The Option of Joint Purchase in Vertically Differentiated Markets

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Abstract

Within the framework proposed by Mussa and Rosen (1978) for modelling quality differentiation, we allow consumers to buy simultaneously different variants of the same indivisible good. We call this the "joint purchase option". We show that this option dramatically affects price competition: while a unique equilibrium always prevails when consumers are assumed to make mutually exclusive purchases, either uniqueness, or multiplicity, or absence of price equilibrium arise when the joint purchase option is opened depending on the utility attached to joint purchase relative to separate purchases.

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1 Introduction

In the theory relating price competition and product selection, it is traditionally assumed that those consumers who decide to buy a product, single out a particular variant of it among the various substitutes provided by the industry, and buy a single unit of that variant[1]. This way of defining the decision set open to consumers reduces the *quantity* decision of households to a binary choice : one or zero unit. Even though this traditional assumption is certainly a good approximation for indivisible durable goods, it often constitutes a too drastic simplification when consumers are sufficiently rich for contemplating the possibility of buying more than a single variant of these goods. This becomes particularly true with the observed improvement of living standards through the population, which allows many households to be equipped with several variants of the same indivisible product. In rich countries today, it is far from seldom to observe households equipped with two or three different cars, or several TV-sets or P.C.'s. Similarly, it is not difficult to identify consumers owning two or three different houses for their personal use only. The above simplifiying assumption prevents to capture such situations, and any model which is based on it cannot explain how price competition among producers of these goods would be influenced by the fact that the market also includes consumers who may be considering buying more than a single unit of the commodity. In this note, we tackle explicitely this problem in the framework of an industry including two firms selling each a product which is vertically differentiated from the other. We keep the property that each variant is consumed in indivisible units. Still, unlike the traditional assumption , we now suppose that consumers who are interested in consuming both variants at the existing price constellation are allowed to do so. The extent to which this possibility of joint purchase influences price competition between duopolists is the problem considered in this note. As we shall see, this possibility may influence in a considerable way the nature of price competition, so that the traditional assumption has to be regarded as far from innocuous.

In a nutshell, we may summarize our findings in the following way. First, when buying the two variants does not add much in utility compared with the utility obtained when buying the best variant only, price competition is not influenced by opening to consumers the option of buying both variants :

[1]Virtually any paper dealing with address-models of product differentiation makes this assumption ; see also Tirole (1988). A noticeable exception is the recent contribution by Caillaud, Grilo and Thisse (2000)

equilibrium prices are the same as those obtained in the "traditional" model, and no consumer in the population takes advantage of the new option available to them (in the following, we refer to this equilibrium as the *exclusive purchase equilibrium).* Second, when buying the two variants starts to add a more substantial amount in utility compared with the utility of the best variant, multiple price equilibria arise, among which the equilibrium of the traditional model is still present. Yet another equilibrium appears along at which some consumers buy both variants of the product. At the new price equilibrium (referred below as the *joint purchase equilibrium)*, both prices are lower than at the standard one, in which no joint purchase is allowed.This reflects the fact that the low-quality seller has to lower his price in order to attract some consumers, who already buy the high-quality product, to purchase as well the low-quality one. Finally, when the utility of buying both variants is close to the sum of the utilities corresponding to consuming each variant separately, no price equilibrium in pure strategies still exists.

The note is organised as follows. In the next section, we introduce briefly the "pure" model of vertical product differentiation, and set how this model has to be adapted in order to take also into account the option of consuming both variants. In section 3, we examine how price competition develops when this new option is opened to consumers. In a short conclusion, we relate the present note to a companion paper dealing with a similar problem, but formulated in a different context.

