Network Competition in a Market where Cross Externalities Induce Vertical Differentiation

Jean Gabszewicz*, Didier Laussel†, Xavier Wauthy‡

March 9, 2007

PRELIMINARY - DO NOT QUOTE

Abstract

Two platforms compete in quantities in a two-sided market where agents' valuation of the cross network externalities are heterogeneous. Cross network effects are shown to generate an endogenous vertical differentiation structure. When agents are only allowed to single-home, there exists a unique equilibrium outcome where two asymmetric platforms co-exist with positive profits. In the case where one side of the market is allowed to multi-home, platforms exhibit asymmetric sizes in equilibrium but they also exhibit inversed hierarchy from one side to the other, i.e. each platform dominates one side of the market.

Keywords: cournot, vertical differentiation,network externalities
JEL Classification: L13

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*CORE, Ucl.

†GREQAM, Université de la Méditerrannée

‡CEREC FUSL and CORE, UCL. Xavier Wauthy gratefully acknowledges financial support from the Belgian federal government under SSTC IAP Program, contract 5/26. We are thankful to Paul Belleflamme for useful suggestions.

1 Introduction

Many industries displaying network effects have been put under scrutiny both by academics and politicians. One reason for this interest is the alleged importance of these industries for economic growth in tertiary economies. A second reason is to be found in the peculiarities of the competitive mechanisms at work in these industries. Software, telecommunication services, e-commerce or media are some prominent examples belonging to the class of (so-called) two-sided markets (see Rochet and Tirole (2004) for a tentative definition). These markets are essentially characterized by cross network externalities whereby the consumption behaviour on one type of the market is essentially determined by the behaviour of those agents located on the other side of the market. Consider for instance the media industry: media owners sell content to readers as well as audience access to advertisers. When deciding the content of a particular medium, media owners directly determine their potential readership, i.e. their hauling on the readers' market, but they also determine the value of their product for advertisers. A key difficulty in understanding the economics of two-sided market is therefore to neatly capture those links between the two sides. Indeed, those links actually "create" the market. While the economic analysis of two-sided markets flourished in the recent years (see Roson (2005) for a tentative survey of the literature), some important issues have been neglected up to now. In particular, the role of agents' heterogeneity regarding the valuation of networks externalities has been fairly neglected[1] as well as the case where platforms actually set quantities rather than prices.

We consider a two-sided market where two platforms are possibly active and supply a service whose value essentially consists of matching agents from one side to those of the other side. The agents are then left to conduct a transaction by themselves. Accordingly, the stand-alone value of a platform's service is nil. Agents are heterogeneous on both sides of the market and differ in their valuation of the network effects. Combining the nature of the service proposed by the platforms and agents' preferences lead to a model of vertical differentiation where product's quality on one side of the market is actually defined by the size of the network on the other side.

In the present note, we model a case where platforms compete in a Cournot manner, i.e. platforms are assumed to commit to network sizes. Given the sizes platforms have committed to, a market mechanism (which is not modelled here) ensures agents coordination by setting the highest prices which are compatible with individual decisions by agents leading to the committed network sizes. Doing this, we ensure that all the network externalities that are at work in the markets are internalized into the platforms' payoffs. We follow in this respect the analysis developed in De Palma, Leruth and Regibeau (1999). A key feature of our framework is that it neatly capture the vertical differentiation dimension which is inherent to markets with cross network externalities, i.e. the willingness to pay of agents on one side is directly tied to the size of the network on the other side which therefore measures the quality of the product. The key feature of a two-sided market is therefore an endogenous but indirect determination of products' quality through market mechanisms. Regarding equilibrium outcomes and market structure, several results are established. First we show that a monopoly platform will tend to implement a strategy which amounts to secure strictly positive margins on the two sides of the market rather than sacrifying margin on one side to better exploit the other side. Second, when agents are allowed to patronize one platform only (single-homing) quantiy competition leads to asymmetric equilibria where one platform dominates the other on the two sides of the market, while both of them secure positive profits. Last, when one side of the market is allowed to multihome, there

[1] Ambrus and Argenziano (2004) and Gabszewicz and Wauthy (2004) are noticeable exceptions, which, within very different set-ups explicitely develop the parallel between two-sided markets and vertically differentiated ones.

exists an equilibrium where one platform dominates on one side of the market while the second platform dominates on the other side. Therefore we observe an inversed hierarchy between platforms across sides. However, the platform which dominates the mutli-homing side of the market is more profitable than the other.

