BERTRAND COMPETITION AND COURNOT OUTCOMES: FURTHER RESULTS

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ABSTRACT

In this note, we extend the classical result of Kreps & Scheinkman [1983] to an oligopolistic setting where capacity is modelled as an imperfect commitment device. To this end, we retain the cost structure put froward in Dixit [1980], i.e. we allow firms to produce beyond installed capacities but in this case they have to incur an additional unit cost. When the unit cost of producing beyond capacityis large enoough, Cournot outcomes always obtain in the unique subgame perfect equilibrium whereas when it is low, there is a continuum of equilibria. Each of them involve an identical aggregate capacity which converges to the competitive one as the extraz unit cost tends to zero.

Keywords : Price Competition, Capacity Commitment

JEL Classification : D43, F13, L13

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1) INTRODUCTION

According to the Bertrand paradox "two is enough for competitive outcomes". This result however is well-known to rely on the hypothesis of constant marginal cost and for this reason lacks any generality (this critique of Bertrand [1883] dates at least back to Edgeworth [1925]). Still, this paradox is often contrasted with the Cournot outcomes and reconciling these two approaches has been the aim of many papers. The most spectacular result in this field is to be found in Kreps & Scheinkman [1983] (hereafter KS). Two firms invest in capacities and then compete in prices with constant marginal cost up to capacity and an abritrarily large marginal cost above capacity; investing into limited capacity has a strategic value because it amounts to commit not to be aggressive in the pricing game. KS reconciliate the Cournot and Bertrand approaches by showing that the Cournot outcome is the unique Subgame Perfect Equilibrium of their game. This result also has been much criticized. In particular because of the particular rationing rule, the efficient one, retained for the analysis. Still, the fact that capacity commitment relaxes price competition and drives equilibrium outcomes towards cournotian ones is much less controversial.

An open question remains: to which extent is the "rigid capacity" assumption central for this result to hold? In other words, would smoothly increasing marginal cost (or weaker forms of quantitative constraints) lead to similar outcomes? This question is interesqting from a purely theoretical point of view but it also belongs to the class of generalizations that seem necessary to pursue robust empirical studies on oligopolistic markets. As argued for instance by Tirole [1988] (chap. 5, p.244), "the Bertrand and Cournot models should not be viewed as two rival models giving contradictory predictions of the outcome of competition in a given market. (After all, firms almost always compete in prices.) Rather, they are meant to depict markets with different cost structures." It seems fair however to say that a rigorous link between the shape of marginal cost and equilibrium price is still missing under strategi price competition. The main goal of the present paper consists precisely in showing how the whole range of prices, from Bertrand towards Cournot ones, can be sustained as equilibrium outcomes in oligopolistic industries, depending on the shape of marginal costs.

Price competition under decreasing returns to scale, or weaker forms of capacity constraints, has been recently studied in the literature. Klemperer and Meyer [1989] have sdhed light on this issue using the concept of supply -function equilibrium. In their model however, the price is not fully strategic in the sense that they are intemately related to supply. Dastidar [1995], [1997] considers a price Bertrand competition with continuously increasing marginal costs and homogeneous products. Maggi [1996] introduces imperfect commitment in capacity games using the set up developed by Dixit [1980] in a market for differentiated products. In his model, marginal cost is modelled as a stepwise, discontinuous function, being constant up to the capacity level where it jumps up to a higher level. Maggi obtain a "cournotian-like" outcome in his unique subgame perfect equilibrium. Still, both Maggi's and Dastidar's result does not compare at all with that of KS and more generally with the standard literature on capacity-constrained pricing games because they totally rule out any form of rationing.

Whether firms are allowed to ration consumers or not is central when studying price competition under decreasing returns to scale. From a technical point of view, it is well-known that rationing tends to destroy payoffs' concavity, and therefore pure strategy equilibria. Under weak forms of capacity constraints (such as those retained by Dastidar and Maggi) producing beyond capacities is always feasible but not necessarily profitable. We are inclined to believe that firms should be allowed not to meet full demand whenever this last strategy is profitable. As a consequence, we will allow for consumers' rationing.

