup> This function is a mapping from  $(\{x_i\}_{i \in N}, g)$  to  $\theta_i$ ,

$$\theta_i = \frac{1}{\bar{c} - x_i - \sum_{k \neq i \in M_i(g)} \phi \cdot x_k - \frac{\alpha}{t(i|l)} \sum_{l \in N_i(g) \setminus M_i(g)} \phi \cdot x_l}, \quad (5)$$

where spillovers are assumed and measured by two parameters  $\phi$  and  $\alpha$ . The parameter  $\phi \in (0, 1]$  measures the spillovers obtained from R&D collaborations. Spillovers from indirect collaborations are not excluded but are smaller than those obtained from direct R&D collaborations,  $\alpha \in [0, 1]$ . Moreover, the spillovers from indirect collaborations deteriorate in the distance of the relationship. Let  $t(ij)$  be the number of links in the shortest path between  $i$  and  $j$  (setting  $t(ij) = \infty$  if there is no path between  $i$  and  $j$ ). Given a network  $g$  and the collection of research outputs  $\{x_i\}_{i \in N}$ , the marginal cost of production for each firm  $i \in N$  becomes

$$c_i(g) = w_i + \bar{c} - x_i - \sum_{k \in M_i(g)} \phi \cdot x_k - \frac{\alpha}{t(i|l)} \sum_{l \in N_i(g) \setminus M_i(g)} \phi \cdot x_l. \quad (6)$$

Let

$$X_i \equiv x_i + \sum_{k \in M_i(g)} \phi \cdot x_k + \frac{\alpha}{t(i|l)} \sum_{l \in N_i(g) \setminus M_i(g)} \phi \cdot x_l \quad (7)$$

be the total cost reduction for firm  $i$  obtained from its own research,  $x_i$ , and from the research knowledge of firms connected with  $i$ , which is partially absorbed depending on  $\phi$  and  $\alpha$ . We refer to this total cost reduction,  $X_i$ , as effective R&D output of firm  $i$ . Then,  $c_i(g) = w_i + \bar{c} - X_i$ . Notice that in Goyal and Moraga-González (2001) the effective R&D is defined as  $X_i = x_i + \sum_{k \in M_i(g)} x_k + \mu \sum_{l \notin M_i(g)} x_l$ . Only when  $\phi = 1$  and  $\alpha = 0$  in our model and  $\mu = 0$  in their model, both models coincide.

Thus, the profits of firm  $i \in N$  in a collaboration network  $g$  are given by

$$\Pi_i(g) = \left[ a - q_i(g) - \sum_{j \neq i} q_j(g) - c_i(g) \right] \cdot q_i(g) - [x_i(g)]^2. \quad (8)$$

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<sup>7</sup>Two distinct ways of modelling knowledge spillovers have emerged. (i) d'Aspremont and Jacquemin (1988) regard leakages in technological know-how as taking place in outputs: each firm's final cost reduction is the sum of its autonomously acquired part and a fraction of other firms' parts. (ii) Kamien, Muller and Zang (1992) postulates the presence of a spillover effect on R&D expenses: each firm's effective R&D investment is the sum of its own expenditure and fixed fraction of the sum of other firms' expenditures. Amir (2000) has shown that the two models are not equivalent from a quantitative and qualitative point of view. Invoking some economic principles, Amir has concluded that the Kamien-Muller-Zang model is fully valid while the d'Aspremont-Jacquemin model appears to be of questionable validity for large values of the spillover parameter. However, the d'Aspremont-Jacquemin model may be adequate for certain industries or R&D processes: for instance, technology parks where the benefits firms draw from larger R&D spillovers outweigh the negative effects of increased competition on their profits.

Wages are settled at the firm-level. Two extreme cases are considered: (i) each firm simultaneously chooses the wage that maximizes profits taken as given the wage chosen by the other firms, (ii) each union simultaneously chooses the wage that maximizes the economic rent taken as given the wage chosen by the other unions.

For any network  $g$ , social welfare is defined as the sum of consumer surplus, producers' profits and unions' rents. Let  $W(g)$  denote aggregate welfare in network  $g$ . Then, social welfare is given by

$$W(g) = \frac{[Q(g)]^2}{2} + \sum_{i \in N} \Pi_i(g) + \sum_{i \in N} U_i(g). \quad (9)$$

Before looking for the stability and efficiency of networks, we derive for each possible network architecture, the equilibrium R&D outputs, quantities produced, profits and wages. See the Appendix.

In presence of unions, any competitive advantage of your rival have to be shared with the union. Thus, the competitive advantage due to increasing research effort will be smaller with unions rather than without unions. For instance, a marginal increase of  $x_j$  will reduce  $j$ 's marginal cost, but in presence of unions part of the marginal cost (wage) will increase with  $x_j$  which partially compensate the reduction in the marginal cost of capital. We could say that unions make research efforts less "substitutes". In the empty network  $g^e$  R&D efforts are always strategic substitutes. In the complete network  $g^c$  R&D efforts are strategic substitutes if spillovers are small and become strategic complements when spillovers are large. However, strategic interactions among R&D efforts of different firms become complex in the star network  $g^s$ : (i) R&D efforts of the two "spoke" firms are strategic substitutes when firms settle wages whatever spillovers are; (ii) but when unions settle wages, R&D efforts of the two "spoke" firms are strategic substitutes if spillovers are small and become strategic complements when spillovers are large.; (iii) finally, R&D efforts of the "hub" firm and a "spoke" firm are strategic substitutes if spillovers are small and become strategic complements when spillovers are large. In general, unionization makes it more "likely" that R&D efforts are strategic complements.

**Proposition 1** *In the empty network  $g^e$ , the star network  $g^s$  and the complete network  $g^c$ , at equilibrium, (i) unions reduce research outputs, profits and quantities; (ii) unions increase wages and prices.*

In the partial network  $g^p$ , R&D efforts for the collaborating firms can also be either strategic substitutes or complements depending on the spillovers parameter  $\phi$ . However, the strategic interaction between R&D efforts of a collaborating firm and the isolated one (or the opposite) is of substitution regardless spillovers size and unionization. When firms settle wages, the isolated firm will tend to be pushed out of the market as spillovers become

larger. But once unions settle wages, part of the benefits due to R&D collaboration goes to the unions which diminishes their competitive advantage with respect to the isolated firm. So as  $\phi$  goes to one, the isolated firms will advocate for a unionized industry in order to avoid being pushed out of the industry.

**Proposition 2** *In the partial network,  $g^p$ , at equilibrium, (i) unions reduce research outputs, profits and quantities of collaborating firms; (ii) unions reduce research outputs of the non-collaborating firm if and only if spillovers are weak ( $\phi < 0.547$ ); (iii) unions reduce profits of the non-collaborating firm if and only if spillovers are weak ( $\phi < 0.633$ ); (iv) unions reduce quantities of the non-collaborating firm if and only if spillovers are very weak ( $\phi < 0.275$ ); and (v) unions increase wages and prices.*

In Figure 2 and Figure 3 we plot the individual R&D outputs when firms settle wages and unions settle wages, respectively. We observe that, if unions choose wages, then R&D output of a firm is decreasing with the number of links the firm has and with the spillover parameter  $\phi$ . If firms settle wages, then individual R&D output still decreases with the spillover parameter  $\phi$ , except for the firms that collaborate in the partial network and for the "hub" firm in the star network. Indeed, the research effort made by the "hub" firm may increase or decrease with  $\phi$  depending on how large spillovers are. As  $\phi$  goes from zero to one, research effort first increases with  $\phi$ , then it starts to decrease with  $\phi$ . But, the relationship between individual R&D output and the number of links becomes much more complex. However, aggregate R&D output is decreasing with the spillover parameter  $\phi$  and with the number of collaborations, whatever the mode of wage settlement and the network architecture.

It is also interesting to analyze the evolution of effective R&D since it is a measure of the reduction in marginal cost. In Figure 4 and Figure 5 we plot effective R&D outputs when firms settle wages and unions settle wages, respectively. We observe that, if unions settle wages, effective R&D output of any firm (except the isolated firm in  $g^p$ ) increases with the spillover parameter  $\phi$ , except for very large spillovers. If firms settle wages, effective R&D output of any firm (except firms in  $g^p$ ) first increases with  $\phi$ , then it decreases with  $\phi$ , and reaches a maximum for values of  $\phi$  close to  $\frac{1}{2}$ . A change in  $\phi$  has a twofold effect: it increases the effect of one unit of R&D output on the whole network and reduces the individual R&D output. Which one of the two effects dominates the other determines the relationship between effective R&D and  $\phi$ .

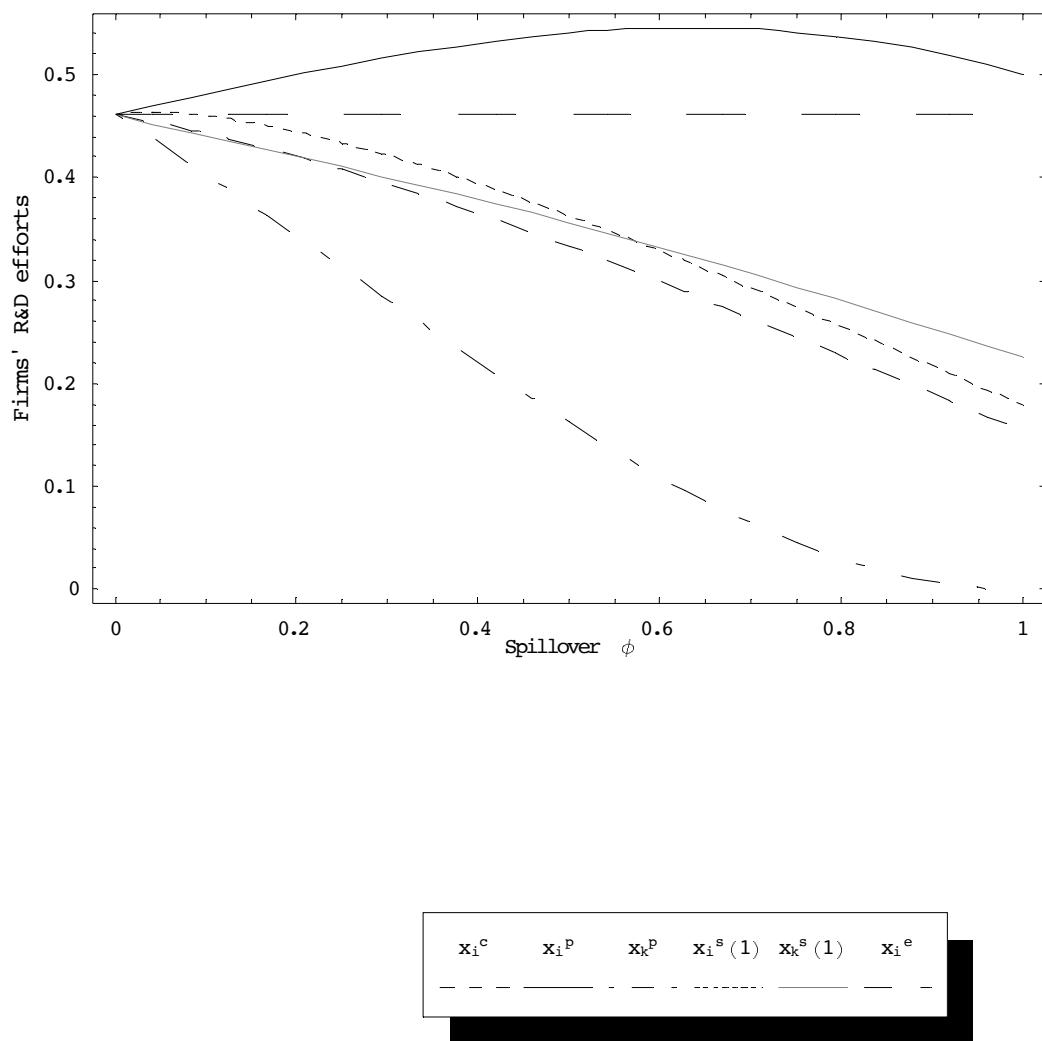


Figure 2: Firms' R&D outputs when firms settle wages and  $\alpha = 1$ .

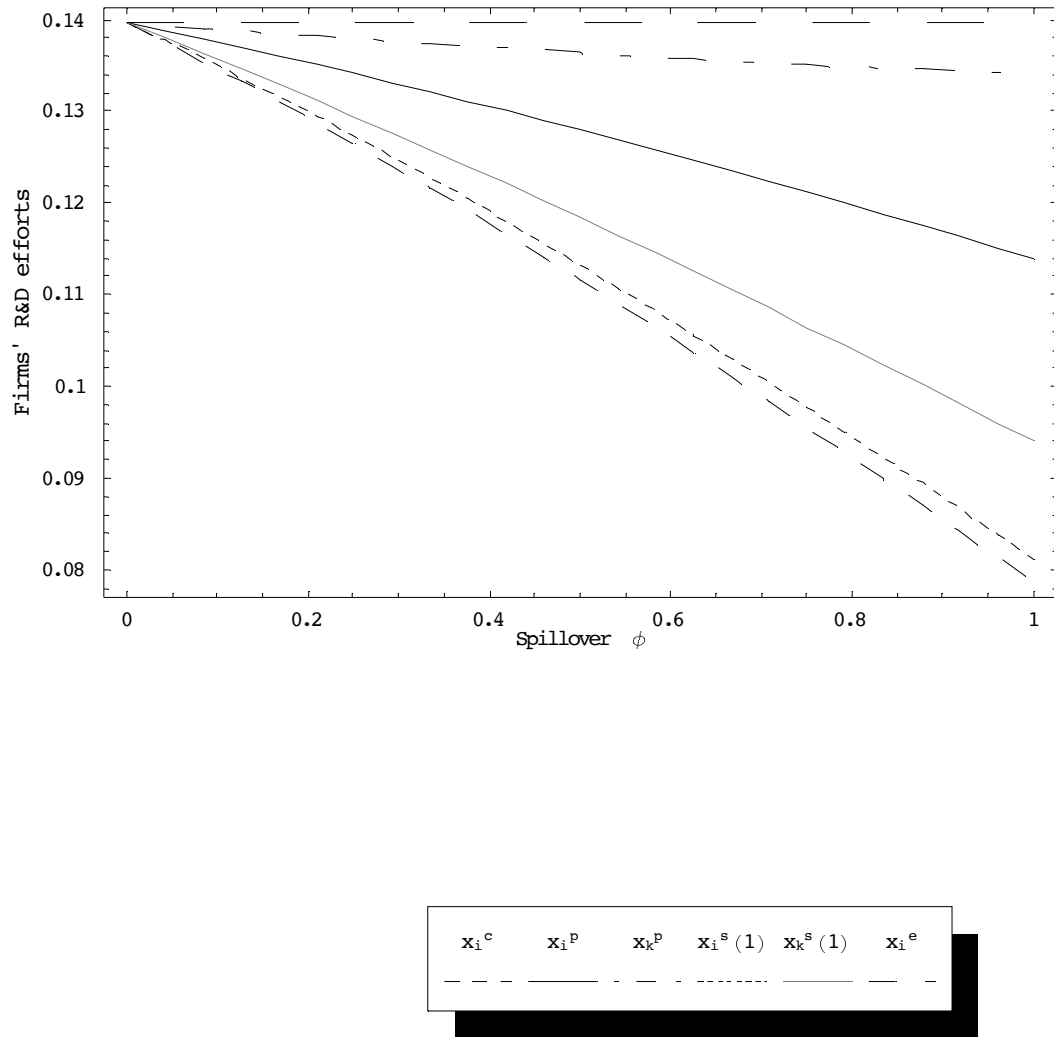


Figure 3: Firms' R&D outputs when unions settle wages and  $\alpha = 1$ .