2 The Model

The basic ingredients of the model are drawn from the standard literature on vertical differentiation. The specification of preferences we retain here are those of Mussa and Rosen (1978). There are three types of agents:

- Platforms: they are denoted by i and sell product $i = 1, 2$. Product i is best viewed as a matching device between agents. For the sake of illustration, we shall refer here to the exhibition centers' metaphor. Then one can think of product i as a commercial fair organized at an exhibition center i. Platforms are the organizers of the fairs in exhibition centers. They sell their product in two markets: the visitors' market and the exhibitors' market. The access permit paid by the visitors, as well as the rental fee paid to the platforms by exhibitors, allow visitors and exhibitors to trade if they succeed to match. Platforms maximize sales revenue by setting committing to network sizes. Prices are then set so that committed sizes are indeed realized ex-post.

- Visitors: they are denoted by their type θ. Types are uniformly distributed in the $[0, 1]$ interval. The total number of visitors is normalized to 1. They possibly buy product $i = 1, 2$ according to a utility function $U_i = \theta x_i^e - p_i$, with x_i^e denoting the expectation visitors have about the number of exhibitors at platform i. When buying the access permits to *both* exhibition centers, a visitor enjoys a utility $U_i = \theta x_3^e - p_1 - p_2$. Parameter x_3^e depends on the expectation visitors have about the number of exhibitors who exhibit in both centers. Holding no access permit yields a utility level normalized to 0.

- Exhibitors: they are denoted by their type γ. Types are uniformly distributed in the $[0, 1]$ interval. Their total number is normalized to 1. They possibly exhibit in both exhibition centers. When they exhibit in center i, $i = 1, 2$, their utility is measured by $U_i' = \gamma v_i^e - \pi_i$, with v_i^e denoting the expectation they form about the number of visitors in center i. When deciding to exhibit in both centers, an exhibitor enjoys a utility $U_i' = \gamma v_3^e - \pi_1 - \pi_2$. Again, v_3^e depends on the expectation about the total number of visitors in both centers. Refraining from exhibiting in any exhibition center yields a utility level normalized to 0.

Notice that our present setup is best viewed as a model where two vertically differentiated markets operate in parallel with the key feature that quality in one of the two markets is determined by the outcomes in the other market. Accordingly, when committing to network sizes, a platform jointly determines quality in each of the two markets.

3 The Monopoly Platform

Recall that under our preferences' setup, we may express the "demand" from visitors as a function of the price they are charged and the number of visitors (and similarly for exhibitors) which has been committed to by the platofrm. We have

$$v = 1 - \frac{p}{x^e} \tag{1}$$

$$x = 1 - \frac{\pi}{v^e} \tag{2}$$

These conditions can be used to identify the prices that must be associated with the committed sizes. Our assumption amounts to consider that agents observe committed sizes before they make their decisions. To this end they correctly anticipate the prices that will ensure theat committed sizes indeed realize. Accordingly, we solve the above system for $v = v^e$ and $x = x^e$, where v and x denotes the size committed by the monopoly platform. Rearranging the two above expressions, we obtain the following expressions:

$$\pi = v(1 - x) \tag{3}$$

$$p = x(1 - v) \tag{4}$$

An equilibrium allocation must satisfy these two conditions simultaneously. Accordingly, these two expressions define the prices such that, given agents' preferences, the two sides of the market "clear" for the committed sizes. Notice then that, by doing this, we capture completely the network externalities that link the two sides of the network: committing to a higher v allows the platform to benefit from a larger price on the x side of the market.