In the next section, we extend the KS result to the case of imperfect commitment and many firms. To this end we retain the cost framework put forward by Dixit [1980] and Maggi [1996]. The chief merit of the cost structure proposed by these authors is that it allows us to parametrize the commitment value of capacities by the height of the upward jump in marginal cost. Firms commit to capacities in a first stage and then compete in price in the second stage. In the second stage, they may produce beyond installed capacity but have to incur for this an extra unit cost, denoted by θ. The subgame perfect equilibria of this game exhibit the following features. If the marginal cost θ of producing beyond capacity is larger than Cournot price, then Cournot equilibrium always obtain in the unique subgame perfect equilibrium. If θ is less than the cournot price, there is a continuum of subgame perfect equilibria but the price on the equilibrium path is always θ which implies that the aggregate quantity converges toward the Bertrand-competitive solution at the limit $\theta = 0$. In other words, capacity has its full commitment value whenever producing beyond capacity entails an additional unit cost at least equal to the corresponding Cournot price. The lower the value of the additional unit cost, the closer the price to the competitive benchmark.

2) BERTRAND-EDGEWORTH COMPETITION AND IMPERFECT COMMITMENT

The game tree we are considering is identical to that of KS. Some **n** ≥ 2 firms choose some costly capacities and then compete in price in the market for an homogeneous product. Rationing, if any, is organized according to the efficient rationing rule. Under the efficient rationing rule, low pricing firms get served first and ties are broken evenly. The firm exhibiting the highest price is left with a residual demand (if any) which is simply defined as a function of the other firms' aggregate capacity.

The cost structure in the pricing game is borrowed from Dixit [1980]. The marginal cost up to the capacity level is w.l.o.g. zero while it is some positive θ beyond (this jump typically measures the legal wage gap for overtime work). Thus, in our capacity-price game tree, firms invest in capacities x_i at cost $c(x_i)$ and then compete in prices for the demand function D(.) with the same cost structure $mc_i(q) = \begin{cases} 0 & \text{if } q \le x_i \\ \theta & \text{if } q > x_i \end{cases}$ for all $i \le n$.

Note that Dixit [1980] uses this set-up under quantity competition whereas Maggi [1996] retains it for analyzing price competition under product differentiation. Note also, that the original KS model corresponds to the particular case where θ is infinite; observe also that any value larger than the zero-demand market clearing price $D^{-1}(0)$ would trivially yield their result.

As a benchmark we consider the basic model of Cournot oligopolistic competition among **n** firms having the same convex cost function c(.). The aggregate consumer demand is D(p), its inverse P(x). Let $x_{-i} \equiv \sum_{j \neq i} x_j$ be the total quantity produced by firm **i**'s opponents. A nil production is clearly optimal if $x_{-i} > D(0)$, otherwise firm **i**'s profit is $x_i P(x_i + x_{-i}) - c(x_i)$. We assume throughout that xP(x+z) is concave in **x** for all **z**. Therefore, the profit maximizing quantity $r_c(x_{-i})$ is unique and decreasing[1] in x_{-i}. We denote $r(x_{-i})$ the best reply with zero production cost, it will play a central role in the study of price competition. The symmetric Cournot-Nash equilibrium is the solution $\bar{x} < D(0)/n$ of $x = r_c((n-1)x)$. For instance, with constant marginal cost **c** and linear demand P(z) = 1 – z, we get $r_c(z) = \frac{1-c-z}{2}$ and $\bar{x} = \frac{1-c}{n+1}$.

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[1] Formally $r_c(z)$ is the solution of $P(x+z) + x\dot{P}(x+z) - \dot{c}(x) = 0$. Let $f(x,z) \equiv xP(x+z) - c(x)$ and differentiate the equation to obtain $\frac{\partial^2 f}{\partial x^2}\frac{\partial r_c}{\partial z} = -\frac{\partial^2 f}{\partial x \partial z} \Rightarrow \frac{\partial r_c}{\partial z} = -\frac{\dot{P}(x+z) + x\ddot{P}(x+z)}{2\dot{P}(x+z) + x\ddot{P}(x+z) - \ddot{c}(x)} \in [-1;0]$.