2 The Joint Purchase Option

Consider a model "à la Mussa-Rosen", with two variants of a product, indexed by their quality u_i, $i = 1, 2$ (Mussa and Rosen (1978)). We assume, without loss of generality, that $u_2 > u_1$. Firms produce at zero cost. They choose prices non-cooperatively in order to maximise revenue. Consumers' types are indexed by a parameter θ which expresses the intensity of their preferences for buying a unit of the good. Types are uniformly distributed in the $[0, 1]$-interval, with one consumer per type. If consumer θ buys one unit of variant i at price p_i, his utility is given by

$$u_i\theta - p_i. \tag{1}$$

We denote by θ_i, $i = 1, 2$, the consumer who is indifferent between the option of buying one unit of variant i at price p_i and the option of not buying. If we assign zero utility to the latter option, we obtain

$$\theta_i = \frac{p_i}{u_i}. \tag{2}$$

Similarly we denote by θ_{12} the consumer who is indifferent between buying one unit of variant 1 at price p_1 and one unit of variant 2 at price p_2, i.e.

$$\theta_{12} = \frac{p_2 - p_1}{u_2 - u_1}. \tag{3}$$

The standard model of vertical product differentiation assumes that consumers, when they buy, select which variant they wish to buy, *at the exclusion of the other.* Using (2) and (3), demands adressed to the sellers are then easily derived as

$$D_1(p_1, p_2) = \theta_{12} - \theta_1 = \frac{p_2 u_1 - p_1 u_2}{u_1(u_2 - u_1)}; \tag{4a}$$

$$D_2(p_1, p_2) = 1 - \theta_{12} = 1 - \frac{p_2 - p_1}{(u_2 - u_1)}. \tag{4b}$$

The corresponding price game has a unique price equilibrium (*exclusive purchase equilibrium*), namely

$$p_1^* = \frac{u_1(u_2 - u_1)}{4u_2 - u_1}, \tag{5a}$$

$$p_2^* = \frac{2u_2(u_2 - u_1)}{4u_2 - u_1}; \tag{5b}$$

in the sequel we simply refer to this equilibrium as p^*.

Now let us assume that, unlike the "traditional" assumption, the quantity decision set of each household is extended to also include the possibility of buying both variants, and denote by u_3 the utility index derived from such a joint consumption. In order to preserve the fact that variants 1 and 2 are *substitutes* of the same product, we shall assume that[2]

$$u_2 < u_3 < u_1 + u_2.$$

In the case of joint purchase , the utility of consumer θ is assumed to be given by $u_3\theta - p_1 - p_2$. As above, we denote by θ_{i3} the consumer who is

[2]The case $u_3 \geq u_1 + u_2$ is considered in Gabszewicz, Sonnac and Wauthy (2000)

indifferent between buying one unit of variant i at price p_i and one unit of *both* variants at prices p_1 and p_2, namely

$$\theta_{13} = \frac{p_2}{u_3 - u_1}; \tag{6a}$$

$$\theta_{23} = \frac{p_1}{u_3 - u_2}. \tag{6b}$$

With these definitions, it is a matter of patience to derive the demand functions of the duopolists, which can be best understood using the following diagram providing a partition of the domain of (p_1, p_2)-prices into four sub-domains $P_i, i = 1, ...4$.

Insert Figure 1 about here

The price -subdomain P_1, which is delimited from below by the line $p_1 = u_3 - u_2$, is defined as

$$P_1 = \{(p_1, p_2) : p_1 \geq u_3 - u_2\}.$$

In this domain, the demand functions D_1 and D_2 are given by (4) : the price p_1 is so high that even consumer $\theta = 1$, who has the highest willingness to pay for consuming both variants, is not willing to buy them at that price. Accordingly, in the domain P_1, demand functions are as in the "pure" vertical differentiation model since nobody in the market is considering to buy both variants. Yet, this changes as soon as $p_1 < u_3 - u_2$: then some consumers - those with the highest θ's - start to buy both variants. Consider then the sub-domain P_2 defined by

$$P_2 = \left\{ (p_1, p_2) : p_1 < u_3 - u_2; p_1 \geq p_2 \frac{u_1}{u_2} \right\}.$$