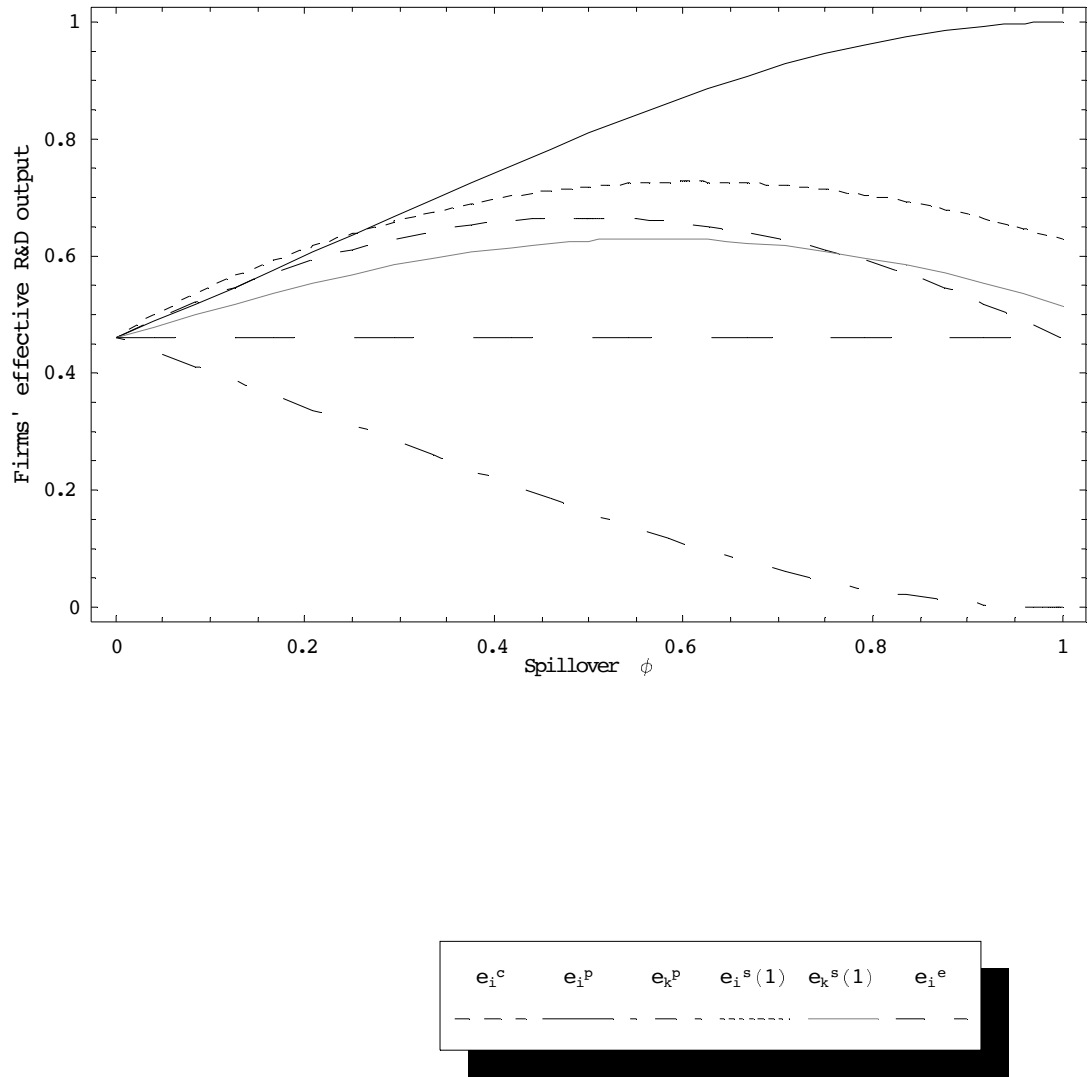


Figure 4: Firms' effective R&D when firms settle wages and  $\alpha = 1$ .

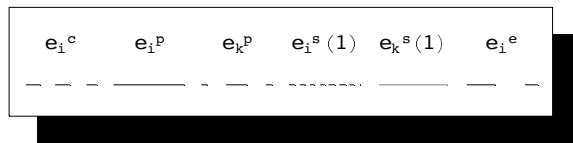
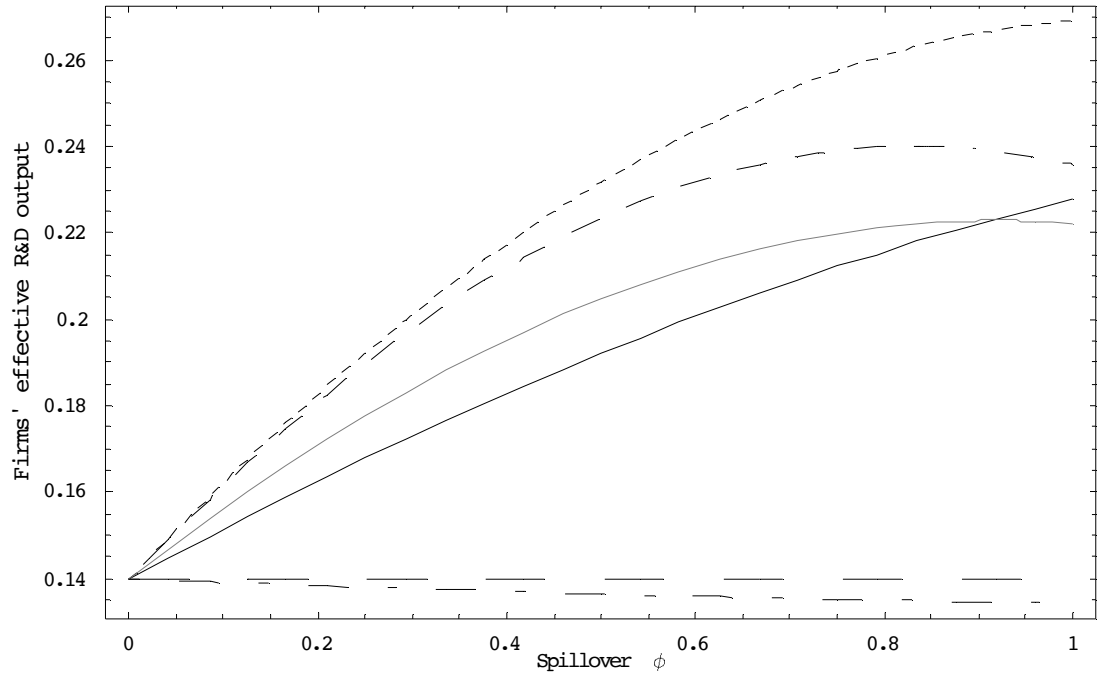


Figure 5: Firms' effective R&D when unions settle wages and  $\alpha = 1$ .

### 3 Stability and efficiency of R&D networks

#### 3.1 Pairwise stable networks

A simple way to analyze the networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that agents not benefit from altering the structure of the network. A weak version of such condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no agent benefits from severing one of their links and no other two agents benefit from adding a link between them, with one benefiting strictly and the other at least weakly.

**Definition 1** *A network  $g$  is pairwise stable if*

- *for all  $ij \in g$ ,  $\Pi_i(g) \geq \Pi_i(g - ij)$  and  $\Pi_j(g) \geq \Pi_j(g - ij)$ , and*
- *for all  $ij \notin g$ , if  $\Pi_i(g) < \Pi_i(g + ij)$  then  $\Pi_j(g) > \Pi_j(g + ij)$ .*

Let us say that  $g'$  is adjacent to  $g$  if  $g' = g + ij$  or  $g' = g - ij$  for some  $ij$ . A network  $g'$  defeats  $g$  if either  $g' = g - ij$  and  $\Pi_i(g') \geq \Pi_i(g)$ , or if  $g' = g + ij$  with  $\Pi_i(g') \geq \Pi_i(g)$  and  $\Pi_j(g') \geq \Pi_j(g)$  with at least one inequality holding strictly. Pairwise stability is equivalent to saying that a network is pairwise stable if it is not defeated by another (necessarily adjacent) network. This definition of stability is quite weak and should be seen as a necessary condition for strategic stability.

We are interested in the networks of R&D collaboration that emerge in two different settings: (i) firms choose wages, (ii) unions choose wages. Throughout the paper we use the symbol  $f$  ( $u$ ) to indicate that the firm (union) chooses the wage. We first study pairwise stable networks when firms settle wages.

**Proposition 3** *Suppose firms settle wages. (i) The complete network  $g^c$  is always pairwise stable, (ii) the partially connected network  $g^p$  is pairwise stable if and only if spillovers are large enough,  $\phi \geq \hat{\phi}(\alpha)$ , (iii) the star and empty networks (respectively,  $g^s$  and  $g^e$ ) are never pairwise stable.*

**Proof.** First we show that the complete network  $g^c$  is always pairwise stable. No pair of firms  $i$  and  $j$  have incentives to delete their link  $ij \in g^c$ . That is,  $\Pi_i^*(g^c, f) > \Pi_i^*(g^s, f)$  and  $\Pi_j^*(g^c, f) > \Pi_j^*(g^s, f)$  with  $ij \notin g^s$ . Let

$$A_1 = 52 + \phi(284 - 20\alpha - (160 + (14 - 5\alpha)\alpha)\phi + 2(36 + \alpha(8 + \alpha))\phi^2 - 8(2 + \alpha)\phi^3).$$

Since

$$\begin{aligned}\Pi_i^*(g^c, f) &= \Pi_j^*(g^c, f) = \frac{(7 + 4(3 - \phi)\phi)(a - \bar{c})^2}{(13 - 4\phi(1 - \phi))^2} > \\ \Pi_i^*(g^s, f) &= \Pi_j^*(g^s, f) = \frac{4(14 - (2 + \alpha)\phi)(2 + (2 + \alpha)\phi)(1 + (5 - 2\phi)\phi)^2(a - \bar{c})^2}{(A_1)^2}\end{aligned}$$

with  $ij \notin g^s$ , it follows that  $g^c$  is pairwise stable. Obviously, the star network  $g^s$  cannot be pairwise stable since firms  $i$  and  $j$  have incentives to form the link  $ij \notin g^s$ .

Second, the empty network  $g^e$  is never pairwise stable. That is,  $\Pi_i^*(g^p, f) > \Pi_i^*(g^e, f)$  and  $\Pi_j^*(g^p, f) > \Pi_j^*(g^e, f)$  with  $ij \in g^p$ . Since

$$\Pi_i^*(g^p, f) = \frac{(7 - \phi)(1 + \phi)(a - \bar{c})^2}{(13 - 5\phi(2 - \phi))^2} > \frac{7(a - \bar{c})^2}{(13)^2} = \Pi_i^*(g^e, f), \text{ with } i \in N(g^p),$$

it follows that  $g^e$  is not pairwise stable.

Third, the partially connected network  $g^p$  is pairwise stable if the spillovers are sufficiently large. Since the empty network is never pairwise stable, the network  $g^p$  is pairwise stable if and only if  $\Pi_i^*(g^p, f) > \Pi_i^*(g^s, f)$  or  $\Pi_j^*(g^p, f) > \Pi_j^*(g^s, f)$  with  $ij \notin g^p$ ,  $ij \in g^s$ , and  $j \notin N(g^p)$ . Since

$$\begin{aligned}\Pi_j^*(g^p, f) &= \frac{7(1 - \phi)^4(a - \bar{c})^2}{(13 - 5\phi(2 - \phi))^2} < \\ \Pi_j^*(g^s, f) &= \frac{4(14 - (2 + \alpha)\phi)(2 + (2 + \alpha)\phi)(1 + (5 - 2\phi)\phi)^2(a - \bar{c})^2}{(A_1)^2};\end{aligned}$$

$g^p$  is pairwise stable if and only if

$$\begin{aligned}\Pi_i^*(g^p, f) &= \frac{(7 - \phi)(1 + \phi)(a - \bar{c})^2}{(13 - 5\phi(2 - \phi))^2} > \\ \Pi_i^*(g^s, f) &= \frac{(7 - 2\alpha)(1 + 2\phi)(4 + 4(7 - \alpha)\phi - (4 - \alpha)(2 + \alpha)\phi^2)^2(a - \bar{c})^2}{(A_1)^2}.\end{aligned}$$

Let  $\hat{\phi}(\alpha)$  be a cutoff function which gives the value of  $\phi$  such that  $\Pi_i^*(g^p, f) = \Pi_i^*(g^s, f)$ ,  $\hat{\phi}(\alpha)$  is decreasing with  $\alpha$ , is bounded above by  $\hat{\phi}(\alpha = 0) = 0.551$ , and is bounded below by  $\hat{\phi}(\alpha = 1) = 0.285$ . Then,  $g^p$  is pairwise stable if and only if  $\phi \geq \hat{\phi}(\alpha)$ . ■

Using Figures 6 and 7 we can study the stability of different networks. (i) The empty network  $g^e$  is never stable because two firms have incentives to collaborate. (ii) The star network  $g^s$  is never stable, because the "spoke" firms that have only one link have incentives to link to each other. Thus, the complete network  $g^c$  is always pairwise stable. (iii) Whether the partially connected network  $g^p$  is stable will depend on spillovers  $\phi$  and  $\alpha$ . If spillovers are large enough, the isolated firm has a significant cost disadvantage