We are now in a position to solve the two-stage game with imperfect capacity commitment and price competition. The particular shape of marginal costs affects the analysis of price subgames. Recall that the Edgeworth's argument consists in showing that upwards price deviation may be profitable when other firms are likely to ration consumers. This requires first that the demand addressed to them exceeds their aggregate capacity and second that they are not willing to meet demand beyond capacity. In the KS model, the second condition is always satisfied since the cost of producing beyond capacity is prohibitive whereas this is no longer the case in our model: a firm is willing to meet any level of demand, beyond its installed capacity provided the price is above θ. It is only for prices below that level that the Edgeworth's argument applies, otherwise the standard Bertrand analysis applies.

Figure 1 below helps to understand the nature of price competition. Consider the case of two fims. A firm will perform rationing if its price is less than θ, which makes it unprofitable to sell beyond capacity, and if the demand it faces is larger than its capacity, thus the relevant threshold for rationing is $\min\{\theta, 1-k_i\}$. In the region where the two firms name price above their respective treshold, a standard Bertrand competition applies.

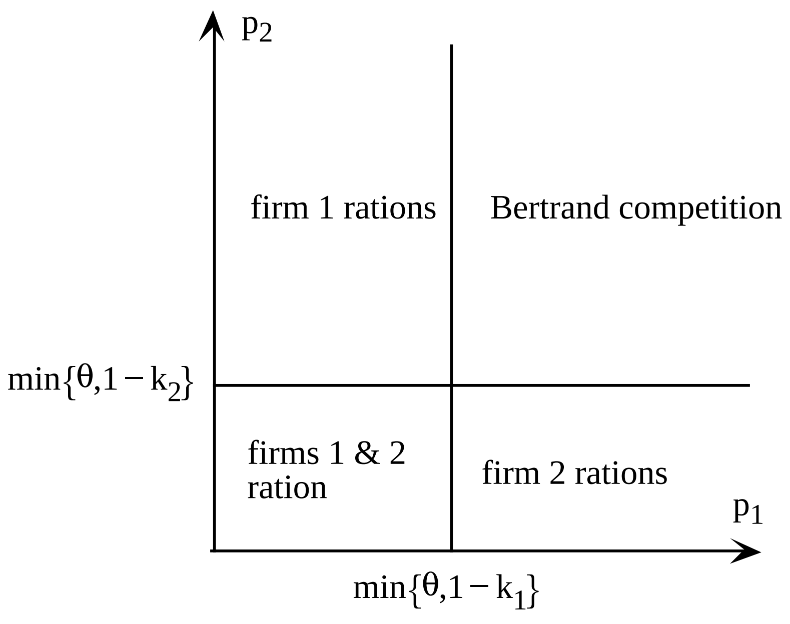

Figure 1

As appears from inspection of Figure 1, the analysis developed by KS for the pricing games under capacity constraints applies here in a truncated part of the strategy space. Lemma 1 characterizes equilibria in all possible price subgames.

Lemma 1

Whenever $\sum_i x_i \le D(\theta)$, $p \equiv \theta$ *is the unique pure strategy equilibrium. Whenever* $\sum_i x_i \ge D(\theta)$ *, if the largest capacity* x_1 *is less than* $r(x_{-1})$*, the equilibrium is the pure strategy* $P(x_1 + x_{-1})$ *, otherwise it is a mixed strategy equilibrium.*

Considering an oligopoly instead of a duopoly requires a different ordering of the arguments used by KS in their lemmas 2 to 5 but no real novelty is introduced. Still, the proof is somewhat technical and has been relegated to the appendix. We present here the line of reasoning.

We start by showing that either the equilibrium is the pure strategy θ or that there are more than two firms playing the lower bound $\underline{p}$ of all equilibrium distributions. Then we show that those firms are constrained at $\underline{p}$ so that their equilibrium payoff is $\Pi_i^* = \underline{p}\, x_i$.[2] Next, we show that if the sum of capacities is lesser than D(θ) then the equilibrium has to be θ because any firm is able to sell all of its capacity at this price. In the remaining cases the equilibrium is in mixed strategies and we can introduce $p^* \equiv P\left(\sum x_i\right) < \theta$. We then show that firms who play the upper bound $\overline{p}$ of all equilibrium distributions satisfy **i)** or **ii)**.

i) $r\left(x_{-i}\right) < x_i$ and $\overline{p} = P\left(r\left(x_{-i}\right) + x_{-i}\right)$: those firms have a large capacity and $\overline{p}$ does not depend on it but on what other firms did choose.

ii) $\overline{p} = p^*$ must hold in which case p^* is the equilibrium.