In P_2, demands are given by

$$D_1(p_1, p_2) = 1 - \theta_{23} = 1 - \frac{p_1}{u_3 - u_2}; \tag{7a}$$

$$D_2(p_1, p_2) = 1 - \theta_{12} = 1 - \frac{p_2}{u_2}. \tag{7b}$$

In the sub-domain P_2, all consumers who buy variant 1 also buy variant 2, so that the market of firm 2 extends up to θ_2. This changes as soon as the inequality $p_1 \geq p_2 \frac{u_1}{u_2}$ is reversed. Then a new class of consumers appears at prevailing prices : those who start to buy only variant 1. Then we enter into the sub-domain P_3 defined by

$$P_3 = \left\{ (p_1, p_2) : p_1 < u_3 - u_2; p_1 \leq p_2 \frac{u_1}{u_2}; p_1 \geq p_2 \frac{u_3 - u_2}{u_3 - u_1} \right\}.$$

In this sub-domain, the demand adressed to firm 2 now coincides with the demand adressed to this firm in the "pure" vertical differentiation model.Yet the demand adressed to firm 1 is made of those consumers who buy both variants (the interval $[\theta_{23}, 1]$), as well as of those who buy variant 1 only (the interval $[\theta_1, \theta_{12}]$), that is

$$D_1(p_1, p_2) = (1 - \theta_{23}) + (\theta_{12} - \theta_1) = 1 + \frac{p_2}{u_2 - u_1} - p_1 K \qquad (8a)$$

$$D_2(p_1, p_2) = 1 - \theta_{12} = 1 - \frac{p_2 - p_1}{u_2 - u_1}, \qquad (8b)$$

with K defined by

$$K = \frac{(u_3 - u_2)(u_2 - u_1) + u_1(u_3 - u_1)}{u_1(u_2 - u_1)(u_3 - u_2)}. \qquad (9)$$

Finally, in the sub-domain P_4 defined by

$$P_4 = \left\{ (p_1, p_2) : p_1 < u_3 - u_2; p_1 \leq p_2 \frac{u_3 - u_2}{u_3 - u_1} \right\},$$

we get

$$D_1(p_1, p_2) = 1 - \theta_1 = 1 - \frac{p_1}{u_1}; \qquad (10a)$$

$$D_2(p_1, p_2) = 1 - \theta_{13} = 1 - \frac{p_2}{u_3 - u_1} : \qquad (10b)$$

now the boundary between markets of firms 1 and 2 corresponds to the consumer who is indifferent between the options of buying only variant 1 or buying both variants, and no longer to the consumer who is indifferent between buying variant 1 only and buying variant 2 only, as it was the case in the sub-domains P_i , $i = 1, 2, 3$, defined above.

At this point, three remarks are in order. Notice first that, compared with the standard vertical differentiation model, allowing for joint purchase essentially amounts to alter the definition of demands when the price of firm 1 is below the value $u_3 - u_2$. The dividing line between region P_1 (where the standard model applies) and regions $P_{2,}$, P_3 and P_4 does not depend on

p_2. When choosing p_1 firm 1 "decides" whether the demands corresponding to those of the standard analysis apply or not, while unilateral deviations of p_2 cannot achieve the same result. Second, it should be noticed that demand addressed to firm 2 in region P_2, and to firm 1 in region P_4, are the standard monopoly demands. Third, the payoffs in this game, obtained as the revenue functions resulting from the demands addressed to each firm in the various sub-domains of prices, are continuous functions throughout the whole space of prices.

Equipped with the above material, we can now tackle the equilibrium analysis assuming that consumers are also allowed to make joint purchases (*joint purchase price game*). This is done in the next section.

3 Equilibrium Analysis

In the situation in which joint purchase does not add much in utility, compared with the utility index corresponding to the top quality variant, a first question which seems natural to raise is whether the equilibrium p^* of the original price game is still an equilibrium in the joint purchase price game. Since the increase in utility obtained by consumers from joint purchase is assumed to be small, it may be conjectured that firms in the latter may have no interest to set prices at equilibrium taking advantage of this new opportunity. Proposition 1 below provides a positive answer to this conjecture.