We may now express the reduced form of the objective function of the monopoly platform:

$$\max_{v,x} vp(v, x) + x\pi(v, x) = v[x(1 - v)] + \pi[v(1 - x)] \tag{5}$$

First order conditions yield the system:

$$x(2 - x - 2v) = 0 \tag{6}$$

$$v(2 - v - 2x) = 0 \tag{7}$$

The system of equations (6) and (7) yields two solutions: $x = v = 0$ and $x^* = v^* = \frac{2}{3}$, which is sustained by prices $p^* = \pi^* = \frac{2}{9}$. Summing up, we have established the following proposition

Proposition 1 *The optimal strategy for the monopoly platform consists in charging* $p^* = \pi^* = \frac{2}{9}$

4 Duopoly Competition under Single-Homing

We assume now that there are two platforms $i = 1, 2$, each committing to sizes v_i, x_i, $i = 1, 2$ simultaneously.

Using the specification of the consumers' preferences, we build the reduced form associated with an equilibrium configuration where $x_2 > x_1$ and $v_2 > v_1$. We have to satisfy simultaneously:

$$v_2 = \frac{x_2 - x_1 - p_2 + p_1}{x_2 - x_1} \tag{8}$$

$$v_1 = \frac{x_2 p_1 - x_1 p_2}{x_1(x_2 - x_1)} \tag{9}$$

$$x_2 = \frac{v_2 - v_1 - \pi_2 + \pi_1}{v_2 - v_1} \tag{10}$$

$$x_1 = \frac{v_2\pi_1 - v_1\pi_2}{v_1(v_2 - v_1)} \tag{11}$$

From which we get

$$p_1 = x_1(1 - v_2 - v_1) \tag{12}$$

$$p_2 = x_2(1 - v_2) - x_1v_1) \tag{13}$$

$$\pi_1 = v_1(1 - x_2 - x_1) \tag{14}$$

$$\pi_2 = v_2(1 - x_2) - x_1v_1) \tag{15}$$

Notice that the above expressions defines for each side of the market the demands and inverse demands of a vertical differentiation model where qualities are given by the sizes of the networks on the other side.

The reduced form for the objective functions of the platforms are therefore the following:

$$\Pi_1 = v_1x_1(2 - v_1 - v_2 - x_1 - x_2) \tag{16}$$

$$\Pi_2 = x_2v_2(2 - v_2 - x_2) - x_1v_1(v_2 + x_2) \tag{17}$$

First order conditions are:

$$v_1 = \frac{2 - x_1 - x_2 - v_2}{2} \tag{18}$$

$$x_1 = \frac{2 - v_1 - v_2 - x_2}{2} \tag{19}$$

$$v_2 = \frac{2 - x_2}{2} - \frac{v_1x_1}{2x_2} \tag{20}$$

$$x_2 = \frac{2 - v_2}{2} - \frac{v_1x_1}{2v_2} \tag{21}$$

Solving the system of equations above we obtain two quadruples of interior solutions, but only one of them satisfies the hierarchy $x_2 > x_1$ and $v_2 > v_1$. Namely:

$$x_1^* = v_1^* = \frac{2}{31}(6 - \sqrt{5}) \cong .242,$$

$$x_2^* = v_2^* = \frac{1}{31}(13 + 3\sqrt{5}) \cong .636.$$

Proposition 2 *Under Single-homing, there exists a duopoly equilibrium at which one platforms dominates the other in the two markets, while both platform enjoy positive profits*

