Monopoly price discrimination and privacy: The hidden cost of hiding*

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Abstract

A monopolist can use a 'tracking' technology to identify a consumer's willingness to pay with some probability. Consumers can counteract tracking by acquiring a 'hiding' technology. We show that consumers may be collectively better off absent this hiding technology.
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1 Introduction

Recent developments in digital technologies (e-commerce, social media and networks, mobile computing, sensor technologies) have not only driven individuals to leave an increasingly long digital trace behind them, but have also made available the tools to assemble, harness and analyse large and complex datasets (so-called 'Big data'). As a consequence, firms are now able to target advertising, product offerings and prices to their customers with an unprecedented precision.

When it comes to prices, firms' enhanced ability to price discriminate implies a reduction in consumer surplus. Yet, the same technological developments have also enabled individuals to protect their privacy (e.g., by erasing their digital trace or by concealing their actions online). Although one would expect that such countermeasures would restore (at least part of) the lost consumer surplus, we show in this note that the opposite may actually happen. Adding insult to injury, the use of privacy-protecting technologies may decrease consumer surplus even further.

We establish this point in a monopoly setting where the firm has access to a 'tracking' technology that allows it to identify the willingness to pay of its consumers with some probability; the firm then charges personalized prices to the consumers it identifies and a common regular price to the consumers it does not identify. Consumers have the possibility to acquire a 'hiding' technology that makes the firm's tracking technology inoperative. Our main result is to show that consumer surplus is often larger when this hiding technology is *not* available. In fact, when the technology is available, the firm has an incentive to limit its use by raising the regular price of its product. As a result, what some consumers gain by protecting their privacy is often more than offset by what the other consumers lose by paying a higher price or by not purchasing any longer.

Compared to the existing literature on privacy (see Acquisti *et al.*, 2016, for a comprehensive and recent survey), the simple setting adopted in this note leaves aside a number of important features: price competition (as, e.g., in Taylor and Wagman, 2014, or in Montes *et al.*, 2015), repeat purchases (as, e.g., in Conitzer *et al.*, 2012), or data intermediaries (as, e.g., in Bergemann and Bonatti, 2015). However, this setting is novel in that it considers a tracking technology whose degree of precision can range between no and full identification of the consumers (in contrast with the existing literature that only considers the two extreme cases).[1]

2 The model

A monopolist produces some product at a constant marginal cost, which is set to zero for simplicity. A unit mass of consumers have a unit demand for the monopolist's product. A consumer's valuation for the product is noted r. The distribution of valuations is given by the cumulative distribution function $F(r)$ with support $[0, \bar{r}]$, where $\bar{r} \in (0, \infty]$, and by a continuous and differentiable density $f(r) \equiv F'(r) \geq 0$.

The monopolist can have access to a 'tracking technology' that allows it to identify the valuation of a consumer with probability λ (with $0 \leq \lambda \leq 1$).[2] The parameter λ can be interpreted as the precision of the tracking technology. In terms of pricing, this means that with probability λ, the monopolist knows the valuation of consumer r and charges this consumer a personalized price $p(r) = r$ (which captures the consumer's entire surplus), whereas with probability $(1 - \lambda)$, the monopolist does not know the consumer's valuation and charges then a 'regular' price p. Arbitrage is supposed to be impossible or prohibitively costly.

Consumers have access to some 'hiding technology' that allows them to prevent the monopolist from discovering their valuation. The technology is assumed to have the following simple form: by paying a cost c, any consumer can make sure that the monopolist cannot identify her valuation, whatever the precision of its tracking technology.

We analyse the following three-stage game. First, the monopolist decides whether or not to use the tracking technology. Second, the monopolist sets its prices (i.e., the regular price p and, possibly, a schedule of personalized prices $p(r)$), while consumers decide whether or not to acquire the hiding technology. Third, consumers observe the price that the monopolist charges them and decide whether or not to buy the product.[3] We solve the game

[1] An exception is Johnson (2013), who allows for gradations in information quality in his model of targeted advertising and advertising avoidance.