Proposition 1. *there exists an interval* $[u_2, u_2 + \varepsilon^*]$, *with* $\epsilon^* > 0$, *such that, whenever* $u_3 \in [u_2, u_2 + \varepsilon^*]$, *the exclusive purchase equilibrium* p^* *is still an equilibrium in the joint purchase price game.*

Proof :

First, it is clear that, for the standard equilibrium $p^* = (p_1^*, p_2^*)$ to be an equilibrium in the joint purchase price game, we must have $p_1^* > u_3 - u_2$ and $p_2^* u_1 > p_1^* u_2$: the first inequality follows from the fact that , at p^*, even consumer $\theta = 1$ should not be willing to buy both variants, while the second inequality follows from $D_1(p_1^*, p_2^*) > 0$. The equilibrium $p^* = (p_1^*, p_2^*)$ is thus located in the (p_1, p_2) - plane as depicted on figure 1.

Notice that no unilateral deviation from the equilibrium p^* which would let the resulting pair of prices in P_1, can be advantageous to any of the two firms : recall that p^* is an equilibrium in the original game which is, in particular, defined in the sub-domain P_1, so that unilateral deviations leaving the pair of prices in this sub-domain cannot be profitable. Since any unilateral deviation of firm 2 from p_2^* maintains the pair of prices in the sub-domain P_1, it cannot be advantageous to firm 2 : in this sub-domain, we know that p_2^* is a best response against p_1^*. To destroy the equilibrium p^* as an equilibrium in the joint purchase price game, we can thus rely only on unilateral deviations p_1 of firm 1 which drive the resulting pair of prices in P_3 orP_4. For deviations in P_3, it follows from (8) that the revenue of firm 1 obtains as

$$R_1(p_1, p_2^*) = p_1(1 + \frac{p_2^*}{u_2 - u_1} - p_1 K),$$

which is maximal in P_3 for p_1 given by

$$p_1^{'} = \frac{u_2 - u_1 + p_2^*}{2(u_2 - u_1)K}. \tag{11}$$

On the other hand, revenue at the equilibrium p^* is given by

$$R_1(p_1^*, p_2^*) = p_1^*(\frac{p_2^* u_1 - p_1^* u_2}{u_1(u_2 - u_1)}).$$

Comparing $R_1(p_1^*, p_2^*)$ and $R_1(p_1^{'}, p_2^*)$ reveals that the former exceeds the latter as long as $u_3 \leq u_2 + \varepsilon^*$, with

$$\varepsilon^* = \frac{4u_1 u_2(u_2 - u_1)}{32u_2^2 - 12u_1 u_2 + u_1^2} > 0,$$

where the last strict inequality follows from the fact that $u_2 > u_1$. Consequently, when $u_3 \in [u_2, u_2 + \varepsilon^*]$, there exists no unilateral advantageous deviation for firm 1 in P_3. Similarly, it can be checked that no unilateral advantageous deviation for firm 1 which would bring the pair of prices in P_4, exists either. Consequently, when $u_3 \in [u_2, u_2 + \varepsilon^*]$, the pair of prices (p_1^*, p_2^*) remains a price equilibrium in the joint purchase price game. Q.E.D.

The above proposition shows that the equilibrium p^* remains robust to the introduction of the joint purchase option, at least when u_3 is in a sufficiently small neighborhood of u_2. But this does not preclude the possibility that, for some u_3-values, another price equilibrium would coexist with p^* when the joint purchase option becomes available. That this is indeed the case follows from the following

Proposition 2. *There exists a non-degenerate interval of values for u_3 in which both the exclusive and the joint purchase equilibria coexist.*

Proof:

Consider the pair of prices which are best responses to each other in the sub-domain P_3, with payoffs (revenues) obtained from the demand functions in P_3 (see (8)), namely

$$\begin{aligned} R_1(p_1,p_2) &= p_1(1+\frac{p_2}{u_2-u_1}-p_1K) \\ R_2(p_1,p_2) &= p_2(1-\frac{p_2-p_1}{u_2-u_1}), \end{aligned}$$

with K as defined by (9). These best responses in P_3 are easily identified from the first-order conditions, i.e.