Notice that, a priori, there might also exists interior equilibrium allocations where hierarchies are reversed from one side to the other. Suppose for instance that we start from the hierarchy $v_2 > v_1$ and $x_2 < x_1$. The reduced form for the payoffs are:

$$\Pi_1 = v_1x_1(1 - v_1 - v_2) + x_1v_1(1 - x_1) - x_1x_2v_2 \tag{22}$$

$$\Pi_2 = x_2v_2(1 - v_2) - v_2v_1x_1 + x_2v_2(1 - x_1 - x_2) \tag{23}$$

First order conditions are:

$$v_1 = \frac{2 - v_1 - v_2}{2} \tag{24}$$

$$x_1 = \frac{2 - v_1 - v_2}{2v_1} - \frac{x_2 v_2}{2v_1} \tag{25}$$

$$v_2 = \frac{2 - x_1 - x_2}{2} - \frac{v_1 x_1}{2x_2} \tag{26}$$

$$x_2 = \frac{2 - v_2 - v_1}{2} \tag{27}$$

Solving this system of equations shows that an acceptable solution must satisfy $x_2 = \frac{2}{3}$, $v_1 = \frac{2}{3}$, and $x_1 = \frac{2}{3} - v_2$. Accordingly, any possible solution to this system violates that initial hierachy.

5 Duopoly and One-Sided Multi-Homing

In this section, we assume that one side of the market is allowed to multi-home and the other side single-homes. Considering the commercial fair metaphor, it seems that a platform may easily require exclusivity from exhibitors, but not from visitors. Therefore we may assume that visitors may multii-home whereas exhibitors are not allowed to do so. Accordingly, a platform can be viewed, from the point of view of visitors as a bottleneck for getting access to the exhibitors located therein. Based on the received literature (see in particular Caillaud ans Jullien (2003) and Armstrong and Wright (2005)) we expect that exhibitors should be priced aggressively whereas visitors should be exploited.

Formally, assuming $v_2 > v_1$ the inverse demand function on the exhibitors'side are given by

$$\pi_1 = v_1(1 - x_1 - x_2) \tag{28}$$

$$\pi_2 = v_2(1 - x_2) - v_1 x_1 \tag{29}$$

On the visitors' side, the decision to attend one fair or the other or both are completely separable because it is known that exhibitors do not multihome (see Gabszewicz and Wauthy (2003)). We may therefore use the monopoly setup to define

$$p_1 = x_1(1 - v_1) \tag{30}$$

$$p_2 = x_2(1 - v_2) \tag{31}$$

Plugging this into the objective function of the platforms, we obtain a unique solution (conditional on the initial assumption $v_2 > v_1$):

$$x_1^* = \frac{12}{23}; v_1^* = \frac{12}{23}; x_2^* = \frac{10}{23}; v_2^* = \frac{18}{23} \tag{32}$$

Fairs are asymmetric in equilibrium with 1 being a bigger fair than 2 in terms of exhibitors. Under our assumption $v_2 > v_1$, platform 2 attracts the visitors displaying a high preference for network size but at the same time she benefits from a lower attendance. Notice that prices are such $\pi_1^* < \pi_2^*$ and $p_1^* > p_2^*$. We thus observe an inversed hierachy: the bigger network on one side of the market attracts a smaller market share on the other side. Still, platform 2 is more profitable than platform 1.

Notice that the above structure only describes a candidate equilibrium. Indeed, we have not checked for deviations that would alter the hierarchy prevailing in the exhibitors' market, i.e. $v2 > v1$. Recall that the hierachy resulting from network size on the exhibitors markets does not matter since visitors multi-home. In order to check for robustness of this candidate, we proceed as follows:

- First, we consider unilateral deviations in v_i along the best reply corresponding to the two regimes and then compare profits to evaluate the critical values so that a jump occurs in the global v_i best reply.
- Second, we consider separetely joint deviations in (v_i, x_i) and evaluate profits at the candidate equilibrium values for (v_j^*, x_j^*).