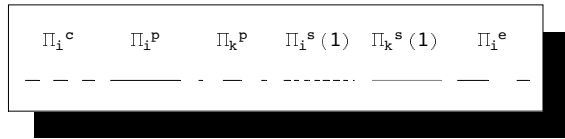
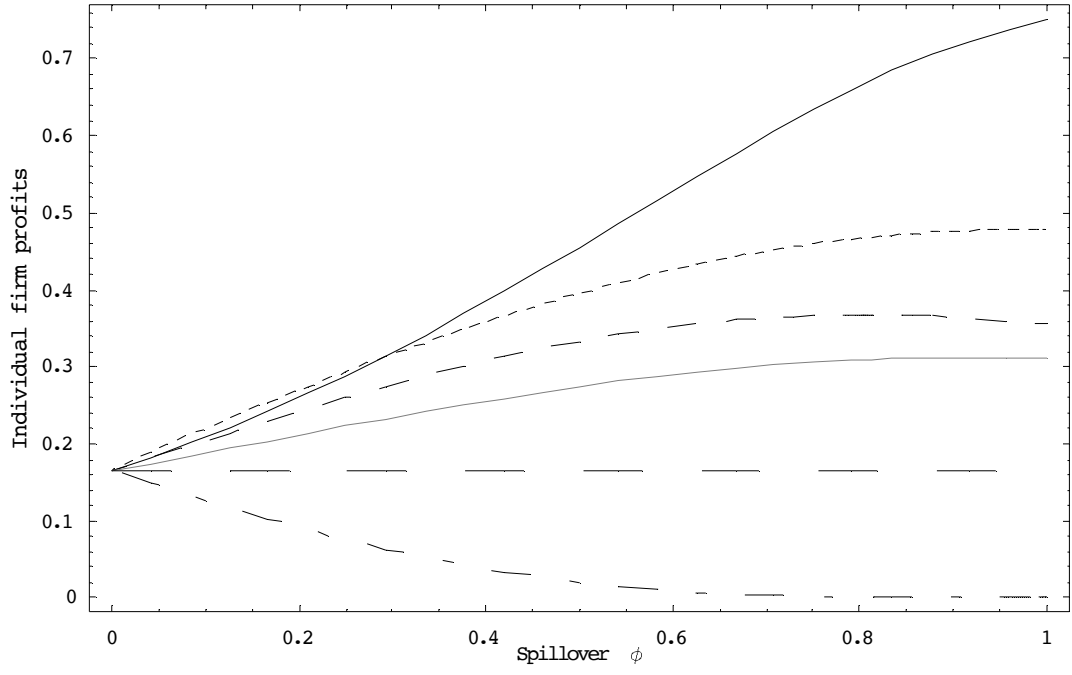


Figure 6: Individual firm profits when firms settle wages and  $\alpha = 1$ .

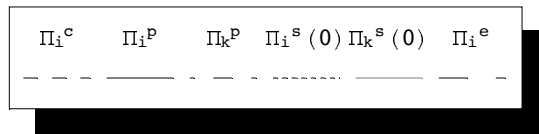
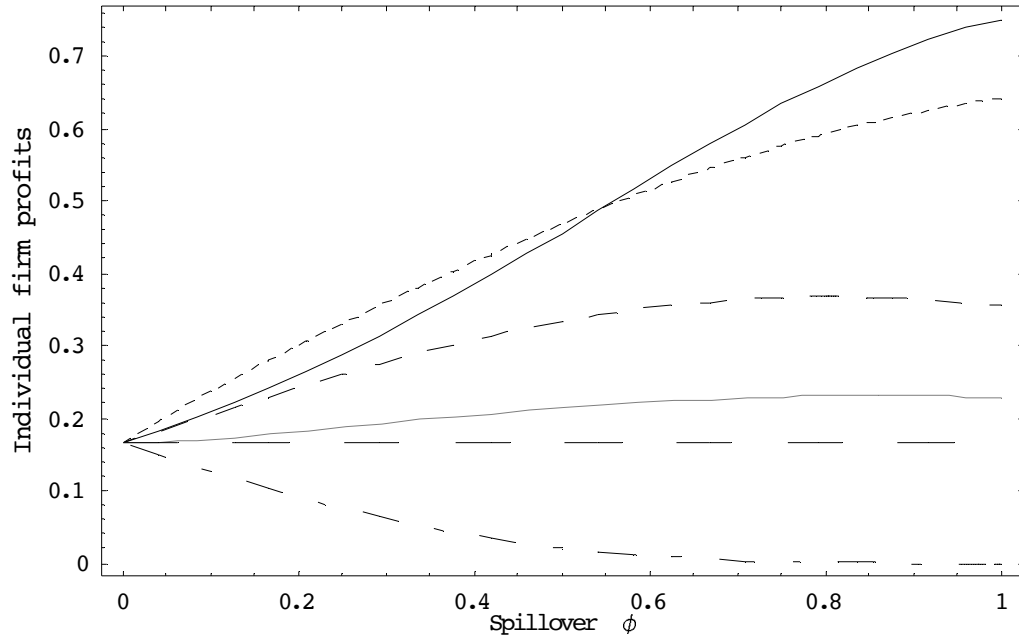


Figure 7: Individual firm profits when firms settle wages and  $\alpha = 0$ .

and it will tend to be pushed out of the market as spillovers become very large. Thus, collaborating firms may decide to keep isolated the third firm and to divide between them most of the market letting only a small share to the isolated firm, rather than forming a star network by offering a collaboration link to the isolated firm. On the contrary, if spillovers are small, collaborating firms have incentives to link with the isolated firm in order to become the "hub" firm in the star network and to benefit from cost reductions due to the increase of effective R&D. The gains due to the increase of effective R&D are not offset by the increase in product competition. The former isolated firm is more competitive under the star network because it benefits from direct spillovers from the "hub" firm and from indirect spillovers from the other "spoke" firm.

As  $\phi$  decreases, the profits of the firms in the different networks become similar, irrespective of the network structure (in the limiting case  $\phi \rightarrow 0$  the profits are all equal). Thus, network structures become more important when direct spillovers are large.<sup>8</sup> Another observation concerns the impact of spillovers on the stability of different networks. Smaller spillovers (direct and indirect) destabilize the partially connected network rapidly.<sup>9</sup> The intuition behind this is that the stability of the partially connected network relies on the great cost asymmetry existing between the linked firms and the isolated firm. It is this asymmetry that discourages a linked firm from forming a link with the isolated firm, for large direct spillovers and large indirect spillovers. As  $\phi$  decreases, this asymmetry reduces, and that destabilizes the partially connected network  $g^p$ . Moreover, the larger  $\phi$  and  $\alpha$  are, the smaller the cost asymmetry existing between firms in the star network is, and the smaller cost advantage the "hub" firm has. In contrast, the complete network remains stable for all values of  $\phi$ ; we note however that the losses from deleting a link diminish as  $\phi$  decreases and as  $\alpha$  increases (in this sense the complete network becomes more vulnerable with decreasing  $\phi$  and increasing  $\alpha$ ).

We now study pairwise stable networks when unions settle wages.

**Proposition 4** *Suppose unions settle wages. The complete network  $g^c$  is the unique pairwise stable network.*

**Proof.** First we show that the complete network  $g^c$  is always pairwise stable. No pair of firms  $i$  and  $j$  have incentives to delete their link  $ij \in g^c$ . That is,  $\Pi_i^*(g^c, u) > \Pi_i^*(g^s, u)$

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<sup>8</sup>Goyal and Moraga-González (2001) found that network structures are more important when public spillovers are modest. This is why we assume no public spillovers.

<sup>9</sup>The smaller the spillovers from indirect collaborations are, the larger the spillovers from direct collaborations have to be in order to make the partially connected network  $g^p$  pairwise stable.

and  $\Pi_j^*(g^c, u) > \Pi_j^*(g^s, u)$  with  $ij \notin g^s$ . Let

$$\begin{aligned} A_2 &= 4468900 + 9\phi(114060 - 20060\alpha - 3(11904 - \alpha(302 + 1003\alpha))\phi \\ &\quad + 54(156 + \alpha(32 + 3\alpha))\phi^2 - 648(2 + \alpha)\phi^3). \end{aligned}$$

Since

$$\begin{aligned} \Pi_i^*(g^c, u) &= \frac{9(151 - 18\phi)(73 + 18\phi)(a - \bar{c})^2}{(675 - 36\phi(5 - 3\phi))^2} > \\ \Pi_i^*(g^s, u) &= \frac{36(302 - 9(2 + \alpha)\phi)(146 + 9(2 + \alpha)\phi)(667 + 9(19 - 6\phi)\phi)^2(a - \bar{c})^2}{(A_2)^2} \\ &= \Pi_j^*(g^s, u) \end{aligned}$$

with  $ij \notin g^s$ , it follows that  $g^c$  is pairwise stable. Obviously, the star network  $g^s$  cannot be pairwise stable since firms  $i$  and  $j$  have incentives to form the link  $ij \notin g^s$ .

Second, the empty network  $g^e$  is never pairwise stable. That is,  $\Pi_i^*(g^p, u) > \Pi_i^*(g^e, u)$  and  $\Pi_j^*(g^p, u) > \Pi_j^*(g^e, u)$  with  $ij \in g^p$ . Since

$$\Pi_i^*(g^p, u) = \frac{4004001(151 - 9\phi)(73 + 9\phi)(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2} > \frac{99207(a - \bar{c})^2}{2805625} = \Pi_i^*(g^e, u),$$

with  $i \in N(g^p)$ , it follows that  $g^e$  is not pairwise stable.

Third, the partially connected network  $g^p$  is never pairwise stable. That is,  $\Pi_i^*(g^s, u) > \Pi_i^*(g^p, u)$  and  $\Pi_j^*(g^s, u) > \Pi_j^*(g^p, u)$  with  $ij \notin g^p$ ,  $ij \in g^s$  and  $i \notin N(g^p)$ . Since we have

$$\begin{aligned} \Pi_i^*(g^s, u) &= \frac{36(302 - 9(2 + \alpha)\phi)(146 + 9(2 + \alpha)\phi)(667 + 9(19 - 6\phi)\phi)^2(a - \bar{c})^2}{(A_2)^2} > \\ \Pi_i^*(g^p, u) &= \frac{99207(667 - 9\phi(10 - 3\phi))^2(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2}, \end{aligned}$$

and

$$\begin{aligned} \Pi_j^*(g^p, u) &= \frac{4004001(151 - 9\phi)(73 + 9\phi)(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2} < \\ \Pi_j^*(g^s, u) &= \frac{9(151 - 18\phi)(73 + 18\phi)(2668 + 36(29 - 5\alpha)\phi - 27(4 - \alpha)(2 + \alpha)\phi^2)^2(a - \bar{c})^2}{(A_2)^2}, \end{aligned}$$

with  $ij \notin g^p$ ,  $ij \in g^s$ ,  $i \notin N(g^p)$ ,  $j \in N(g^p)$ ,  $g^p$  is never pairwise stable. ■

Using Figure 8 for  $\alpha = 1$  we can study the stability of different networks. This analysis goes through for all values of  $\alpha \in [0, 1]$ .<sup>10</sup> (i) The empty network  $g^e$  is still never stable. (ii) The star network  $g^s$  is never stable either. Indeed, "spoke" firms that have only one

<sup>10</sup>Notice that  $\alpha = 1$  makes the star network less asymmetric than with  $\alpha < 1$ . If the partially connected network  $g^p$  is not pairwise stable for  $\alpha = 1$ , then for sure  $g^p$  is not pairwise stable for  $\alpha < 1$ .



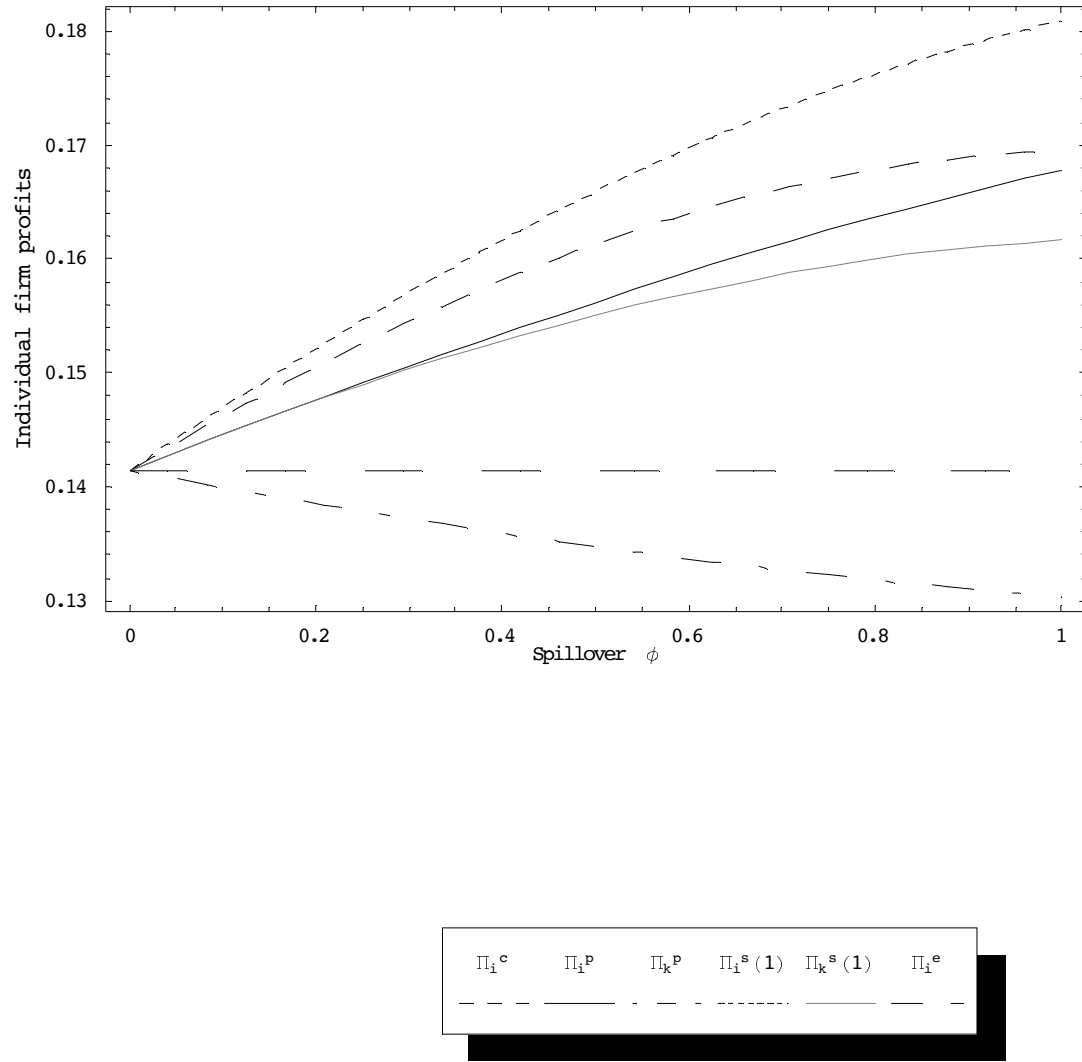


Figure 8: Individual firm profits when unions settle wages and  $\alpha = 1$ .

link have still incentives to link to each other. Thus, the complete network  $g^c$  is pairwise stable. (iii) But, once the unions settle wages, the partially connected network  $g^p$  is no longer stable even when spillovers  $\phi$  are large. Without unions, the isolated firm will tend to be pushed out of the market as spillovers become very large. However, under unionization, a large share of the benefits of the linked firms thanks to cost reductions due to R&D collaborations goes to the unions which diminishes their competitive advantage with respect to the isolated firm. As a consequence, collaborating firms have less incentives to make R&D, meanwhile the isolated firm may even make more R&D effort in presence of unions. Even when  $\phi$  goes to one the isolated firm maintains a significant market share. In fact unionization reduces considerably the asymmetry between the linked firms and the isolated firm. Thus, unionization destabilizes  $g^p$  making  $g^c$  the unique pairwise stable network.