$$\phi_1(p_2) = \frac{u_2-u_1+p_2}{2(u_2-u_1)K} \tag{12}$$

for firm 1, and

$$\phi_2(p_1) = \frac{u_2-u_1+p_1}{2} \tag{13}$$

for firm 2. Combining these best responses yields a candidate price equilibrium $(\overset{\circ}{p_1},\overset{\circ}{p_2})$ which is given by

$$\begin{aligned} \overset{\circ}{p_1} &= \frac{3(u_2-u_1)}{4(u_2-u_1)K-1}; \\ \overset{\circ}{p_2} &= \frac{(2K(u_2-u_1)+1)(u_2-u_1)}{4(u_2-u_1)K-1}. \end{aligned}$$

Now we study the necessary and sufficient conditions under which this candidate is, indeed, a price equilibrium. First, it is easy to check that $\overset{\circ}{p_1} < u_3-u_2$, so that $\overset{\circ}{p_1} \in P_3$ (the candidate equilibrium is indeed defined in Region P_3) and, by definition, no unilateral deviation can be advantageous if it leaves the pair of prices in this sub-domain. Let us then consider deviations that lead us outside region P_3.

To remain robust against unilateral deviations of firm 2 driving the pair of prices in P_4, it is necessary and sufficient that

$$R_2(\overset{\circ}{p_1},\overset{\circ}{p_2}) \geq R_2(p_1{}^{\circ},\psi_2(\overset{\circ}{p_1})),$$

with $R_2(\overset{\circ}{p_1},\psi_2(\overset{\circ}{p_1}))$ denoting the revenue of firm 2 at its best response against $p_1{}^{\circ}$ in P_4. Using (10), a direct comparison between these two numbers reveals that the desired inequality holds if, and only if, $u_3-u_2<\eta$, with η as defined in footnote 3. [3] Ruling out an advantageous deviation for firm 1 driving the pair of prices in P_2 follows from applying the same method

[3]Computations show that
$\eta = \frac{u_1(u_2-u_1)(11u_1-8u_2+\sqrt{17u_1^2-48u_1u_2+64u_2^2})}{2(4u_2-u_1)^2}$

as the one used above for deviations of firm 2 driving the pair of prices in P_4. The resulting comparison reveals the existence of a particular value η', $\eta < \eta'$, such that no deviation for firm 1 in P_2 can be profitable if, and only if, $u_3 - u_2 < \eta'$.[4] Whenever $u_3 - u_2 < \text{Min}\{\eta, \eta'\}$, no profitable deviations from $(\overset{\circ}{p}_1, \overset{\circ}{p}_2)$ exist towards regions $P_2$and P_4. It then remains to exclude the possibility of profitable deviations from $\overset{\circ}{p}_1$ for firm 1 which would lead the pair of prices in P_1. Such advantageous deviations are excluded if, and only if, the inequality

$$R_1(\overset{\circ}{p}_1, \overset{\circ}{p}_2) \geqq R_1(\psi_1(\overset{\circ}{p}_2), \overset{\circ}{p}_2)$$

holds, with $\psi_1(\overset{\circ}{p}_2)$ denoting the best response of firm 1 to $\overset{\circ}{p}_2$ in the standard model of vertical differentiation (remind that the demand function of firm 1 is defined in P_1 as in this model). An additional computation shows that the above inequality holds if, and only if, $u_3 > u_2 + \delta^*$, with δ^* as defined in footnote 5 ; furthermore, it is easily checked that $0 < \delta^* < \varepsilon^*$.[5] A direct comparison between the numbers δ^* and $\eta = \min\{\eta, \eta'\}$ shows that $\delta^* < \eta$. Consequently, when the difference $u_3 - u_2$ starts to be larger than the number δ^*, the pair of prices $(\overset{\circ}{p}_1, \overset{\circ}{p}_2)$ is indeed a price equilibrium, namely the joint purchase equilibrium, in the interval of u_3-values $[u_2 + \delta^*, u_2 + \eta]$. This interval includes the non-degenerate interval $[u_2 + \delta^*, u_2 + \varepsilon^*]$ in which the pair of prices (p_1^*, p_2^*) defined by (5) has been already shown to be a price equilibrium (see proposition 1). This completes the proof of proposition 2. Q.E.D.