[2] Alternatively, we can assume that each valuation r is shared by a unit mass of consumers and that the technology allows the monopolist to identify a fraction λ of those consumers.

[3] This formulation implies, quite realistically, that (i) the firm is unable to observe a consumer's hiding decision before setting its prices, and (ii) consumers have to decide whether or not to hide before observing the price they are charged. We also considered

for its perfect Bayesian Nash equilibria.

We consider two benchmarks. First, *if the monopolist decides not to use the tracking technology* at the first stage of the game, then it charges the regular price to all consumers. Its problem is given by $\max_p p\,(1 - F\,(p))$. The FOC for profit-maximization allows us to determine implicitly the optimal price p_0 by solving $1 - F\,(p_0) - p_0 f\,(p_0) = 0$. We assume that the distribution of valuations satisfies the monotone hazard rate condition: $(1 - F\,(r))\,/f\,(r)$ is monotonically non-increasing for all r; this guarantees that the monopolist's objective function is quasi-concave and the SOC is satisfied. It follows that the monopolist sells a quantity $1 - F\,(p_0)$ at price p_0. The consumer surplus is then computed as

$$CS_0 = \int_{p_0}^{\bar{r}} (r - p_0)\, f\,(r)\, dr.$$

Second, *if no hiding technology were available*, the monopolist would charge p_0 to unidentified consumers and their valuation r to identified consumers. Hence the consumer surplus would be equal to:

$$CS_n\,(\lambda) = \lambda \times 0 + (1 - \lambda) \int_{p_0}^{\bar{r}} (r - p_0)\, f\,(r)\, dr = (1 - \lambda)\, CS_0. \qquad (1)$$

Unsurprisingly, when consumers have no way to hide their identity, the consumer surplus decreases when the precision of the tracking technology (i.e., λ) increases.[4]

3 Equilibrium

Suppose that the monopolist uses the tracking technology and that consumers can counteract tracking by acquiring some hiding technology at a

an *alternative timing* in which the monopolist first sets and commits to its regular price, after which consumers observe this price and decide whether to hide or not. Then the tracking technology is applied and the monopolist sets personalized prices to the identified consumers. Finally, consumers observe the price they are charged (either personalized of regular) and decide to buy or not. Here, the monopolist is able to influence directly the consumers' hiding decision by committing to the regular price. This alternative timing yields, nevertheless, qualitatively equivalent results: the monopolist charges a larger regular price when both tracking and hiding are possible; the monopolist is better off with the tracking technology and consumers may be collectively better off by not having access to a hiding technology.

[4] When $\lambda = 1$, the monopolist captures the entire consumer surplus, which corresponds to the case of perfect price discrimination.

constant cost c. At stage 2, consumers anticipate that they will pay a price p^e if they are not identified or a personalized price equal to their valuation if they are. Given this expectation, which determines the mass of consumers who decide to hide, the monopolist chooses its optimal price p. It is then imposed that the expectations be fulfilled at equilibrium.

Hence, any consumer r with $r \geq p^e$ will have a surplus of $(1-\lambda)(r-p^e)$ if she does not acquire the hiding technology and a surplus of $r - p^e - c$ if she does. It is thus worth acquiring the hiding technology if and only if $c \leq \lambda(r-p^e)$, i.e., if the cost of hiding one's valuation (c) is inferior to the benefit of hiding it (i.e., to keep the surplus $r-p^e$ when the tracking technology would discover one's valuation if it is not hidden).[5] The latter inequality can be rewritten as $r \geq p^e + c/\lambda$. Consumers with such valuations will hide and will thus pay, with certainty, the regular price p^e; consumers with a lower valuation will pay their valuation with probability λ or will pay p^e with probability $(1-\lambda)$ if their valuation is larger than p^e. The monopolist's profit can thus be expressed as[6]

$$\begin{aligned}\pi_h &= \lambda \int_0^{p^e+c/\lambda} rf(r)\,dr + (1-\lambda)\int_p^{p^e+c/\lambda} pf(r)\,dr + \int_{p^e+c/\lambda}^{\bar{r}} pf(r)\,dr \\ &= \int_p^{\bar{r}} pf(r)\,dr + \lambda\left(\int_0^p rf(r)\,dr + \int_p^{p^e+c/\lambda}(r-p)\,f(r)\,dr\right).\end{aligned}$$

We can now establish the following result (all proof are relegated to the appendix).