### 3.2 Strongly stable networks

While pairwise stability is natural and quite easy to work with, there are some limitations of the concept. First, it is a weak notion in that it only considers deviations on a single link at a time. For instance, it could be that an agent would not benefit from severing any single link but would benefit from severing several links simultaneously, and yet the network would still be pairwise stable. Second, pairwise stability considers only deviations by at most a pair of agents at a time. It might be that some group of agents could all be made better off by some complicated reorganization of their links, which is not accounted for under pairwise stability.

Alternatives to pairwise stability that allow for larger coalitions than just pairs of agents to deviate were first considered by Dutta and Mutuswami (1997). The definition of strong stable networks is in that spirit, and is due to Jackson and van den Nouweland (2004). A strongly stable network is a network which is stable against changes in links by any coalition of agents.

A network  $g' \in G$  is obtainable from  $g \in G$  via deviations by  $S$  if

- (i)  $ij \in g'$  and  $ij \notin g$  implies  $ij \subset S$ , and
- (ii)  $ij \in g$  and  $ij \notin g'$  implies  $ij \cap S \neq \emptyset$ .

The above definition identifies changes in a network that can be made by a coalition  $S$ , without the need of consent of any agents outside of  $S$ . Part (i) requires that any new links that are added can only be between agents in  $S$ . This reflects the fact that consent of both agents is needed to add a link. Part (ii) requires that at least one agent of any

deleted link be in  $S$ . This reflects the fact that either agent in a link can unilaterally sever the relationship.

**Definition 2** *A network  $g$  is strongly stable if for any  $S \subset N$ ,  $g'$  that is obtainable from  $g$  via deviations by  $S$ , and  $i \in S$  such that  $\Pi_i(g') > \Pi_i(g)$ , there exists  $j \in S$  such that  $\Pi_j(g') < \Pi_j(g)$ .*

Strong stability provides a powerful refinement of pairwise stability. The concept of strong stability mainly makes sense in smaller network situations where agents have substantial information about the overall structure and potential payoffs and can coordinate their actions. That is, it makes sense to model agreements between firms in an oligopoly.

**Proposition 5** *Suppose firms settle wages. If  $\phi \geq \hat{\phi}(\alpha)$  the partially connected network  $g^p$  is the unique strongly stable network. Otherwise, no network  $g \in G$  is strongly stable.*

**Proof.** First, since strong stability is a refinement of pairwise stability, we have that the empty and star networks are never strongly stable. Second, we show that the complete network  $g^c$  is never strongly stable. Indeed, we have  $\Pi_i^*(g^p, f) > \Pi_i^*(g^c, f)$  and  $\Pi_j^*(g^p, f) > \Pi_j^*(g^c, f)$  with  $ij \in g^p$ , where

$$\begin{aligned}\Pi_i^*(g^c, f) &= \Pi_j^*(g^c, f) = \frac{(7 + 4(3 - \phi)\phi)(a - \bar{c})^2}{(13 - 4\phi(1 - \phi))^2} < \\ \Pi_i^*(g^p, f) &= \Pi_j^*(g^p, f) = \frac{(7 - \phi)(1 + \phi)(a - \bar{c})^2}{(13 - 5\phi(2 - \phi))^2}.\end{aligned}$$

Third, from Proposition 3 we know that if  $\phi < \hat{\phi}(\alpha)$  then the partially connected network is not pairwise stable, and so is not strongly stable; where  $\hat{\phi}(\alpha)$  is a cutoff function which gives the value of  $\phi$  such that  $\Pi_i^*(g^p, f) = \Pi_i^*(g^s, f)$ , with  $i \in N(g^p)$  and  $i$  having two links in  $g^s$ . But, if  $\phi \geq \hat{\phi}(\alpha)$ , then  $g^p$  is pairwise stable. Is  $g^p$  strongly stable too? Since  $g^p$  is pairwise stable, it suffices to show that no coalition has incentives to add links to form the complete network  $g^c$ . The answer is no since  $\Pi_i^*(g^p, f) > \Pi_i^*(g^c, f)$  and  $\Pi_j^*(g^p, f) > \Pi_j^*(g^c, f)$  with  $ij \in g^p$  as shown above. So, if  $\phi \geq \hat{\phi}(\alpha)$  then  $g^p$  is the unique strongly stable network, and if  $\phi < \hat{\phi}(\alpha)$  then no network is strongly stable. ■

Since a strongly stable network is a pairwise stable network, the only two candidates to be strongly stable are  $g^p$  and  $g^c$  when firms settle wages. First, we consider the case when both  $g^p$  and  $g^c$  are pairwise stable. That is, if  $\phi \geq \hat{\phi}(\alpha)$ . Using Figures 6 and 7 we see that the complete network  $g^c$  is not strongly stable because two firms have incentives to form a coalition and to delete their links with the third firm; so moving to the partially connected network  $g^p$ . Such deviation was not allowed with pairwise stability. Thus,  $g^p$

is the unique strongly stable network when spillovers are large,  $\phi \geq \widehat{\phi}(\alpha)$ . Second, we consider the case when only  $g^c$  is pairwise stable. That is, if  $\phi < \widehat{\phi}(\alpha)$ . From Figures 6 and 7 we observe that  $g^c$  is never strongly stable.

We now consider the situation when unions settle wages.

**Proposition 6** *Suppose unions settle wages. The complete network  $g^c$  is the unique strongly stable network.*

**Proof.** First, since strong stability is a refinement of pairwise stability, we have that the empty, partially connected and star networks are never strongly stable. Second, we show that the complete network  $g^c$  is always strongly stable. From Proposition 4 we know that the complete network is always pairwise stable. It suffices to show that no coalition of firms have incentives to delete links to form either the partially connected network or the empty network. Since

$$\begin{aligned}\Pi_i^*(g^c, u) &= \frac{9(151 - 18\phi)(73 + 18\phi)(a - \bar{c})^2}{(675 - 36\phi(5 - 3\phi))^2} > \\ \Pi_i^*(g^p, u) &= \frac{4004001(151 - 9\phi)(73 + 9\phi)(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2} > \\ \Pi_i^*(g^e, u) &= \frac{99207(a - \bar{c})^2}{1675^2} > \\ \Pi_j^*(g^p, u) &= \frac{99207(667 - 9\phi(10 - 3\phi))^2(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2},\end{aligned}$$

we have that  $\Pi_i(g^c) > \Pi_i(g^p) > \Pi_i(g^e) > \Pi_j(g^p)$ , with  $i \in N(g^p)$  and  $j \notin N(g^p)$ , and so the complete network  $g^c$  is strongly stable for  $\phi \in (0, 1]$ . ■

Using Figure 8 we observe that  $g^c$  is strongly stable whatever  $\phi > 0$  since a coalition of two firms never has incentives to form and to delete its links with the third firm. The intuition is that unionization again reduces the asymmetry of the partially connected network  $g^p$ . Thus, the strongly stable network that will emerge in the long run is different whether firms settle wages or unions settle wages.

What would happen if unions had a word to say in the decision about R&D collaborations? One extreme case is a situation where unions decide about links instead of firms. Using Figure 9 we observe that (i)  $g^e$ ,  $g^p$ , and  $g^s$  are never pairwise, (ii)  $g^c$  is the unique pairwise stable network. Is  $g^c$  strongly stable too? If  $\phi < 0.663$  then  $g^c$  is strongly stable, otherwise no network is strongly stable. We conclude that in terms of network architecture, firms and unions aspirations are very close.

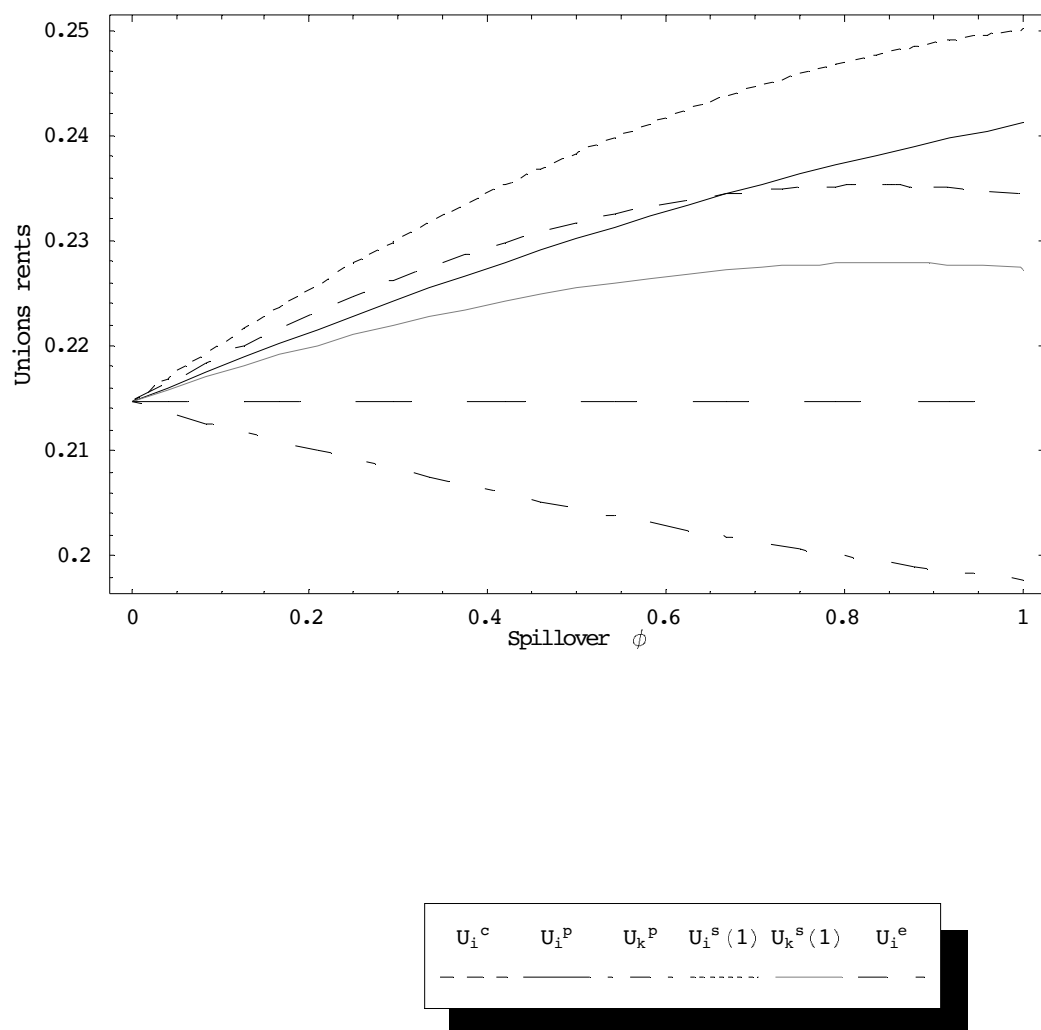


Figure 9: Unions' rents when  $\alpha = 1$ .

### 3.3 Aggregate performance of networks

We now explore the aggregate performance of different networks. In Figure 10 and Figure 11 we plot the aggregate profits of firms when firms settle wages and unions settle wages, respectively. Remember that the symbol  $f$  ( $u$ ) indicates that firms (unions) settle wages. Define  $\phi_{TP}$  as the solution to equation  $\sum_i \Pi_i(g^c, f) = \sum_i \Pi_i(g^p, f)$ . Figure 10 shows that  $\phi_{TP}$  exists and is unique, and reveals that if  $\phi < \phi_{TP}$  then  $g^c$  is the network that maximizes aggregate profits when firms settle wages, otherwise it is  $g^p$ . Notice that aggregate profits are not always increasing with the number of collaborations. We now provide some intuition for this pattern. When spillovers are large, the isolated firm tends to be pushed out of the market and the collaborating firms will obtain profits close to the duopoly case which are greater than those obtained in the complete network where all firms have equal market share. As  $\phi \rightarrow 1$  we converge to a situation where in  $g^p$  two firms collaborate in R&D and share the whole market, while in  $g^c$  three firms collaborate in R&D and share the whole market. However, we observe in Figure 11 that the complete network  $g^c$  dominates in terms of aggregate profits when unions settle wages. Moreover, aggregate industry profits are increasing with the number of collaborations and with the spillover parameter  $\phi$ .

In Figure 12 and Figure 13 we plot the aggregate production of the industry when firms settle wages and unions settle wages, respectively. Define  $\phi_{Q1}$  as the solution to equation  $Q(g^c, f) = Q(g^s, f, \alpha)$  and  $\phi_{Q2}$  as the solution to equation  $Q(g^p, f) = Q(g^s, f, \alpha)$ . We have that, if  $\phi < \phi_{Q1}$  then  $g^c$  is the network which maximizes aggregate production. Aggregate production is increasing with the number of collaborations. If  $\phi \in (\phi_{Q1}, \phi_{Q2})$  then  $g^s$  is the network which maximizes aggregate production. Finally, if  $\phi > \phi_{Q2}$  then  $g^p$  maximizes aggregate production. So, when spillovers are large, intermediate levels of collaborations maximize aggregate production of the industry. Notice that if spillovers are small, aggregate production is increasing with the spillover parameter  $\phi$ . But, when spillovers become large, aggregate production is decreasing with  $\phi$ , except for  $g^p$ . In case of unionization, aggregate production is increasing with the number of collaborations except for very large spillovers. Finally, notice that total effective R&D and aggregate unions rents have a shape very close to the plot of the aggregate production.