Proposition 1 indicates that, when joint purchase only adds little utility to the utility of the high quality variant, the traditional model and the joint purchase price game give the same outcome to price competition. Yet, proposition 2 shows that this is no longer true when the increase in utility corresponding to joint purchase becomes more significant. Even if the exclusive purchase equilibrium (p_1^*, p_2^*) still belongs to the set of equilibria, another price equilibrium, - the joint purchase equilibrium $(p_1^\circ, \overset{\circ}{p}_2)$ -, starts to coexist. However, this pair of prices does not remain an equilibrium for *all* values of u_3 in the admissible range $]u_2, u_1 + u_2[$: we know from the above proof that, when $u_3 - u_2 > \eta$, firm 2 has an advantageous deviation from $\overset{\circ}{p}_2$ by letting the pair of prices to enter into P_4. Accordingly, for values

[4]Computations show that $\eta' = \frac{2u_1(u_2-u_1)(2u_1+u_2+3\sqrt{u_1^2-4u_1u_2+9u_2^2})}{(4u_2-u_1)^2}$

[5]The explicit value of δ^* obtains as

$$\delta^* = \frac{-2(u_1^3 - 8u_1^2u_2 + 7u_1u_2^2 - 3\sqrt{u_1^2u_2(9u_2 - u_1)(u_2 - u_1)})}{32u_2^2 - 4u_1u_2 + u_1^2}$$

of u_3 exceeding $u_2 + \eta$, neither (p_1^*, p_2^*), nor $(\overset{\circ}{p_1}, \overset{\circ}{p_2})$ are still price equilibria. That no other price equilibrium exists in that case follows from the following reasoning. First, due to the concavity of the revenue functions of both firms when restricted to the sub-domains P_1 and P_3, it is clear that no pair of prices differing from (p_1^*, p_2^*) or $(\overset{\circ}{p_1}, \overset{\circ}{p_2})$ could be a price equilibrium in these sub-domains. Furthermore, given the definition (10) of demands in the sub-domain P_4, any candidate equilibrium in this sub-domain is excluded by the fact that the best response $\frac{u_1}{2}$ of firm 1 lies outside the projection of the sub-domain P_4 on the p_1-axis. A similar argument reveals that no candidate equilibrium could exist either in P_3. Since a direct comparison between $u_2 + \eta$ and $u_1 + u_2$ shows that $u_2 + \eta < u_1 + u_2$, we obtain the following

Proposition 3 *In the non-degenerate interval* $]u_2 + \eta, u_1 + u_2[$ *of* u_3*-values, there exists no price equilibrium in pure strategies.*

It is important to notice that proposition 3 only precludes the existence of *pure-strategy* price equilibria in the relevant domain, but does not do it for *mixed* strategies. In fact,since the revenue functions of both firms are continuous, we know that mixed-strategy price equilibria must exist whenever $u_3 \in]u_2 + \eta, u_1 + u_2[$. Furthermore, since these revenue functions are piecewise concave, one should expect that equilibrium mixed strategies must have only a finite support in prices. Exploiting this property, we have been able to identify mixed-strategy equilibria in the relevant range. Without entering into detail,[6] let us simply notice that these mixed-strategy equilibria consist, for one firm, in playing a pure strategy and, for the other, in playing with some probability a "low" price and, with the complementary probability, a "high" price. Furthermore, the closer u_3 to $u_1 + u_2$, the higher the probability assigned to the "high" price option, and the closer this option to the pure monopoly price. This is interesting because, at the only price equilibrium corresponding to the limiting case $u_3 = u_1 + u_2$, both firms set their monopoly price $\frac{u_i}{2}$ (for a formal proof, see Gabszewicz, Sonnac and Wauthy (2000)). In other words, the sequence of mixed-strategy equilibria which we have identified, converges to the pair $(\frac{u_1}{2}, \frac{u_2}{2})$ of monopoly prices when $u_3 \to u_1 + u_2$.