Lemma 1 *The monopolist charges a larger regular price, p_h, when he uses the tracking technology and consumers can hide their identity. Moreover, as long as some consumers hide, this price increases with the level of precision of the tracking technology: $\partial p_h/\partial\lambda > 0$.*

The intuition behind this result is clear. Two reasons push the monopolist to raise the regular price when consumers can hide: on the one hand, raising p discourages hiding (as only those consumers with $r \geq p + c/\lambda$ finds it profitable to hide); on the other hand, hiding consumers are high-valuation consumers and it is profitable for the monopolist to charge them

[5]Consumers with $r < p^e$ do not find it profitable to hide: if they do not hide, their surplus is zero, whereas if they hide, their surplus is $-c$ (as they do not buy the good).

[6]Imposing $p^e = p$; it is verified ex post that $p^e + c/\lambda \geq p$.

a higher price. In addition, a more precise tracking technology makes consumers want to hide more, leading the monopolist to increase the regular price even further so as to discourage hiding.

Moving now to the first stage of the game, we need to check whether using the tracking technology brings a larger profit than not using it. Writing $\pi_h^*(\lambda, c)$ for the optimal profit when the monopolist uses a tracking technology of precision λ and when the hiding technology is available at cost c, we decompose the difference with the profit the monopolist can achieve without tracking (π_0) as

$$\begin{aligned}\pi_h^*(\lambda, c) - \pi_0 &= \int_0^{p_0} \lambda r f(r)\, dr + \int_{p_0}^{p_h} (\lambda r - p_0) f(r)\, dr \\ &\quad + \int_{p_h}^{p_h + c/\lambda} (\lambda r + (1-\lambda) p_h - p_0) f(r)\, dr \\ &\quad + \int_{p_h + c/\lambda}^{\bar{r}} (p_h - p_0) f(r)\, dr.\end{aligned}$$

Among the four terms, only the second can be negative: consumers with valuations between p_0 and p_h buy in the absence of tracking but do not buy under tracking and hiding (as the monopolist sets a higher price) unless they are identified; tracking leads then to lower revenues for consumers with valuation $r < p_0/\lambda$ (possibly all consumers in the relevant range if $\lambda p_h < p_0$). For all other consumers, tracking brings larger revenues to the monopolist. Unless most of the mass of consumers would be concentrated between p_0 and p_h, we can be fairly confident that $\pi_h^*(\lambda, c) > \pi_0$, meaning that using the tracking technology is profitable.

4 How does hiding affect consumers?

We saw above that, absent any hiding technology, improved tracking (i.e., larger λ) reduces the consumer surplus: $CS_n(\lambda)$ is a decreasing function of λ. We now want to evaluate the extent to which the availability of the hiding technology challenges this result. A priori, we expect a move in the opposite direction: the possibility to hide one's valuation should increase the consumer surplus as it reduces the monopolist's ability to extract it. However, the result of Lemma 1 has to be factored in: the larger regular price that the monopolist sets for its product under tracking and hiding inevitably contributes to reduce consumer surplus.

To evaluate the balance between these two effects, we express the consumer surplus under hiding as[7]

$$CS_h(\lambda, c) = (1-\lambda)\int_{p_h}^{p_h+c/\lambda}(r-p_h)\, f(r)\, dr + \int_{p_h+c/\lambda}^{\bar{r}}(r-p_h-c)\, f(r)\, dr.$$

Using expression (1) and rearranging terms, we have that $CS_h(\lambda, c) \leq CS_n(\lambda)$ if and only if

$$\begin{aligned}(1-\lambda)\left(\textstyle\int_{p_0}^{\bar{r}}(r-p_0)\, f(r)\, dr - \int_{p_h}^{\bar{r}}(r-p_h)\, f(r)\, dr\right)\\ \geq \textstyle\int_{p_h+c/\lambda}^{\bar{r}}(\lambda(r-p_h)-c)\, f(r)\, dr.\end{aligned} \tag{2}$$

On the left-hand side, we have the lost surplus for unidentified consumers because of the price increase; on the right-hand side, we have the (potential) gain in surplus for hiding consumers (they gain the surplus $(r-p_h)$ with probability λ and pay the cost c with certainty). We now show that, for relatively precise tracking technologies, consumers are collectively better off when the hiding technology is not available.