### 3.4 Efficient networks

We now examine social welfare under the different networks. To compute social welfare  $W(g)$  under a network  $g$  we substitute equilibrium quantities and profits in the social welfare expression (9). These computations are given in the appendix. We say that a network  $g$  is efficient if and only if  $W(g) \geq W(g')$  for all  $g'$ . In Figure 14 we plot the

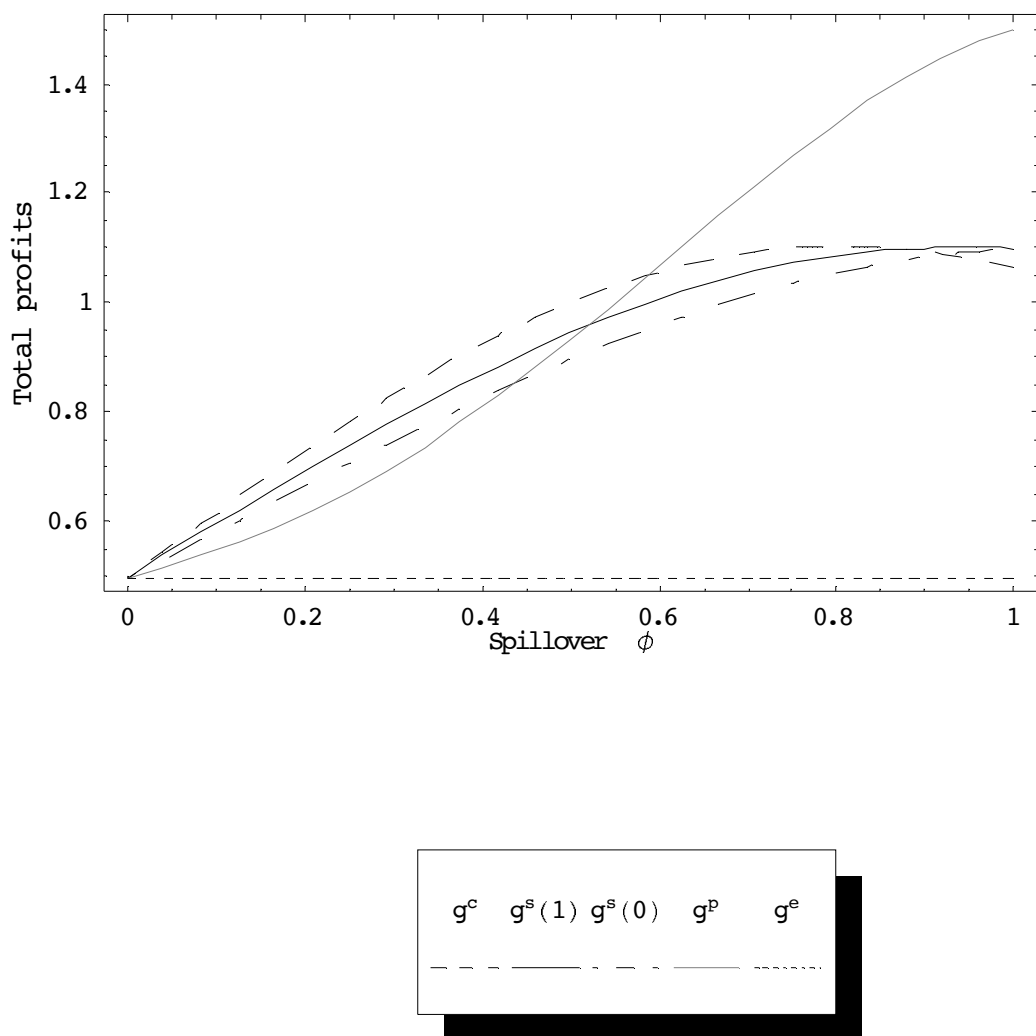


Figure 10: Total profits when firms settle wages.

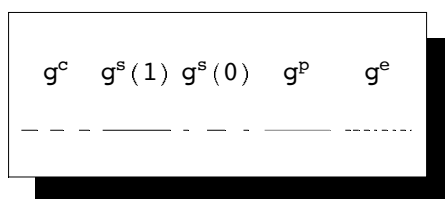
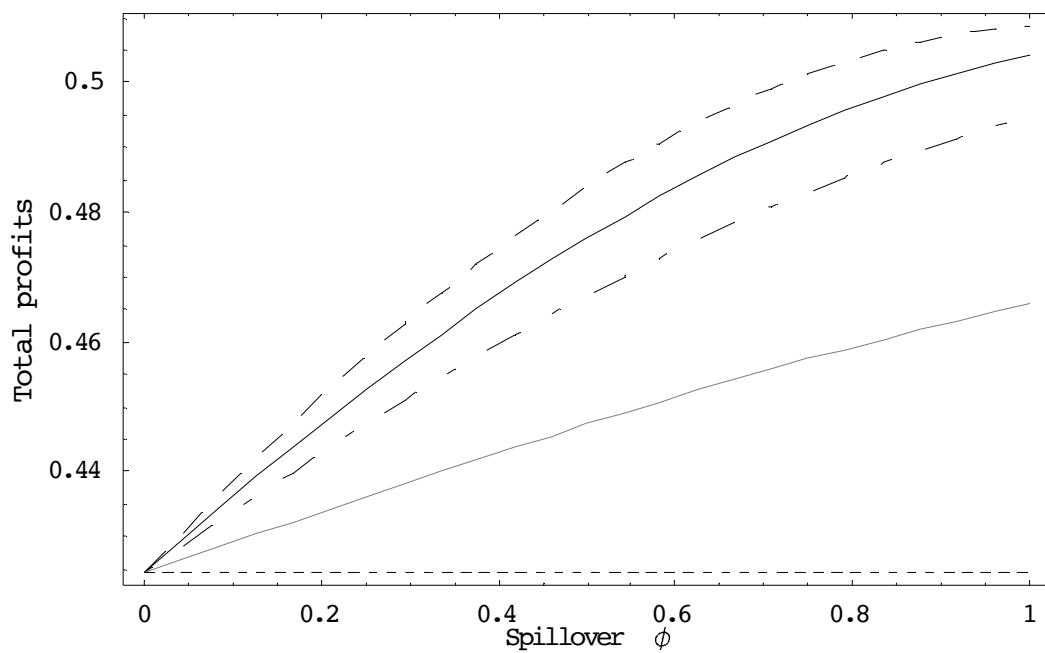


Figure 11: Total profits when unions settle wages.



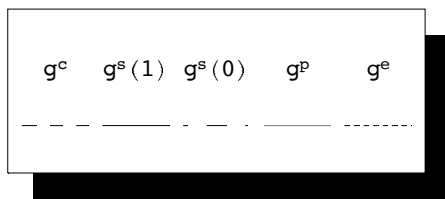
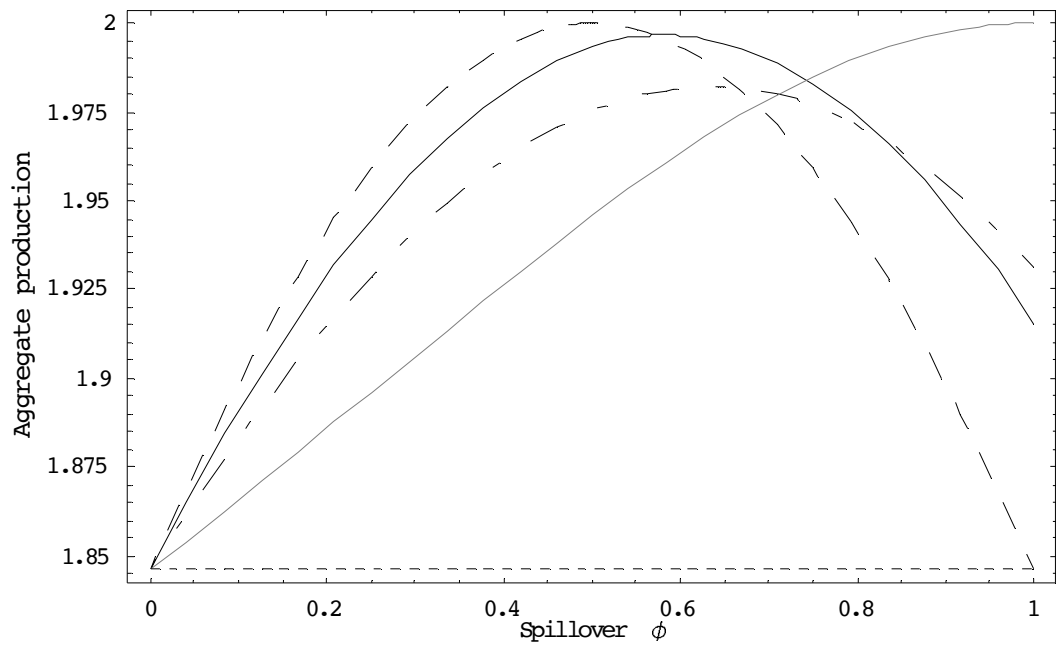


Figure 12: Aggregate production when firms settle wages.

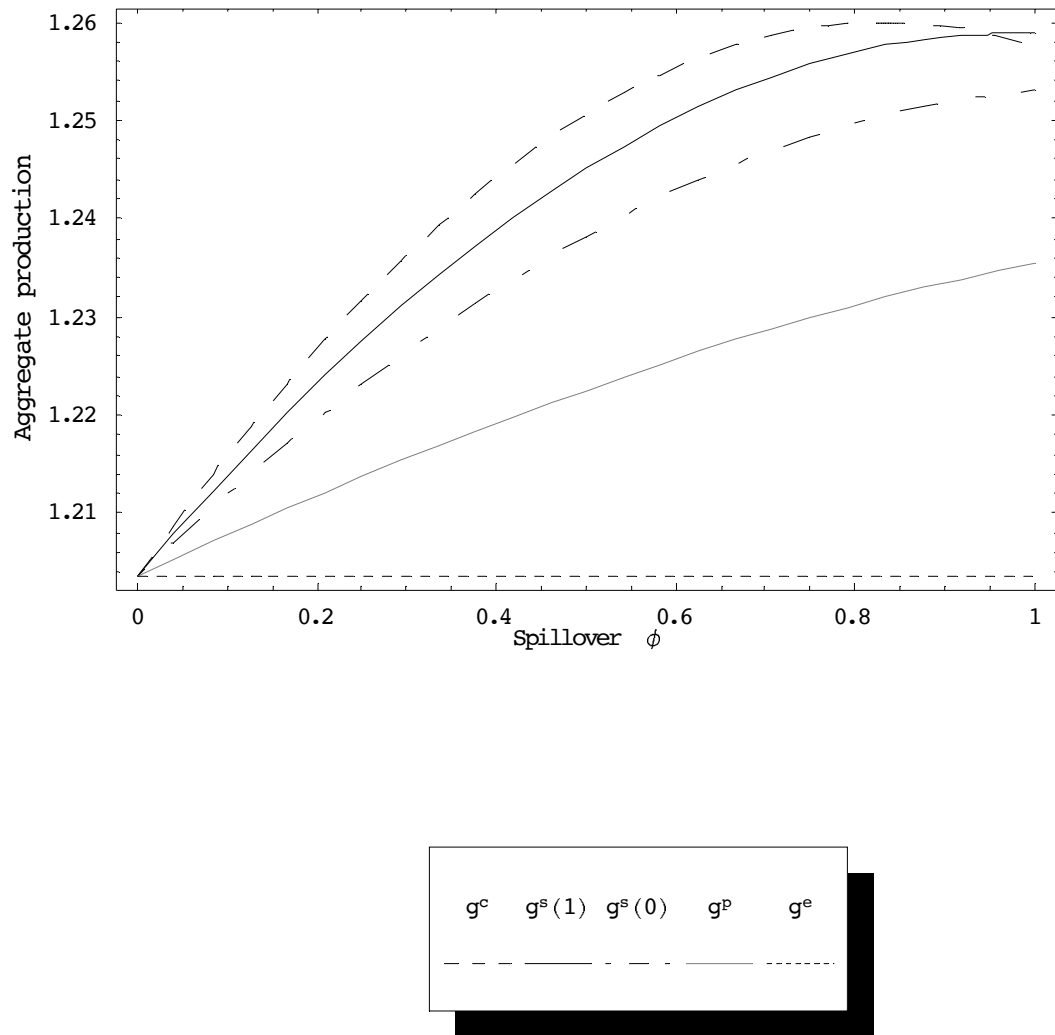


Figure 13: Aggregate production when unions settle wages.

welfare levels under the different networks without unions. Define  $\bar{\phi}$  as the solution to equation  $W(g^p) = W(g^c)$ . The figure shows that  $\bar{\phi}$  exists and is unique:  $\bar{\phi} = 0.6305$ . We are ready to state the following proposition (see the Appendix for details).

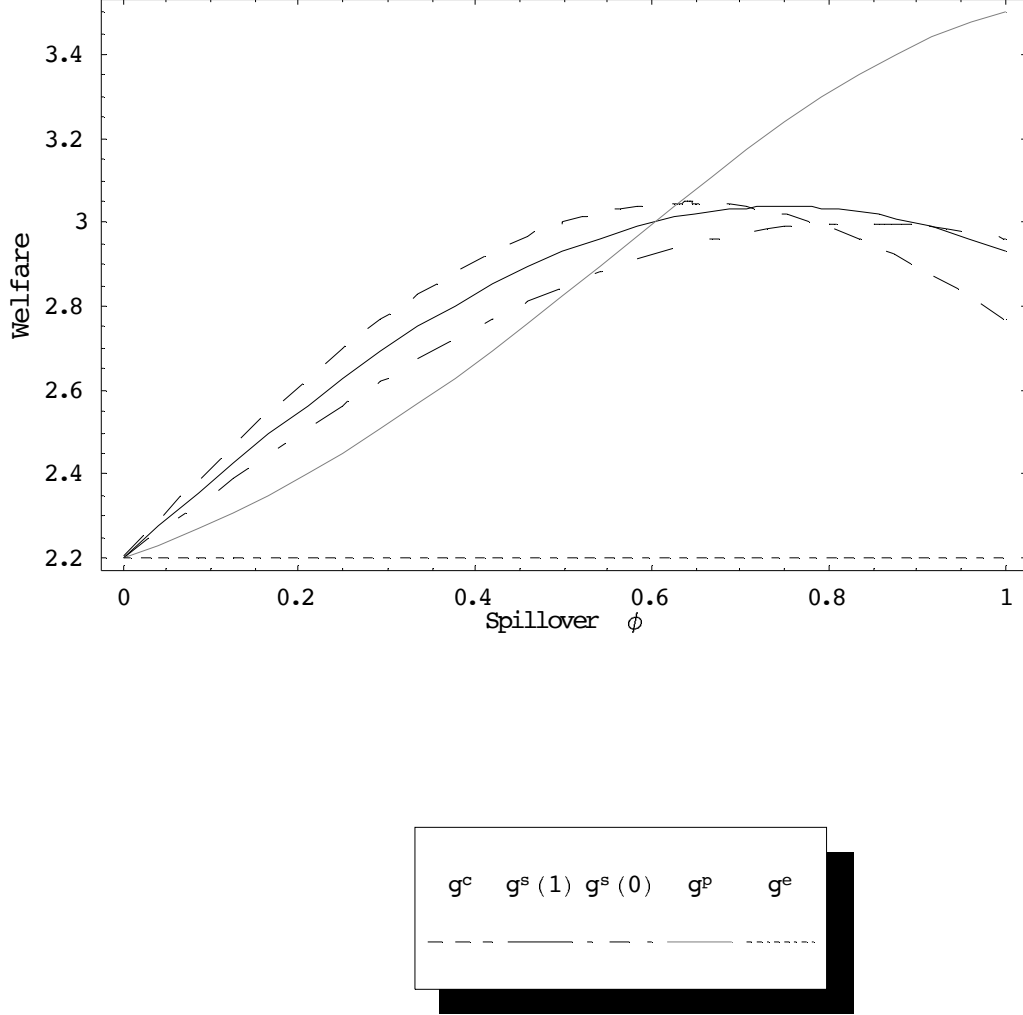


Figure 14: Social welfare when firms settle wages.