4 Conclusion

As a conclusion to the above analysis, it seems fair to say that introducing the joint purchase option considerably enriches the nature of price competi-

[6]the derivation of such a mixed strategy equilibrium is given in the appendix.

tion between firms, compared with the standard model of vertical product differentiation. The natural next step to pursue research in this field would consist in studying the implications of joint purchase on *quality selection* by firms. With this respect, it is interesting to put the present analysis in perspective with a companion paper (Gabszewicz and Wauthy (2000)) dealing with a closely related topic in the context of monopoly. In this paper, we consider a monopolist selling a homogeneous product to a population of consumers, starting with the assumption that consumers, when they decide to buy the good, only buy a single unit of it. We show that, under this assumption, it is always optimal for the monopolist to select the highest quality which is available to him when no cost of any sort is attached to quality improvement. This simply reflects the fact that, if cost does not increase with quality, monopolist's payoff must necessarly increase with it. Then we extend the decision set of the consumers to allow the option of buying as well *two* units of the same indivisible good, supposing that the utility index corresponding to the consumption of two units of the good is smaller than the utility index corresponding to the consumption of a single unit of it.[7] Of course, the price monopoly solution may be influenced by this extension of consumers' decision sets, since the monopolist can now attract households who would be willing to buy two units of the good if the unit price is sufficiently low. Surprisingly enough, this may entail that the above statement concerning quality selection is no longer necessarly true. Indeed, we build an example in which the monopolist does not select to produce the highest quality which is available, in spite of the fact that we have assumed that cost does not increase with quality! Consequently, introducing the double purchase option may have dramatic effects on quality selection in the case of monopoly. It seems natural to wonder whether such effects on quality could as well appear as a consequence of introducing the joint purchase option in the framework of a vertically differentiated market.

[7] We recognise here the analog of our assumption $u_3 < u_1 + u_2$.

References

[1] Caillaud B., I. Grilo and J. Thisse (2000), Stratégies de différenciation sur le Marché du Positionnement par Systèmes satellitaires, CERAS, mimeo.

[2] Gabszewicz J. and X.Y. Wauthy (2000), Another Perverse Effect of Monopoly Power, CORE DP 20..

[3] Gabszewicz J., N. Sonnac and X.Y.Wauthy (2000), Price Competition with Complementary Goods, *Economics Letters*, forthcoming.

[4] Tirole J. (1988),*The Theory of Industrial Organisation*, MIT Press.

[5] Mussa M. and S. Rosen (1978), Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301-17.

Appendix: A mixed strategy equilibrium

In this appendix, we characterize an equilibrium in mixed strategies in which firm 2 randomizes while firm 1 plays a pure strategy. The alternative equilibrium where firm 1 randomizes can be derived using the same methodology.

Recall first that each firm's payoff is concave, region by region. Therefore, against a pure strategy of firm 1, we may identify a unique best reply in each region of the price space. Let us then consider firm 2's best reply candidate against a "low" p_1 (i.e. region P_1 irrelevant).

As already argued in section 3, firm 2's best reply candidate in region P_2 is defined as the frontier between between region P_2 and P_3. By continuity of firm 2's payoffs, this best reply candidate must be dominated by the best reply candidate in region P_3, which is given by $\phi_2(p_1)$ as defined by (13). Using (10b), it is immediate to derive firm's candidate best reply in region P_4 as $p_2 = \frac{u_3-u_1}{2}$.

In order to identify which of $\phi_2(.)$ or $\frac{u_3-u_1}{2}$ is the "true" best reply against p_1 we only need to compare firm 2's payoffs in the two cases and identify the critical level of p_1 which makes firm 2 indifferent between the two strategies. Solving $\frac{u_3-u_1}{4} = \phi_2(p_1)(1 - \frac{\phi_2(p_1)-p_1}{u_2-u_1})$, for p_1, we obtain the critical value $\hat{p_1}$.

A candidate equilibrium may therefore be identified as follows: firm 2 randomizes over $\phi_2(\hat{p_1})$ and $\frac{u_3-u_1}{2}$ with probability $(\mu, 1-\mu)$ while firm 1 plays the pure strategy $\hat{p_1}$. In order to prove that this is indeed an equilibrium, we only need to show that there exists a μ such that $\hat{p_1}$ is a best reply for firm 1 against firm 2's mixed strategy $(\mu, 1-\mu)$.