Proposition 1 *Let $\bar{r} < \infty$ be the choking price of demand. Then, there exists $\bar{\lambda} < 1$ such that $CS_h(\lambda, c) < CS_n(\lambda)$ for all $\lambda \in (\bar{\lambda}, 1)$, meaning that consumers are collectively better off when the hiding technology is not available.*

The uniform distribution on the unit interval illustrates that for some log concave distributions, this inequality is *always* satisfied (i.e., for any $\lambda \in (0,1)$) even when the hiding technology is free (i.e., for $c = 0$). In this case we find: $p_0 = 1/2$ and $p_h = 1/(2-\lambda)$. With $c = 0$, we also have that Condition (2) boils down to

$$(1-\lambda)\int_{p_0}^{1}(r-p_0)\, f(r)\, dr > \int_{p_h}^{1}(r-p_h)\, f(r)\, dr \Leftrightarrow \tfrac{1-\lambda}{8} > \tfrac{(1-\lambda)^2}{2(2-\lambda)^2},$$

which holds for any $\lambda \in (0,1)$.

A corollary of Proposition 1 is that the consumers who hide their valuation may exert a negative externality on those who do not. In such a case, only an association representing all consumers would be able to internalize this externality. This association's best conduct would be to prevent

[7] If λ tends to one (i.e., if tracking becomes perfect), we can see that $CS_h(\lambda, c)$ tends to zero: the first term clearly vanishes, and so does the second (we show indeed in the appendix that p_h tends to $\bar{r}$ as λ tends to one).

individual consumers from acquiring the hiding technology, thereby securing a consumer surplus of $CS_n(\lambda) = (1-\lambda) CS_0$. The association could even improve consumer surplus further by acquiring itself μ units of the hiding technology (with $0 \leq \mu \leq 1$) and distributing them *randomly* to the consumers. This would reduce the precision of the tracking technology from λ to $\lambda(1-\mu)$, generating an increase in consumer surplus equal to $CS_n(\lambda(1-\mu)) - CS_n(\lambda) = \lambda\mu CS_0$. However, for such tactic to be profitable, the increase in surplus must be larger than μc, which is the total cost of the hiding technology for the association. This is so as long as $\lambda\mu CS_0 \geq \mu c$ or $c \leq \lambda CS_0$. As this condition is independent of μ, the association's optimum would then be to set $\mu = 1$, i.e., to distribute the hiding technology to each and every consumer, thereby annihilating the monopolist's price discrimination abilities. Yet, if $c > \lambda CS_0$, the consumer association cannot do better than preventing the individual use of the hiding technology.

5 Conclusion

In this note, we have shown that when a monopolist has some probability to identify the consumers' valuation and, thereby, charge them personalized prices, the possibility for consumers to hide their valuation may reduce consumer surplus (even when hiding can be done at no cost). The reason is that the monopolist raises the regular price that it charges to unidentified consumers, which harms consumers who choose not to hide their valuation. Hiding generates thus a hidden cost as consumers who hide exert a negative externality on consumers who do not. In future research, we aim at extending our analysis to a duopoly situation; in particular, we want to allow sellers to choose the precision of the tracking technology (parametrized by λ in our setting), a decision that existing studies (e.g., Montes *et al.*, 2015) are ill-equipped to analyze.