Lastly, using a complex formula borrowed from KS, we show that firms playing $\overline{p}$ in equilibrium have the same capacity and also the largest one.

Relying on Lemma 1, we may state our theorem which extends the result of KS to oligopoly and imperfect commitment. For **n** symmetric firms and efficient rationing, the Cournot outcome emerge as the subgame perfect equilibrium outcome of the capacity-pricing game as soon as the ex-post marginal cost θ is larger than the Cournot price.

Theorem 1

If $\theta > P(n\overline{x})$*, the symmetric Cournot-Nash investment* $\overline{x}$ *followed by* $P(n\overline{x})$ *is the unique subgame perfect equilibrium of Γ. If* $\theta < P(n\overline{x})$*, there is a continuum of SPE who nevertheless satisfy* $D(\theta) = \sum_{i \le n} x_i$*. It converges toward the Bertrand solution.*

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[2] In the KS setting, there are only two firms thus all firms derive this payoff which eases the rest of the proof.

Proof If $x_i \le D(\theta) - x_{-i}$, the equilibrium of the pricing game played after $(x_i)_{i\le n}$ is the pure strategy θ and firm **i**'s payoff in Γ is θx_i. If $D(\theta) - x_{-i} < x_i \le r(x_{-i})$, the equilibrium is the pure strategy $P(x_{-i} + x_i)$ and firm **i** is paid $f(x_{-i}, x_i) = x_i P(x_{-i} + x_i) - c(x_i)$. This function is concave with a maximum at $r_c(x_{-i}) < r(x_{-i})$. If $x_i > r(x_{-i})$, the equilibrium is in mixed strategies and firm **i** earns $g(x_{-i}, x_i) = R(x_{-i}) - c(x_i)$.

Notice that $P(x_{-i} + x_i) = \theta$ when $x_i = D(\theta) - x_{-i}$ and $g(x_{-i}, r(x_{-i})) = f(x_{-i}, r(x_{-i}))$. Hence the first period payoff as a function of x_i is continuous. Moreover at $x_i = D(\theta) - x_{-i}$, $\frac{\partial f}{\partial x_i} = \theta + x_i \dot{P}(x_{-i} + x_i) - \dot{c}(x_i) < \theta$ and the slope of **g** is steeper than that of **f** as the second period payoff becomes constant. The payoff function is thus concave in x_i for any x_{-i}; its average over the equilibrium distributions of the others firms is concave too, meaning that the best reply of firm **i** is always a pure strategy. Because this applies for all firms, the equilibrium is in pure strategies and satisfies $x_i = \max\{D(\theta) - x_{-i}, r_c(x_{-i})\}$ for all **i**.

If $D(\theta) - x_{-j} > r_c(x_{-j})$ for some x_j then $D(\theta) = x_{-j} + x_j$. From this we deduce that $D(\theta) = x_{-i} + x_i$ must hold for all other firms, so that $x_i = r_c(x_{-i}) > D(\theta) - x_{-i}$ is impossible. Therefore the candidate equilibria are all vectors $(x_i)_{i\le n}$ satisfying $D(\theta) = \sum_{i\le n} x_i$ and $D(\theta) > r_c(x_{-j}) + x_{-j}$ for all **j**. The symmetric equilibrium $D(\theta)/n$ exists if it is larger than the Cournot candidate $\bar{x}$ i.e., if $\theta \le P(n\bar{x})$ the Cournot price. Solving for $D(\theta) = r_c(y) + y$ yields a value y^* that circumvents the range of asymmetric equilibria; they are given by the constraint $\forall i \le n, x_i \ge y^*$ in addition to $D(\theta) = \sum_{i\le n} x_i$, thus this set is a simplex.