**Proposition 7** *Suppose firms settle wages. If spillovers are weak,  $\phi < \bar{\phi}$ , then the complete network  $g^c$  is the unique efficient network. If spillovers are strong,  $\phi > \bar{\phi}$ , then the partially connected network  $g^p$  is the unique efficient network.*

The above result shows that the welfare-maximizing number of collaborations declines with respect to the spillover parameter. For low spillover parameter  $\phi$ , the complete network  $g^c$  is efficient. But for large spillover parameter,  $\phi > 0.6305$ , the partially connected

network  $g^p$  is efficient. It is efficient because when spillovers are large, the isolated firm tends to be pushed out of the market and the collaborating firms will obtain profits close to the duopoly case which are greater than those obtained in the complete network where all firms have equal market share. Moreover, consumer surplus is also maximized with the partially connected network when spillovers are large. The reason is that the increase in effective R&D output by the collaborating firms results in an increase in their output that more than compensate the reduction in the isolated firm's output. The partially connected network is the only network where the collaborating firms are able to reduce drastically the rival's market share when spillovers are very large.

Define  $\phi_0$  as the solution to equation  $W(g^p) = W(g^s, \alpha = 0)$ . The figure shows that  $\phi_0$  exists and is unique:  $\phi_0 = 0.526$ .

**Corollary 1** *Suppose firms settle wages. If spillovers are weak,  $\phi < \phi_0$ , then social welfare is increasing with the number of collaborative links.*

Notice that, only if spillovers are weak,  $\phi < \phi_0$ , then social welfare is increasing with the number of collaborative links whenever firms settle wages. Indeed, when spillovers are strong, intermediate levels of collaborations are preferred from a social point of view.

Figure 15 contrasts the efficient and pairwise stable networks. We observe that a conflict between pairwise stability and efficiency may occur when firms settle wages. Meanwhile the efficient network is always pairwise stable, the reverse is not true. For instance, the partially connected network may be stable when the complete network is efficient, and the complete network is stable when the partially connected network is efficient. Notice that there is always a unique efficient network.

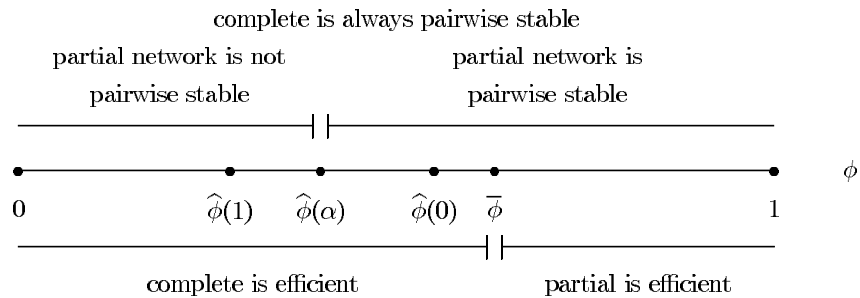


Figure 15: A conflict between stability and efficiency when firms settle wages.

This conflict is much stronger when we consider the notion of strongly stable network. The efficient network may not be strongly stable. More precisely, the complete network is the efficient network for  $\phi < \bar{\phi} = 0.6305$  but the complete network is never strongly

stable. However, if  $\phi \geq \bar{\phi}$  then the partially connected network is the efficient network and is the unique strongly stable.

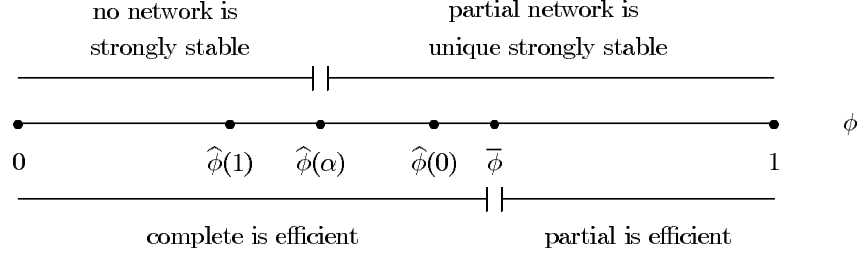


Figure 16: A conflict between strong stability and efficiency when firms settle wages.

We turn now to the case where unions settle wages. In Figure 17 we plot the welfare levels under the different networks with unions. We observe that the complete network  $g^c$  is the efficient network. Moreover, social welfare is increasing with the number of collaborative links (see the Appendix for details).

**Proposition 8** *Suppose unions settle wages. The complete network  $g^c$  is the unique efficient network and social welfare is increasing with the number of collaborative links:  $W^*(g^c) > W^*(g^s) > W^*(g^p) > W^*(g^e)$ .*

That is, whenever unions settle wages, there is no conflict between stability and efficiency. The complete network  $g^c$  is both the unique pairwise stable network and the efficient network. It is also the unique strongly stable network. Thus, unionization reconciles the private incentives to form R&D collaborations with the social welfare viewpoint.

## 4 Conclusion

We have developed a model of strategic networks in order to analyze how unions will affect the stability and efficiency of R&D collaboration networks in an oligopolistic industry with three firms. We have found that, whenever firms settle wages, the complete network is always pairwise stable and the partially connected is stable if and only if spillovers are large enough. However, the complete network is the efficient network if spillovers are small, while the partially connected network is the efficient network if spillovers are large. Thus, a conflict between stability and efficiency may occur: efficient networks are pairwise stable, but the reverse is not true. Strong stability even reinforces this conflict. But, once unions settle wages such conflict disappears: the complete network is the unique pairwise

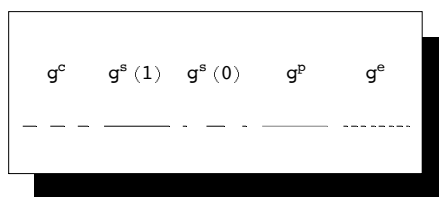
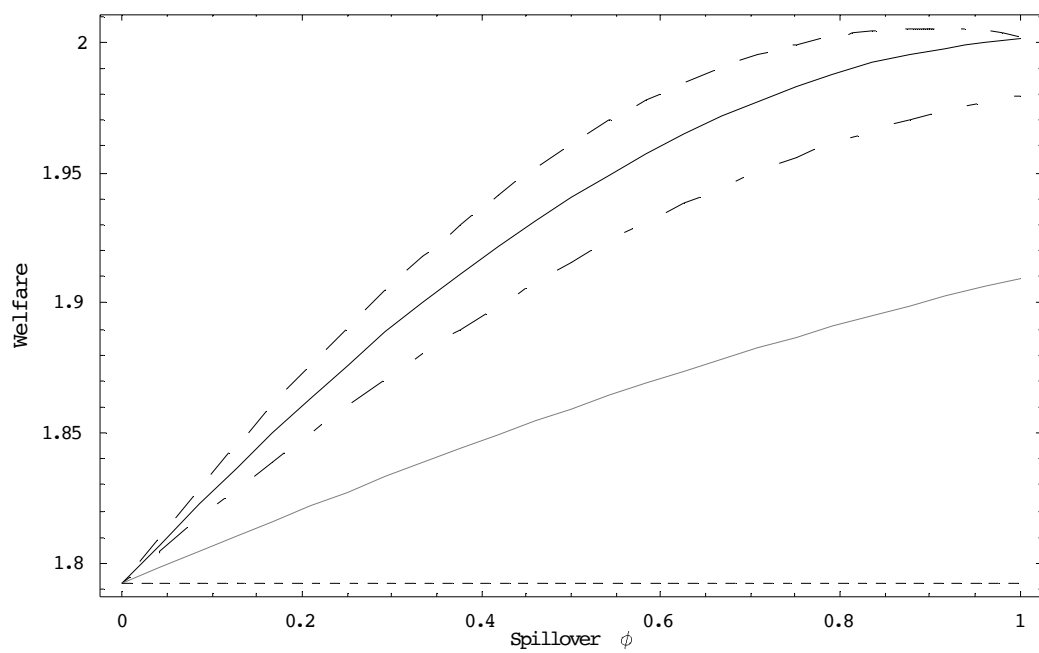


Figure 17: Social welfare when unions settle wages.

and strongly stable network and is the efficient network whatever the spillovers.

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## Appendix A: Empty network

In the last stage of the game, the R&D collaboration links have already been chosen, the wage levels have already been determined and the research efforts have already been chosen. Under Cournot competition the firms compete by choosing simultaneously their outputs to maximize profits with price adjusting to clear the market. The unique Nash equilibrium of this stage game is

$$q_i^*(g^e, f) = \frac{1}{4} (a - \bar{c} + 3x_i - x_j - x_k), i \in N,$$

if the firm settles the wage, and

$$q_i^*(g^e, u) = \frac{1}{4} (a - \bar{c} - 3w_i + w_j + w_k + 3x_i - x_j - x_k), i \in N,$$

if the union settles the wage. The symbol  $f$  ( $u$ ) indicates that the firm (union) chooses the wage. In the third stage, wages are settled at the firm-level. We have  $w_i^*(g^e, f) = 0$ . Standard computations give us

$$w_i^*(g^e, u) = \frac{1}{28} (7(a - \bar{c}) + 13x_i - 3(x_j + x_k)).$$

Then, we obtain the profits as function of R&D outputs:

$$\begin{aligned} \Pi_i^*(g^e, f) &= \frac{1}{16} (a - \bar{c} + 7x_i - (x_j + x_k)) \cdot (a - \bar{c} - x_i - (x_j + x_k)), \\ \Pi_i^*(g^e, u) &= \frac{1}{12544} (21(a - \bar{c}) + 151x_i - 9(x_j + x_k)) \cdot \\ &\quad (21(a - \bar{c}) - 73x_i - 9(x_j + x_k)), \end{aligned}$$

It follows that marginal benefits from R&D are decreasing with the research outputs from the other firms. Indeed,

$$\frac{\partial \Pi_i(g^e, f)}{\partial x_i \partial x_j} = -\frac{3}{8} < 0 \text{ and } \frac{\partial \Pi_i(g^e, u)}{\partial x_i \partial x_j} = -\frac{9}{6272} < 0.$$

Then,  $x_i$  and  $x_j$  are strategic substitutes. Moreover, we observe that marginal benefits from R&D are decreasing less with the research outputs from the other firms when unions settle wages;

$$\left| \frac{\partial \Pi_i(g^e, f)}{\partial x_i \partial x_j} \right| > \left| \frac{\partial \Pi_i(g^e, u)}{\partial x_i \partial x_j} \right|.$$

In the second stage, the firms choose simultaneously their research outputs to maximize profits anticipating perfectly wages and outputs. The unique (symmetric) Nash equilibrium of this stage game is

$$x_i^*(g^e, f) = \frac{3(a - \bar{c})}{13}, x_i^*(g^e, u) = \frac{117(a - \bar{c})}{1675}, i \in N.$$

Since there is no collaboration, firm  $i$ 's own R&D output is its effective R&D output. One can easily obtain the equilibrium outputs, profits, and wages:

$$q_i^*(g^e, f) = \frac{4(a - \bar{c})}{13}, \Pi_i^*(g^e, f) = \frac{7(a - \bar{c})^2}{169}, i \in N,$$

in case the firm settles the wage;

$$q_i^*(g^e, u) = \frac{336(a - \bar{c})}{1675}, \Pi_i^*(g^e, u) = \frac{99207(a - \bar{c})^2}{1675^2}, w_i^*(g^e, u) = \frac{448(a - \bar{c})}{1675},$$

in case the union settles the wage. In  $g^e$  the global effective R&D effort is given by

$$X^*(g^e, f) = \frac{9(a - \bar{c})}{13}, X^*(g^e, u) = \frac{351(a - \bar{c})}{1675}.$$

Unions payoffs are

$$U_i^*(g^e, f) = 0, U_i^*(g^e, u) = \frac{150528(a - \bar{c})^2}{1675^2}.$$

## Appendix B: Partial network

Let  $k$  be the firm which is isolated and has no link. Firm  $i$  and firm  $j$  are linked to each other, and share R&D activities. The unique Nash equilibrium of the Cournot competition stage game is

$$\begin{aligned} q_i^*(g^p, f) &= \frac{1}{4}(a - \bar{c} + x_i(3 - \phi) - x_k - x_j(1 - 3\phi)), \\ q_k^*(g^p, f) &= \frac{1}{4}(a - \bar{c} + 3x_k - (x_i + x_j)(1 + \phi)), \end{aligned}$$



if the firm settles the wage, and

$$\begin{aligned} q_i^*(g^p, u) &= \frac{1}{4}(a - \bar{c} - 3w_i + w_k + w_j + x_i(3 - \phi) - x_k - x_j(1 - 3\phi)), \\ q_k^*(g^p, u) &= \frac{1}{4}(a - \bar{c} - 3w_k + w_i + w_j + 3x_k - (x_i + x_j)(1 + \phi)), \end{aligned}$$

if the union settles the wage.