The profit function of firm 1 against firm 2's mixed strategy defines as

$$\pi_1(p_1, p_2, \mu) = p_1[\mu(1 - \frac{p_1}{u_1}) + (1-\mu)(1 + \frac{p_2}{u_2 - u_1} - p_1 K)].$$

In order for $\hat{p_1}$ to be part of an equilibrium, it must be true that the first order condition for the above function is satisfied at $(\hat{p_1}, \phi_2(p_1))$, i.e.

$$\mu(1 - \frac{2\hat{p_1}}{u_1}) + (1-\mu)(1 + \frac{\phi_2(\hat{p_1})}{u_2 - u_1} - \hat{p_1}K) = 0.$$

The first term is positive while the second is negative, so that there must exist some μ^* which satisfies the previous equation. Straightforward computations yield

$$\mu^* = \frac{1 + \frac{\phi_2(\hat{p_1})}{u_2-u_1} - 2\hat{p_1}K}{1 + \frac{\phi_2(\hat{p_1})}{u_2-u_1} - 2\hat{p_1}K - (1 - \frac{2\hat{p_1}}{u_1})}$$

Aditionnal computations show that $\mu \geq 0$ whenever the pure strategy equilibrium candidate defined in region P_3 ceases to exist whereas it is less than 1 whenever $\hat{p}_1 < p_1^m$.

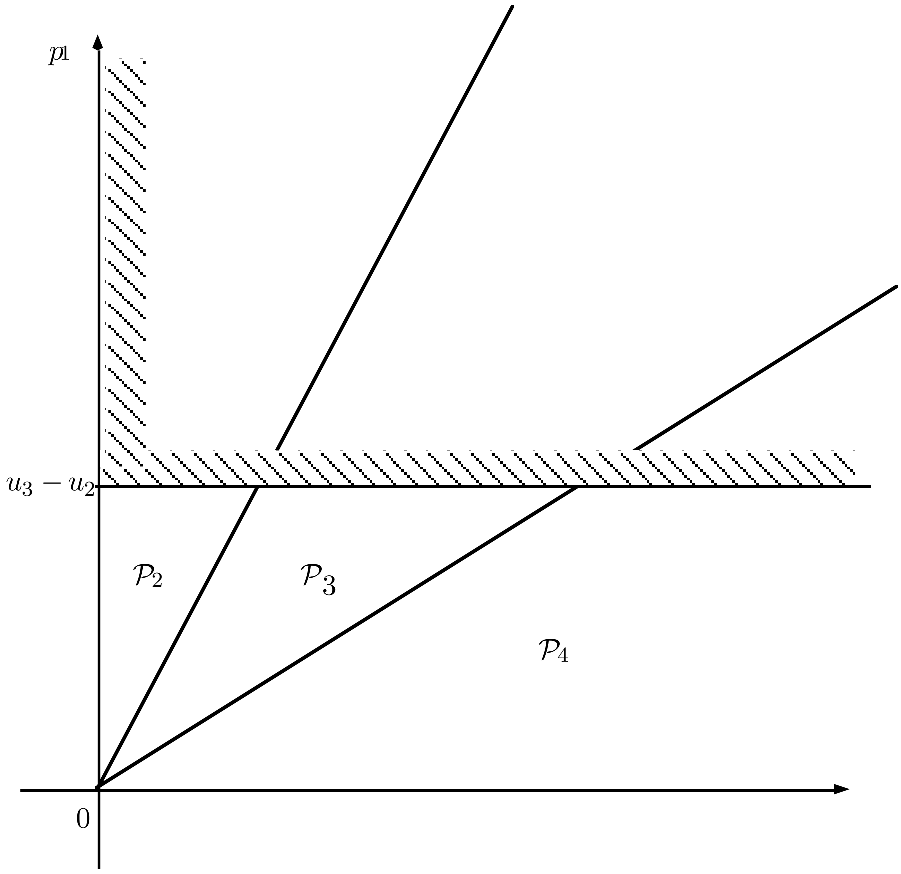

Figure 1