Now if $\theta > P(n\bar{x})$, the equilibrium is unique. The case for n = 2 is KS, thus consider n > 2 and let $\bar{m} \equiv x_{-1-2}$. If $x_1 = r_c(\bar{m}+x_2)$ and $x_2 = r_c(\bar{m}+x_1)$, then x_1 and x_2 are solutions of $z = h(z) \equiv r_c(\bar{m} + r_c(\bar{m} + z))$. But since $0 > \dot{r}_c(z) > -1$ (cf. footnote 5) it must be the case that $\dot{h}(z) = \dot{r}_c(m + r_c(\bar{m} + z))\dot{r}_c(\bar{m} + z) < 1$, thus **h** has a unique fixed point so that $x_1 = x_2$. By repetition of the argument to all pairs, we conclude that the equilibrium is unique and symmetric: it is the Cournot quantity $\bar{x}$.♦

Figure 2 below illustrates our findings for n = 2, a linear demand D(p) = 1 − p and zero marginal cost. The Cournot quantity 1/3 is found at the intersection of the two best reply functions (dashed lines on Figure 2). If the sum of quantities $x_1 + x_2$ is less than $D(\theta) = 1 - \theta$ then firms are not able to avoid the traditionnal Bertrand competition, it is only for large aggregate capacitites that the price equilibrium result in Cournot payoffs. Thus for $\theta >$ 1/3 (recall that the KS hypothesis was $\theta > 1$), very low capacity choices push firms toward the line $D(\theta) = x_1 + x_2$, then for larger capacities the Cournot competition applies and leads to the symmetric equilibrium choice of 1/3. For a θ' smaller than 1/3, the area where Bertrand competition applies incorporates the previous equilibrium meaning that firms are induce to build more capacity because the fierce price competition yield too small margins. There is now a continüm of equilibria where firms share the market but not too asymmetrically as the Cournot best replies provide lower bounds on one's equilibrium capacity.

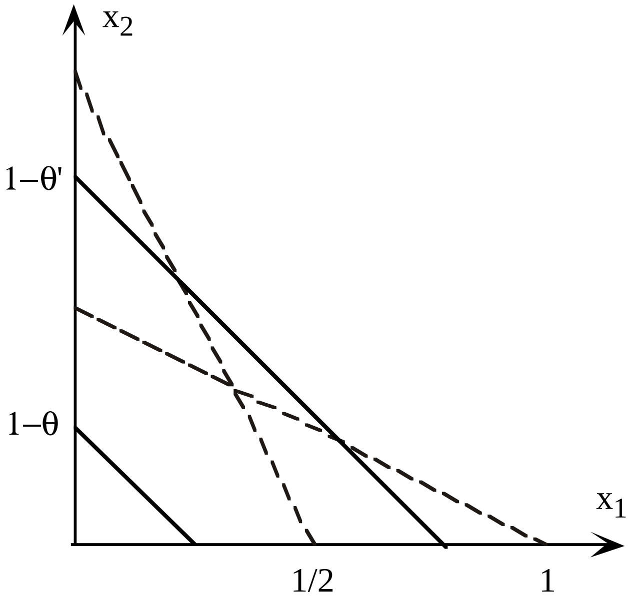

Figure 2

4) FINAL REMARKS

Many researches have aimed at reconciling Cournotian outcomes with the explicit price mechanism involved in the Bertrand model. These researches have been successful to the extent that they have been able to combine the two features of oligopolistic industries which are limited scales of production or increasing marginal costs and price setting behaviour. The main challenge in this respect consists in dealing with the issue of quantitative constraints (non-constant marginal cost) at the price competition stage which tends to make it unprofitable for the firms to meet full demand. This in turn generates rationing possibilities which are at the heart of the Edgeworth's critique.

The issue of rationing in pricing games is best understood by studying closely the allocation process of a non competitive market with price-setting firms. Three stages are needed to correctly describe this process. In the first stage firms name prices and consumers address demand to firms. In the second one, firms decide on their sales and possibly ration consumers. In the third stage, rationed consumers possibly report their demand to non-rationing firms who may or may not accept them. In the case of homogeneous products and perfect display of prices, the low pricing firm receives all the demand at the end of the first stage. Under constant returns to scale, this firm is willing to meet any demand level so that it always choose to serve all consumers in the second stage; the third stage is then irrelevant. With decreasing returns to scale, things are quite different because it may not be optimal to meet full demand in the first stage.