In the third stage, wages are settled at the firm-level. We have  $w_i^*(g^p, f) = w_k^*(g^p, f) = 0$ . Standard computations give us

$$\begin{aligned} w_i^*(g^p, u) &= \frac{1}{28}(7(a - \bar{c}) + x_i(13 - 3\phi) - 3x_k - x_j(3 - 13\phi)), \\ w_k^*(g^p, u) &= \frac{1}{28}(7(a - \bar{c}) + 13x_k - 3(x_i + x_j)(1 + \phi)). \end{aligned}$$

Incorporating the equilibrium outputs and wages into profits, we get

$$\begin{aligned} \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_j} &= -\frac{1}{8}(3 - \phi)(1 - 3\phi) < 0 \text{ if and only if } \phi < \frac{1}{3}, \\ \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_j} &= -\frac{9}{6272}(13 - 3\phi)(3 - 13\phi) < 0 \text{ if and only if } \phi < \frac{3}{13}, \\ \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_k} &= -\frac{1}{8}(3 - \phi) < 0, \\ \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_k} &= -\frac{27}{6272}(13 - 3\phi) < 0, \\ \frac{\partial \Pi_k(g^p, f)}{\partial x_k \partial x_i} &= -\frac{3}{8}(1 + \phi) < 0, \\ \frac{\partial \Pi_k(g^p, u)}{\partial x_k \partial x_i} &= -\frac{351}{6272}(1 + \phi) < 0. \end{aligned}$$

More precisely,

- (i)  $\frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_j} < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_j} < 0$  if  $\phi < \frac{3}{13}$ ;
- (ii)  $\frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_j} < 0 < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_j}$  if  $\frac{3}{13} < \phi < \frac{1}{3}$ ;
- (iii)  $0 < \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_j} < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_j}$  if  $\frac{1}{3} < \phi < \frac{3119 - 32\sqrt{5590}}{2001} \simeq 0.363$ ;
- (iv)  $0 < \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_j} < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_j}$  if  $\frac{3119 - 32\sqrt{5590}}{2001} < \phi < 1$ .

Notice that R&D efforts for the collaborating firms can be either strategic substitutes or complements depending on the spillovers parameter  $\phi$ . However, the strategic interaction between R&D efforts of a collaborating firm and the isolated one (or the opposite) is of substitution regardless spillovers size and unionization. Moreover, we observe that: (i) marginal benefits from R&D for the isolated firm is decreasing more with R&D done by

a collaborating firm than marginal benefits from R&D for the collaborating firm do with R&D done by the isolated firm,

$$\frac{\partial \Pi_k(g^p, f)}{\partial x_k \partial x_i} < \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_k} \text{ and } \frac{\partial \Pi_k(g^p, u)}{\partial x_k \partial x_i} < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_k}$$

(ii) Marginal benefits from R&D for a collaborating firm are decreasing more with R&D done by the isolated firm than with R&D done by its research partner,

$$\frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_k} < \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_j} \text{ and } \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_k} < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_j}$$

(iii) Marginal benefits from R&D for a firm are decreasing much more with R&D done by a firm which is not linked to it whenever firms settle wages,

$$\frac{\partial \Pi_k(g^p, f)}{\partial x_k \partial x_i} < \frac{\partial \Pi_k(g^p, u)}{\partial x_k \partial x_i}, \frac{\partial \Pi_i(g^p, f)}{\partial x_i \partial x_k} < \frac{\partial \Pi_i(g^p, u)}{\partial x_i \partial x_k}.$$

In the second stage, the firms choose simultaneously their research outputs to maximize profits anticipating perfectly wages and outputs. Invoking symmetry for the firms linked to each other, i.e.  $x_i = x_j$ , the unique Nash equilibrium of this stage game is

$$\begin{aligned} x_i^*(g^p, f) &= \frac{(3 - \phi)(a - \bar{c})}{13 - 5(2 - \phi)\phi}, x_k^*(g^p, f) = \frac{3(1 - \phi)^2(a - \bar{c})}{13 - 5(2 - \phi)\phi}, \\ x_i^*(g^p, u) &= \frac{6003(13 - 3\phi)(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}, x_k^*(g^p, u) = \frac{117(667 - 9\phi(10 - 3\phi))(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}. \end{aligned}$$

We observe that research efforts are decreasing with spillovers ( $\phi$ ) when the union settles the wage. That is,  $\frac{\partial x_i^*(g^p, u)}{\partial \phi} < 0$  and  $\frac{\partial x_k^*(g^p, u)}{\partial \phi} < 0$ . In case the firm settles the wage, research efforts made by the isolated firm  $k$  are always decreasing with  $\phi$ , while research efforts made by firm  $i$  and firm  $j$  are decreasing with  $\phi$  if and only if spillovers are strong enough. That is,  $\frac{\partial x_k^*(g^p, f)}{\partial \phi} < 0$  and  $\frac{\partial x_i^*(g^p, f)}{\partial \phi} < 0$  if and only if  $\phi > \frac{1}{5}(15 - 2\sqrt{35})$ .

One can easily obtain the equilibrium outputs, profits and wages:

$$\begin{aligned} q_i^*(g^p, f) &= \frac{4(a - \bar{c})}{13 - 5(2 - \phi)\phi}, q_k^*(g^p, f) = \frac{4(1 - \phi)^2(a - \bar{c})}{13 - 5(2 - \phi)\phi} \\ q_i^*(g^p, u) &= \frac{224112(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}, q_k^*(g^p, u) = \frac{336(667 - 9\phi(10 - 3\phi))(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}. \end{aligned}$$

$$\begin{aligned} \Pi_i^*(g^p, f) &= \frac{(7 - \phi)(1 + \phi)(a - \bar{c})^2}{(13 - 5(2 - \phi)\phi)^2}, \\ \Pi_k^*(g^p, f) &= \frac{7(1 - \phi)^4(a - \bar{c})^2}{(13 - 5(2 - \phi)\phi)^2}, \\ \Pi_i^*(g^p, u) &= \frac{4004001(151 - 9\phi)(73 + 9\phi)(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2}, \\ \Pi_k^*(g^p, u) &= \frac{99207(667 - 9\phi(10 - 3\phi))^2(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2}. \end{aligned}$$

$$w_i^*(g^p, u) = \frac{298816(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}, w_k^*(g^p, u) = \frac{448(667 - 9\phi(10 - 3\phi))(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}.$$

The global effective R&D effort is given by

$$\begin{aligned} X^*(g^p, f) &= \frac{(9 - (2 - \phi)\phi)(a - \bar{c})}{13 - 5(2 - \phi)\phi}, \\ X^*(g^p, u) &= \frac{9(26013 + 1217\phi(10 - 3\phi))(a - \bar{c})}{1117225 - 9027\phi(10 - 3\phi)}. \end{aligned}$$

Unions payoffs are

$$\begin{aligned} U_i^*(g^p, u) &= \frac{66968251392(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2}, \\ U_k^*(g^p, u) &= \frac{150528(667 - 9\phi(10 - 3\phi))^2(a - \bar{c})^2}{(1117225 - 9027\phi(10 - 3\phi))^2}. \end{aligned}$$

## Appendix C: Star network

Let  $i$  be the "hub" firm linked to the "spoke" firms  $j$  and  $k$ . The unique Nash equilibrium of the Cournot competition stage game is

$$\begin{aligned} q_i^*(g^s, f) &= \frac{1}{8}(2(a - \bar{c}) + x_i(6 - 4\phi) - (x_j + x_k)(2 - (6 - \alpha)\phi)), \\ q_j^*(g^s, f) &= \frac{1}{8}(2(a - \bar{c}) + x_j(6 - (2 + \alpha)\phi) - x_i(2 - 4\phi) - x_k(2 + (2 - 3\alpha)\phi)), \end{aligned}$$

and

$$\begin{aligned} q_i^*(g^s, u) &= \frac{1}{8}(2(a - \bar{c}) - 6w_i + 2(w_j + w_k) + x_i(6 - 4\phi) - (x_j + x_k)(2 - (6 - \alpha)\phi)), \\ q_j^*(g^s, u) &= \frac{1}{8}(2(a - \bar{c}) - 6w_j + 2(w_i + w_k) + x_j(6 - (2 + \alpha)\phi) - x_i(2 - 4\phi) \\ &\quad - x_k(2 + (2 - 3\alpha)\phi)). \end{aligned}$$

In the third stage, wages are settled at the firm-level. We have  $w_i^*(g^s, f) = w_j^*(g^s, f) = 0$ . Standard computations give us

$$\begin{aligned} w_i^*(g^s, u) &= \frac{1}{56}(14(a - \bar{c}) + x_i(26 - 12\phi) - (x_j + x_k)(6 - (26 - 3\alpha)\phi)), \\ w_j^*(g^s, u) &= \frac{1}{56}(14(a - \bar{c}) + x_j(26 - 3(2 + \alpha)\phi) - x_i(6 - 20\phi) - x_k(6 + (6 - 13\alpha)\phi)). \end{aligned}$$

Incorporating the equilibrium outputs and wages into profits, we get

$$\begin{aligned}
\frac{\partial \Pi_i(g^s, f)}{\partial x_i \partial x_j} &= -\frac{1}{16}(3-2\phi)(2-(6-\alpha)\phi) < 0 \text{ if and only if } \phi < \frac{2}{6-\alpha}, \\
\frac{\partial \Pi_i(g^s, u)}{\partial x_i \partial x_j} &= -\frac{9}{12544}(13-6\phi)(6-(26-3\alpha)\phi) < 0 \text{ if and only if } \phi < \frac{6}{26-3\alpha}, \\
\frac{\partial \Pi_j(g^s, f)}{\partial x_j \partial x_i} &= -\frac{1}{16}(1-2\phi)(6-(2+\alpha)\phi) < 0 \text{ if and only if } \phi < \frac{1}{2}, \\
\frac{\partial \Pi_j(g^s, u)}{\partial x_j \partial x_i} &= -\frac{9}{12544}(3-10\phi)(26-3(2+\alpha)\phi) < 0 \text{ if and only if } \phi < \frac{3}{10}, \\
\frac{\partial \Pi_j(g^s, f)}{\partial x_j \partial x_k} &= -\frac{1}{32}(2+(2-3\alpha)\phi)(6-(2+\alpha)\phi) < 0, \\
\frac{\partial \Pi_j(g^s, u)}{\partial x_j \partial x_k} &= -\frac{9}{25088}(6+(6-13\alpha)\phi)(26-3(2+\alpha)\phi) < 0 \text{ if either } \alpha < \frac{12}{13} \\
&\text{or } \alpha > \frac{12}{13} \text{ and } \phi < \frac{6}{13\alpha-6}.
\end{aligned}$$

In the second stage, the firms choose simultaneously their research outputs to maximize profits anticipating perfectly wages and outputs. We write  $x_i^*(g^s, f, 1)$  for  $x_i^*(g^s, f, \alpha = 1)$  and  $x_i^*(g^s, f, 0)$  for  $x_i^*(g^s, f, \alpha = 0)$ . Invoking symmetry for the firms at the spokes, i.e.  $x_j = x_k$ , the unique Nash equilibrium of this stage game is

$$\begin{aligned}
x_i^*(g^s, f, 1) &= \frac{(3-2\phi)(4+3\phi(8-3\phi))(a-\bar{c})}{52+\phi(264-\phi(169-6\phi(15-4\phi)))}, \\
x_j^*(g^s, f, 1) &= \frac{6(2-\phi)(1+\phi(5-2\phi))(a-\bar{c})}{52+\phi(264-\phi(169-6\phi(15-4\phi)))}, \\
x_i^*(g^s, u, 1) &= \frac{9(13-6\phi)(2668+27\phi(32-9\phi))(a-\bar{c})}{4468900+9\phi(94000-9\phi(3533-6\phi(191-36\phi)))}, \\
x_j^*(g^s, u, 1) &= \frac{18(26-9\phi)(667+9\phi(19-6\phi))(a-\bar{c})}{4468900+9\phi(94000-9\phi(3533-6\phi(191-36\phi)))}, \\
x_i^*(g^s, f, 0) &= \frac{(3-2\phi)(1+\phi(7-2\phi))(a-\bar{c})}{13+\phi(71-2\phi(20-\phi(9-2\phi)))}, \\
x_j^*(g^s, f, 0) &= \frac{(3-\phi)(1+\phi(5-2\phi))(a-\bar{c})}{13+\phi(71-2\phi(20-\phi(9-2\phi)))}, \\
x_i^*(g^s, u, 0) &= \frac{9(13-6\phi)(667+9\phi(29-6\phi))(a-\bar{c})}{1117225+27\phi(9505-6\phi(496-9\phi(13-2\phi)))}, \\
x_j^*(g^s, u, 0) &= \frac{9(13-6\phi)(667+9\phi(19-6\phi))(a-\bar{c})}{1117225+27\phi(9505-6\phi(496-9\phi(13-2\phi)))}.
\end{aligned}$$

We have that research efforts are decreasing with spillovers ( $\phi$ ) when the union settles the wage. That is,  $\frac{\partial x_i^*(g^s, u, 1)}{\partial \phi} < 0$ ,  $\frac{\partial x_j^*(g^s, u, 1)}{\partial \phi} < 0$ ,  $\frac{\partial x_i^*(g^s, u, 0)}{\partial \phi} < 0$  and  $\frac{\partial x_j^*(g^s, u, 0)}{\partial \phi} < 0$ . In case the firm settles the wage, research efforts made by firm  $j$  and firm  $k$  are decreasing with  $\phi$ ,  $\frac{\partial x_j^*(g^s, f, 1)}{\partial \phi} < 0$  and  $\frac{\partial x_k^*(g^s, f, 1)}{\partial \phi} < 0$ ; but the research effort made by the "hub" firm may

increase or decrease with  $\phi$  depending on how large spillovers are. As  $\phi$  goes from zero to one, research effort first increases with  $\phi$ , then it starts to decrease with  $\phi$ .