• Suppose we have $v_2 > v1$, then the payoff of 1 is equal to $v_1 x_1[2 - v_1 - x_1 - x_2]$. The best reply in v_1 is then equal to $v_1 = \frac{2-x_1-x_2}{2}$. The payoff of 1 in this regime along its best reply is then equal to $W_1 = \frac{(2-x_1-x_2)^2}{4} x_1$. Suppose instead that $v_2 < v_1$ the payoff of 1 is $x_1[v_1(2 - x_1 - x_2) - v_2 x_2]$. The best reply under this hierarchy is $v_1' = \frac{2-x_1}{2}$. The corresponding payoff is then $W_1' = (\frac{(2-x_1)^2}{4} - v_2 x_2)x_1$. Direct computations show that $W_1 - W_1' > 0$ whenever $v_2 > 1 - \frac{x_1}{2} - \frac{x_1}{4}$. It is then direct to show that this condition is satisfied at our equilibrium candidate. This implies that platform 1 does not gain by switching to the regime where it would reverse the v_i hierarchy. In order to check that 2 does not gain either, it is sufficient to reverse the indices in the last condition and check that the condition is satisfied at our candidate. Which, again, is the case.

• We know check for joint deviation in (v_i, x_i). To this end, we start with the expression of payoff for firm 1 when $v_2 > v_1$: $v_1 x_1[2 - v_1 - x_1 - x_2]$. However, we need now to replace x_2 and v_2 by their best reply values. Since $v_1 = \frac{2-x_1-x_2}{2}$ and $x_1 = \frac{2-v_1-x_2}{2}$, we obtain by solving the system $v_1 = x_1 = \frac{2-x_2}{3}$. Accordingly, the payoff of firm 1 along these best replies is given by $Z_1 = \frac{(2-x_2)^3}{27}$. A similar argument is then developed under the assumption that $v_2 < v_1$. We have here that $v_1' = \frac{2-x_1}{2}$ and $x_1' = 1 - \frac{v_1}{2} - \frac{v_2 x_2}{2v_1}$. Solving the system, we obtain $x_1' = \frac{4}{3} - \frac{2\sqrt{1+3v_2x_2}}{3}$ and $v_1' = \frac{1}{3} + \frac{\sqrt{1+3v_2x_2}}{3}$. The payoff along the best replies is therefore given by $Z_1' = \frac{4}{27}(1 - 9v_2x_2 + (1 + 3v_2x_2)^{\frac{3}{2}}$. Evaluating numerically the expressions Z_1 and Z_1' at the equilibrium candidate, we can check that $Z_1 > Z_1'$. Accordingly, platform 1 does not gain by deviating from the equilibrium. A similar computation can be performed for platform 2 by reversing the indices and a comparable result applies: 2 does not gain from deviating.

Summing up, we have proved

Proposition 3 *Under One-sided Multi-homing, there exists a duopoly equilibrium at which both platforms enjoy positive profits, the prices hierarchy is reversed from one side to the other*

References

[1] Ambrus A. and R. Agrenziano (2004), Network Markets and consumer Coordination, Mimeo, Cowles Foundation DP 1481

[2] Armstrong M. and J. Wright (2004), Two-sided markets with multihoming and exclusive dealing, Mimeo, University College London

[3] Caillaud B. and B. Jullien (2003), Chicken and Eggs: Competition among Intermediation Service Providers, *Rand Journal of Economics*, 34-2, 309-328.

[4] De Palma A., L. Leruth and P. Regibeau (1999), Partial compatibility with network externalities and double purchase, *Information Economics and Policy*, 11, 209-227.

[5] Gabszewicz J. and X. Wauthy (2003), The Option of Joint Purchase in Vertically Differentiated Markets, *Economic Theory*, 22, 817-829.

[6] Gabszewicz J. and X. Wauthy (2004), Two-Sided Markets and Price Competition with Multi-Homing, Mimeo, CORE DP 2004/30

[7] Mussa M. and S. Rosen (1978), Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301-317

[8] Rochet J-C. and Tirole J. (2004), Defining Two-sided Markets, mimeo, IDEI

[9] Roson R. (2005), Two-Sided Markets: a Tentative Survey, *Review of Network Economics*, 4, 142-160