Curiously enough, Edgeworth's classical approach has been abridged in several recent papers. Their authors consider decreasing returns to scale; hence they have to deal with the fact that firms are not always willing to meet demand. Yet, it is generally asserted that "*firms meet demand"* without a word of explanation. There is neither reference to an external mechanism that forces firms to meet demand nor reference to some reputation effect in a larger game that would make this restriction tenable. Rationing is thus ruled out by assumption. Replaced in our three stages process, we can interpret the equilibria of these price competition games as Nash equilibria which are *not subgame perfect*. In those equilibria indeed, each firm names a price and threaten its opponents to take the non optimal decision to serve all of its clientele in the subsequent stage. Still the threat is never carried out on the equilibrium path. Kuhn [1994], Maggi [1996], Bulow, Geanakoplos & Klemperer [1985], Vives [1990], and Dastidar [1995], [1997] are prominent examples[3] where such an odd vision of price competition is endorsed. [4]

Yet what an economist probably has in mind when introducing quantitative constraints into pricing models is not that firms commit to incur losses on high levels of sales but rather commit not to call for such large sales, precisely because this would entail losses. Obviously, when a firm names a price **p**, it does not threaten the other firms to make losses by later selling units having a marginal cost larger than **p**. A firm has thus two basic options when setting its price: it either undercuts opponents to receive a large demand and possibly serves only a fraction of it or it charges a high price in order to benefit from rationing spillovers.

Allowing for rationing drives us back to the analysis initiated by Edgeworth [1925] and popularized by KS. In the setting of these last authors, the quantitative restriction may seem too effective since it is physically impossible to produce beyond capacities at the pricing stage. In the present paper we have thus considered the imperfect commitment of Dixit [1980] within an oligopolistic framework. Our framework thus exhibits as its two

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[3] The first two papers state the hypothesis implicitly, the next two use a footnote but without justification; only Dastidar makes an effort by referring to Dixon [1990] (see the preceding footnote).

[4]What is lost exactly of the competition process when rationing is forbidden is largely an open question. In a companion paper however we show that when rationing is forbidden within the KS framerwork, many SPE outcomes , including the collusive ones can be sustained.

polar cases the Bertrand and the KS's cost structure. We have then generalized of the findings of KS in this framework to show that, depending on the value of the commitment, the whole range of prices between Bertrand and Cournot prices can be sustained as part of subgame perfect equilibria.

Proof of Lemma 1

If the large capacity x_1 is less than $r(x_{-1})$, the equilibrium is the pure strategy $P(x_1 + x_{-1})$, otherwise it is a mixed strategy equilibrium.

Proof W.l.o.g. firm 1 has the largest capacity in the subgame following the play of $(x_i)_{i \le n}$. Let $\underline{p}_i$ and $\overline{p}_i$ be the lower and upper bounds of firm **i**'s equilibrium distribution F_i. Let $\underline{H} \equiv \arg\min_{i \le n} \underline{p}_i$, $\underline{p} \equiv \min_{i \le n} \underline{p}_i$, $\overline{H} \equiv \arg\max_{i \le n} \overline{p}_i$ and $\overline{p} \equiv \max_{i \le n} \overline{p}_i$.

Existence of an equilibrium is guaranteed by theorem 5 of Dasgupta & Maskin [1986]. By lowering its price a firm always benefits from an increase in demand (this property is not influenced by our rationing rule), its payoff is therefore left lower semi-continuous (l.s.c.) in its price, thus weakly l.s.c.. The sum of payoffs is u.s.c. because discontinuous shifts in demand occur only when two firms or more derive the same profit.

Claim 1 $\#\underline{H} > 1$ or the equilibrium is θ.

If $\#\underline{H} = 1$, some firm **k** enjoys demand $D(p_k)$ on $\left[\underline{p}; \min_{i \notin \#\underline{H}} \underline{p}_i\right]$.

i) $\underline{p} < \theta$. If firm **k** is constrained at $\underline{p}$ its revenue is the strictly increasing function $p_k x_k$, a contradiction to $\underline{p}$ being in the support of the equilibrium distribution F_k. Thus, $D(\underline{p}) < x_k$ and because $p_k D(p_k)$ is a non-constant function, it must be the case that $\underline{p}$ is the monopoly price $P(r(0))$. Since no other price (irrespective of what may play the other firms) can yield the monopoly payoff, firm **k** must be playing the pure strategy $P(r(0))$ but then any other firm **i** undercuts it, contradicting the optimality of its own equilibrium strategy.

ii) $\underline{p} \ge \theta$. We are contemplating the classical Bertrand price competition whose outcome is pricing at the marginal cost θ.