One can easily obtain the equilibrium outputs, profits, and wages:

$$\begin{aligned}
q_i^*(g^s, f, 1) &= \frac{4(4 + 3\phi(8 - 3\phi))(a - \bar{c})}{52 + \phi(264 - \phi(169 - 6\phi(15 - 4\phi)))}, \\
q_j^*(g^s, f, 1) &= \frac{16(1 + \phi(5 - 2\phi))(a - \bar{c})}{52 + \phi(264 - \phi(169 - 6\phi(15 - 4\phi)))}, \\
q_i^*(g^s, f, 0) &= \frac{4(1 + \phi(7 - 2\phi))(a - \bar{c})}{13 + \phi(71 - 2\phi(20 - \phi(9 - 2\phi)))}, \\
q_j^*(g^s, f, 0) &= \frac{4(1 + \phi(5 - 2\phi))(a - \bar{c})}{13 + \phi(71 - 2\phi(20 - \phi(9 - 2\phi)))}, \\
\Pi_i^*(g^s, f, 1) &= \frac{(7 - 2\phi)(1 + 2\phi)(4 + 3(8 - 3\phi)\phi)^2(a - \bar{c})^2}{(52 + \phi(264 - \phi(169 - 6\phi(15 - 4\phi))))^2}, \\
\Pi_j^*(g^s, f, 1) &= \frac{2(14 - 3\phi)(2 + 3\phi)(1 + (5 - 2\phi)\phi)^2(a - \bar{c})^2}{(52 + \phi(264 - \phi(169 - 6\phi(15 - 4\phi))))^2}, \\
\Pi_i^*(g^s, f, 0) &= \frac{(7 - 2\phi)(1 + 2\phi)(1 + (7 - 2\phi)\phi)^2(a - \bar{c})^2}{(13 + \phi(71 - 2\phi(20 - \phi(9 - 2\phi))))^2}, \\
\Pi_j^*(g^s, f, 0) &= \frac{(7 - \phi)(1 + \phi)(1 + (5 - 2\phi)\phi)^2(a - \bar{c})^2}{(13 + \phi(71 - 2\phi(20 - \phi(9 - 2\phi))))^2},
\end{aligned}$$

when firms settle wages, and

$$\begin{aligned}
q_i^*(g^s, u, 1) &= \frac{336(2668 + 27\phi(32 - 9\phi))(a - \bar{c})}{4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi)))}, \\
q_j^*(g^s, u, 1) &= \frac{1344(667 + 9\phi(19 - 6\phi))(a - \bar{c})}{4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi)))}, \\
q_i^*(g^s, u, 0) &= \frac{336(667 + 9\phi(29 - 6\phi))(a - \bar{c})}{1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi)))}, \\
q_j^*(g^s, u, 0) &= \frac{336(667 + 9\phi(19 - 6\phi))(a - \bar{c})}{1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi)))}, \\
\Pi_i^*(g^s, u, 1) &= \frac{3(151 - 18\phi)(73 + 18\phi)(2668 + 27\phi(32 - 9\phi))^2(a - \bar{c})^2}{(4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi))))^2}, \\
\Pi_j^*(g^s, u, 1) &= \frac{6(302 - 27\phi)(146 + 27\phi)(667 + 9\phi(19 - 6\phi))^2(a - \bar{c})^2}{(4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi))))^2}, \\
\Pi_i^*(g^s, u, 0) &= \frac{3(151 - 18\phi)(73 + 18\phi)(667 + 9\phi(29 - 6\phi))^2(a - \bar{c})^2}{(1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi))))^2}, \\
\Pi_j^*(g^s, u, 0) &= \frac{3(151 - 9\phi)(73 + 9\phi)(667 + 9\phi(19 - 6\phi))^2(a - \bar{c})^2}{(1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi))))^2},
\end{aligned}$$

$$\begin{aligned}
w_i^*(g^s, u, 1) &= \frac{448(2668 + 27\phi(32 - 9\phi))(a - \bar{c})}{4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi)))}, \\
w_j^*(g^s, u, 1) &= \frac{1792(667 + 9\phi(19 - 6\phi))(a - \bar{c})}{4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi)))}, \\
w_i^*(g^s, u, 0) &= \frac{448(667 + 9\phi(29 - 6\phi))(a - \bar{c})}{1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi)))}, \\
w_j^*(g^s, u, 0) &= \frac{448(667 + 9\phi(19 - 6\phi))(a - \bar{c})}{1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi)))},
\end{aligned}$$

when unions settle wages. The global effective R&D effort is given by

$$\begin{aligned}
X^*(g^s, f, 1) &= \frac{(36 + \phi(232 + \phi(107 - 18\phi(15 - 4\phi))))(a - \bar{c})}{52 + \phi(264 - \phi(169 - 6\phi(15 - 4\phi)))}, \\
X^*(g^s, f, 0) &= \frac{(9 + \phi(59 + 6\phi(4 - \phi)(1 - 2\phi)))(a - \bar{c})}{13 + \phi(71 - 2\phi(20 - \phi(9 - 2\phi)))}, \\
X^*(g^s, u, 1) &= \frac{9(104052 + \phi(162416 - 3\phi(13003 + 54\phi(191 - 36\phi))))(a - \bar{c})}{4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi)))}, \\
X^*(g^s, u, 0) &= \frac{9(26013 + \phi(34519 - 18\phi(304 + 27\phi(13 - 2\phi))))(a - \bar{c})}{1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi)))}.
\end{aligned}$$

Unions payoffs are

$$\begin{aligned}
U_i^*(g^s, u, 1) &= \frac{150528(2668 + 27\phi(32 - 9\phi))^2(a - \bar{c})^2}{(4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi))))^2}, \\
U_j^*(g^s, u, 1) &= \frac{2408448(667 + 9\phi(19 - 6\phi))^2(a - \bar{c})^2}{(4468900 + 9\phi(94000 - 9\phi(3533 - 6\phi(191 - 36\phi))))^2}, \\
U_i^*(g^s, u, 0) &= \frac{150528(667 + 9\phi(29 - 6\phi))^2(a - \bar{c})^2}{(1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi))))^2}, \\
U_j^*(g^s, u, 0) &= \frac{150528(667 + 9\phi(19 - 6\phi))^2(a - \bar{c})^2}{(1117225 + 27\phi(9505 - 6\phi(496 - 9\phi(13 - 2\phi))))^2}.
\end{aligned}$$

## Appendix D: Complete network

The unique Nash equilibrium of the Cournot competition stage game is either

$$q_i^*(g^c, f) = \frac{1}{4}(a - \bar{c} + 3x_i - x_j - x_k + 2(x_j + x_k - x_i)\phi), i \neq j \neq k,$$

or

$$q_i^*(g^c, u) = \frac{1}{4}(a - \bar{c} - 3w_i + w_j + w_k + 3x_i - x_j - x_k + 2(x_j + x_k - x_i)\phi), i \neq j \neq k.$$

In the third stage, wages are settled at the firm-level. We have  $w_i^*(g^c, f) = 0$ . Standard computations give us

$$w_i^*(g^c, u) = \frac{1}{28}(7(a - \bar{c}) + x_i(13 - 6\phi) - (x_j + x_k)(3 - 10\phi)), i \neq j \neq k.$$

We obtain

$$\begin{aligned}\Pi_i^*(g^c, f) &= \frac{1}{16} (a - \bar{c} + x_i (3 - 2\phi) - (x_j + x_k) (1 - 2\phi))^2 - x_i^2, \\ \Pi_i^*(g^c, u) &= \frac{1}{12544} (21(a - \bar{c}) + x_i(151 - 18\phi) - 3(x_j + x_k)(3 - 10\phi)) \cdot \\ &\quad (21(a - \bar{c}) - x_i(73 + 18\phi) - 3(x_j + x_k)(3 - 10\phi)),\end{aligned}$$

$i \neq j \neq k$ . It follows that marginal benefits from R&D are decreasing with the research outputs from other firms if and only if spillovers are small, or even smaller if unions settle wages. Indeed, we have

$$\begin{aligned}\frac{\partial \Pi_i(g^c, f)}{\partial x_i \partial x_j} &= -\frac{3}{8} + \phi - \frac{1}{2}\phi^2 < 0 \text{ if and only if } \phi < \frac{1}{2}, \\ \frac{\partial \Pi_i(g^c, u)}{\partial x_i \partial x_j} &= -\frac{9}{6272} (13 - 6\phi)(3 - 10\phi) < 0 \text{ if and only if } \phi < \frac{3}{10}.\end{aligned}$$

More precisely,

- (i)  $\frac{\partial \Pi_i(g^c, f)}{\partial x_i \partial x_j} < \frac{\partial \Pi_i(g^c, u)}{\partial x_i \partial x_j} < 0$  if  $\phi < \frac{3}{10}$ ;
- (ii)  $\frac{\partial \Pi_i(g^c, f)}{\partial x_i \partial x_j} < 0 < \frac{\partial \Pi_i(g^c, u)}{\partial x_i \partial x_j}$  if  $\frac{3}{10} < \phi < \frac{1}{2}$ ;
- (iii)  $0 < \frac{\partial \Pi_i(g^c, f)}{\partial x_i \partial x_j} < \frac{\partial \Pi_i(g^c, u)}{\partial x_i \partial x_j}$  if  $\frac{1}{2} < \phi < \frac{69}{118}$ ;
- (iv)  $0 < \frac{\partial \Pi_i(g^c, u)}{\partial x_i \partial x_j} < \frac{\partial \Pi_i(g^c, f)}{\partial x_i \partial x_j}$  if  $\frac{69}{118} < \phi < 1$ .

In the second stage, the firms choose simultaneously their research outputs to maximize profits anticipating perfectly wages and outputs. The unique (symmetric) Nash equilibrium of this stage game is

$$x_i^*(g^c, f) = \frac{(3 - 2\phi)(a - \bar{c})}{13 - 4\phi(1 - \phi)}, \quad x_i^*(g^c, u) = \frac{9(13 - 6\phi)(a - \bar{c})}{1675 - 36\phi(5 - 3\phi)}, \quad i \in N.$$

We observe that research efforts are decreasing with spillovers ( $\phi$ ). Then, one can easily obtain the equilibrium outputs, profits, and wages:

$$\begin{aligned}q_i^*(g^c, f) &= \frac{4(a - \bar{c})}{13 - 4\phi(1 - \phi)}, \quad \Pi_i^*(g^c, f) = \frac{(7 + 4(3 - \phi)\phi)(a - \bar{c})^2}{(13 - 4\phi(1 - \phi))^2}, \\ q_i^*(g^c, u) &= \frac{336(a - \bar{c})}{1675 - 36\phi(5 - 3\phi)}, \quad \Pi_i^*(g^c, u) = \frac{9(151 - 18\phi)(73 + 18\phi)(a - \bar{c})^2}{(1675 - 36\phi(5 - 3\phi))^2}, \\ w_i^*(g^c, u) &= \frac{448(a - \bar{c})}{1675 - 36\phi(5 - 3\phi)}.\end{aligned}$$

The global effective R&D effort is given by

$$X^*(g^c, f) = \frac{3(3 - 2\phi)(1 + 2\phi)(a - \bar{c})}{13 - 4\phi(1 - \phi)}, \quad X^*(g^c, u) = \frac{27(13 - 6\phi)(1 + 2\phi)(a - \bar{c})}{1675 - 36\phi(5 - 3\phi)}.$$

Unions payoffs are

$$U_i^*(g^c, u) = \frac{150528(a - \bar{c})^2}{(1675 - 36\phi(5 - 3\phi))^2}.$$

## Appendix E: Social welfare

In case firms settle wages, the equilibrium welfare in each network configuration is given by

$$\begin{aligned} W^*(g^e) &= \frac{93(a - \bar{c})^2}{169}, \\ W^*(g^p) &= \frac{(93 - \phi(112 - 5\phi(8 - (4 - \phi)\phi)))(a - \bar{c})^2}{(13 - 5\phi(2 - \phi))^2}, \\ W^*(g^s) &= \frac{A_3(a - \bar{c})^2}{(A_1)^2}, \\ W^*(g^c) &= \frac{3(31 + 4(3 - \phi)\phi)(a - \bar{c})^2}{(13 - 4\phi(1 - \phi))^2}, \end{aligned}$$

where

$$\begin{aligned} A_3 &= 1488 + 224(77 - 4\alpha)\phi + 16(2915 - \alpha(367 - 30\alpha))\phi^2 \\ &\quad - 8(2784 + \alpha(254 - \alpha(241 - 15\alpha)))\phi^3 \\ &\quad - (6816 + \alpha(1216 - \alpha(164 - 3\alpha(52 - 5\alpha))))\phi^4 \\ &\quad + 4(2 + \alpha)(496 - (6 - \alpha)\alpha(8 + 3\alpha))\phi^5 \\ &\quad - 4(2 + \alpha)^2(24 - (8 - \alpha)\alpha)\phi^6. \end{aligned}$$

**Proof of Proposition 7.** Suppose firms settle wages. Simple computations show that, first, the empty network is the less efficient network:  $W^*(g^c) > W^*(g^e)$ ,  $W^*(g^s) > W^*(g^e)$ , and  $W^*(g^p) > W^*(g^e)$ . Second, if  $\phi > \phi_1 = 0.6035$ , then  $W^*(g^p) > W^*(g^s)$ . Third,  $W^*(g^c) > W^*(g^p)$  if and only if  $\phi < \phi_2 = 0.6305$ . Let  $\bar{\phi} \equiv \phi_2$ . Fourth, if  $\phi < \phi_3 = 0.7112$ , then  $W^*(g^c) > W^*(g^s)$ . Finally, (i) if  $\phi > \phi_4 = 0.788$  then  $W^*(g^s) > W^*(g^c)$ ; (ii) if  $\phi < \phi_0 = 0.5258$  then  $W^*(g^s) > W^*(g^p)$ . ■

In case unions settle wages, the equilibrium welfare in each network configuration is given by

$$\begin{aligned} W^*(g^e) &= \frac{1257237(a - \bar{c})^2}{2805625}, \\ W^*(g^p) &= \frac{9(a - \bar{c})^2 A_4}{(1117225 - 9027\phi(10 - 3\phi))^2}, \\ W^*(g^s) &= \frac{9(a - \bar{c})^2 A_5}{(A_2)^2}, \\ W^*(g^c) &= \frac{3(419079 + 972(13 - 3\phi)\phi)(a - \bar{c})^2}{(1675 - 36\phi(5 - 3\phi))^2}, \end{aligned}$$



where

$$\begin{aligned}
A_4 &= 62147879077 + 3\phi(-1655296568 + 3\phi(208960760 + 918549\phi(-20 + 3\phi))), \\
A_5 &= 994366065232 + 3\phi(-618976(-333503 + 21394\alpha) + 48(-408494973 + \\
&\quad \alpha(-202998331 + 52240190\alpha))\phi - 216(80192576 + \alpha(4374462 + \alpha(-4849629 \\
&\quad + 510305\alpha)))\phi^2 + 81(23047968 + \alpha(4756160 + \alpha(-1509972 + \alpha(-464404 + \\
&\quad 102061\alpha))))\phi^3 + 26244(2 + \alpha)(2064 + (-6 + \alpha)\alpha(40 + 13\alpha))\phi^4 - 78732(2 + \alpha)^2 \\
&\quad (24 + (-8 + \alpha)\alpha)\phi^5).
\end{aligned}$$

Simple computations lead to Proposition 8.

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