Claim 2 $\forall i \in \underline{H}, \Pi_i^* = \underline{p} x_i$

i) $\underline{p} < \theta$. If firm $i \in \underline{H}$ deviates to $\underline{p}^-$ (this is a shorthand for $\underline{p} - \varepsilon$ where ε is a small positive real number) its demand may jump upward; in order for $\underline{p}$ to be in the support of an equilibrium distribution it must be the case that this does not happen, thus the payoff is continuous at $\underline{p}$. If the demand at $\underline{p}^-$ is $D(\underline{p}^-)$ it must be the case that $\underline{p}^- < P(r(0))$ for otherwise firm **i** would deviate downward, thus $D(\underline{p}^-) > r(0)$ which is an upper bound on capacity investment[5] as it is the optimal quantity with zero cost for a monopoly. Therefore firm **i** is constrained at $\underline{p}$ and we get $\Pi_i^* = \underline{p}\,x_i$.

ii) $\underline{p} \geq \theta$. We saw that θ is the unique possible price equilibrium. The claim is even valid for all firms since sales beyond capacity neither generate losses nor benefits.

Claim 3 If $x_1 + x_{-1} \leq D(\theta)$, θ is the unique price equilibrium.

The only case we need to consider is $\underline{p} < \theta$. If firm **i** plays $\theta^- > \underline{p}$ then the other firms that are less expensive receives full demand but serve only their capacities so that firm **i** receives more than $D(\theta) - x_{-i} \geq x_i$ thus $\Pi_i\left(\theta^-, F_{-i}\right) = \theta^- x_i > \Pi_i^*$ a contradiction.

From now on we study the case where $p^* \equiv P\left(x_1 + x_{-1}\right) < \theta$.

Claim 4 $\overline{H} = \overline{H}^A \cup \overline{H}^B$ where $j \in \overline{H}^A$ if $r\left(x_{-j}\right) < x_j$ and $j \in \overline{H}^B$ if $\overline{p} = p^*$

Let $\Psi_j(p_j) \equiv p_j.\min\left\{x_j, \max\left\{0, D(p_j) - x_{-j}\right\}\right\}$ be the payoff to firm **j** when it names a price $p_j > \max_{i \notin \overline{H}}\{\overline{p}_i\}$. If $p_j \leq \max_{i \neq j}\{\overline{p}_i\}$ then firm **j** gets at least the payoff $\Psi_j(p_j)$, thus this function must be maximal at $\overline{p}$ to sustain this price as a member of the support of an equilibrium strategy. Firm **j** cannot be fully served at $\overline{p}^+$ for otherwise it would deviate upward thus it will only sell units with zero marginal cost. Two cases can occur. If $\Psi_j(p_j)$ = $p_j\left(D(p_j) - x_{-j}\right)$ in a neighbourhood of $\overline{p}$; we study the alternate formulation of profits $yP(y + x_{-j})$. The argmax is $r(x_{-j})$ so that $\overline{p} = P\left(r(x_{-j}) + x_{-j}\right)$ and since firm **j** is not constrained at $\overline{p}$, it must be true that $r(x_{-j}) < x_j$. Furthermore the equilibrium payoff in that case is $\Pi_j^* = R(x_{-j})$ where $R(x) \equiv r(x)P\left(r(x) + x\right)$. Using the envelope theorem we obtain $\dot{R}(x) = r(x)\dot{P}\left(r(x) + x\right) < 0$. If on the other hand, $\Psi_j(p_j) = x_j p_j$ at $\overline{p}^-$ then the upper price is $\overline{p} = P\left(x_j + x_{-j}\right)$ and we have $x_j \leq r(x_{-j})$.

Claim 5 If $\overline{H}^B \neq \varnothing$, the equilibrium is $p^* \equiv P\left(x_1 + x_{-1}\right)$

[5] In a Subgame Perfect Equilibrium we can eliminate strictly dominated strategies in the first stage.

Observe that p^* guarantees the revenue $p^* x_i$ to any firm **i**. Indeed, if all other firms are less expensive, they are served first but the residual demand addressed to firm **i** is precisely its capacity. If $\overline{H}^B \neq \varnothing$, the equilibrium must be the pure strategy p^* for all firms.

Claim 6 If $\overline{H}^B = \varnothing$ then $\overline{H} = \overline{H}^A = \{1\}$ i.e., the large capacity firm

Let $j \in \overline{H} = \overline{H}^A$. If $x_1 > x_j$ then $x_{-1} < x_{-j} \Rightarrow r(x_{-1}) + x_{-1} < r(x_{-j}) + x_{-j}$ as $\dot{r}(z) > -1$ (cf. footnote 5). Hence, firm 1 obtains a payoff $R\left(x_{-1}\right) \leq \Pi_1^*$ by naming $P\left(r(x_{-1}) + x_{-1}\right) > P\left(r(x_{-j}) + x_{-j}\right) = \overline{p}$. We now prove that $x_1 R\left(\overline{m} + x_1\right) < x_j R\left(\overline{m} + x_j\right)$ where $\overline{m} = x_{-1-j}$ (zero if n = 2). Let us define $\Theta(z) \equiv zR(\overline{m} + z) = zr(\overline{m} + z)P(\overline{m} + z + r(\overline{m} + z))$. The envelope theorem implies $\dot{\Theta}(z) = (r(\overline{m} + z) - z)P(\overline{m} + z + r(\overline{m} + z))$. If $r(\overline{m} + x_j) < x_j$ then $\dot{\Theta} < 0$ and we are done. Otherwise $r(\overline{m} + x_j) > x_j$ implies that $r(\overline{m} + x) = x$ is solved for a x^* greater than x_j since r(.) is decreasing. By the same token, $x^* > x_j$ implies that the solution y^* to $r(\overline{m} + x) = x_j$ has to be greater than x^*. Finally, $r(\overline{m} + y^*) = x_j < x_1$ implies $y^* < x_1$. This is crucial because the positiveness of $\dot{\Theta}$ on $[x_j;x^*]$ will be offset by its negativeness on the large interval $[x^*;x_1]$ as the following development shows.

$$\Theta(x_1) - \Theta(x_j) = \int_{x_j}^{x^*} \dot{\Theta} + \int_{x^*}^{x_1} \dot{\Theta} \leq \int_{x_j}^{x^*} \dot{\Theta} + \int_{x^*}^{y^*} \dot{\Theta} = \Theta(y^*) - \Theta(x_j)$$

$$= y^* R\left(r^{-1}(x_j)\right) - x_j R(\overline{m} + x_j) = y^* x_j P\left(x_j + r^{-1}(x_j)\right) - x_j R(\overline{m} + x_j)$$

$$= x_j \left(y^* P(x_j + \overline{m} + y^*) - R(\overline{m} + x_j)\right) < 0 \text{ by definition of } R(.).$$

So far we have proved that $\Pi_j^* = R(x_{-j}) < \frac{x_j}{x_1} R(x_{-1}) \leq \frac{x_j}{x_1} \Pi_1^* < \Pi_1^*$ which is equal to $\underline{p}_1 \left[F_{-1}(\underline{p}_1)\left(D(\underline{p}_1) - x_{-1}\right) + \left(1 - F_{-1}(\underline{p}_1)\right) D(\underline{p}_1)\right]$. If firm **j** plays $\underline{p}_1^-$ it obtains demand $F_{-1-j}(\underline{p}_1)\left(D(\underline{p}_1) - x_{-1}\right) + \left(1 - F_{-1-j}(\underline{p}_1)\right) D(\underline{p}_1)$ which is larger than the demand of firm 1 because there is more weight on the monopolistic demand term, therefore $\Pi_j\left(\underline{p}_1, F_{-j}\right) > \Pi_j^*$ the desired contradiction. We have thus shown that $1 \in \overline{H}^A$. Since $\overline{p} = P\left(r(x_{-j}) + x_{-j}\right)$ holds true for any $j \in \overline{H}^A$, members of $\overline{H}^A$ must have the same (largest) capacity.

The two cases that occur in claim 4 now make sense: if a firm names prices larger than other firms ($j \in \overline{H}^A$), it must be the one with the greatest capacity and furthermore,the excess must be large enough. ♦

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