6 Appendix

6.1 Proof of Lemma 1

To prove that $p_h > p_0$, let us first determine p_h. Deriving the expected profit with respect to p yieds

$$\frac{d\pi_h}{dp} = 1 - F(p) - pf(p) + \lambda\left[F(p) + pf(p) - F(p^e + c/\lambda)\right], \quad (3)$$

while the second order condition, $-(1-\lambda)(2f(p) + pf'(p)) < 0$, is satisfied by log concavity. Setting $d\pi_h/dp = 0$ and imposing $p^e = p_h$, we have that p_h is implicitly defined by

$$p_h = \frac{1 - F(p_h) - \lambda\left[F(p_h + c/\lambda) - F(p_h)\right]}{(1-\lambda)f(p_h)}. \quad (4)$$

We now evaluate the derivative (3) at $p = p_0$. Imposing $p^e = p_h$ and recalling that $1 - F(p_0) - p_0 f(p_0) = 0$, we find

$$\left.\frac{d\pi_h}{dp}\right|_{p=p_0} = \lambda\left[1 - F(p_h + c/\lambda)\right].$$

The latter expression is strictly positive as long as $\lambda > 0$ and some consumers decide to hide ($F(p_h + c/\lambda) < 1$). It follows that $p_h > p_0$.

To show that the regular price increases with λ, we derive equation (4) with respect to λ:

$$\frac{\partial p_h}{\partial \lambda} = \frac{F(p_h) + p_h f(p_h) - F(p_h + c/\lambda) + \frac{c}{\lambda} f(p_h + c/\lambda)}{(1-\lambda)(p_h f'(p_h) + 2f(p_h)) + \lambda f(p_h + c/\lambda)}. \quad (5)$$

Using the first-order condition $d\pi_h/dp = 0$ and expression (3), we have that

$$F(p_h) + p_h f(p_h) - F(p_h + c/\lambda) = -(1/\lambda)\left[1 - F(p_h) - pf(p_h)\right].$$

Because $d\pi_h/dp|_{p=p_0} > 0$ and $p_h > p_0$, we have

$$\left.\frac{dpF(p)}{dp}\right|_{p=p_h} = 1 - F(p_h) - p_h f(p_h) < 0.$$

It follows that $F(p_h) + p_h f(p_h) - F(p_h + c/\lambda) > 0$ and as $\frac{c}{\lambda} f(p_h + c/\lambda) > 0$, the numerator of expression (5) is positive. From the second order condition, we have that $2f(p_h) + p_h f'(p_h) > 0$, which implies that the denominator is also positive. It follows that $\partial p_h/\partial\lambda > 0$, which completes the proof.

6.2 Proof of Proposition 1

Let $\triangle(\lambda, c) \equiv CS_n(\lambda) - CS_h(\lambda, c)$. First remark that $\triangle(1, c) = \triangle(0, c) = 0$. To see that $\triangle(1, c) = 0$, note that when $\lambda \to 1$, the FOC can only be fulfilled when $F(p + \frac{c}{\lambda}) \to 1$. But then $p \to \bar{r} < \infty$ and we obtain $CS_n(\lambda) = CS_h(\lambda, c) = 0$. To see that $\triangle(0, c) = 0$, note that in this case the firm has no tracking technology and (the cost of) hiding is irrelevant: the firm will always charge p_0 and consumers do not hide. We need to show that $\partial\triangle(\lambda, c)/\partial\lambda < 0$ when evaluated at $\lambda = 1$. We write

$$\begin{aligned}\triangle(\lambda, c) &= (1-\lambda)\int_{p_0}^{\bar{r}} (r - p_0)\, f(r)\, dr \\ &\quad - (1-\lambda)\int_{p_h}^{p_h + c/\lambda} (r - p_h)\, f(r)\, dr \\ &\quad - \int_{p_h + c/\lambda}^{\bar{r}} (r - p_h - c)\, f(r)\, dr,\end{aligned}$$

and we obtain

$$\begin{aligned}\frac{\partial\triangle(\lambda, c)}{\partial\lambda} &= -\int_{p_0}^{\bar{r}} (r - p_0)\, f(r)\, dr + \int_{p_h}^{p_h + c/\lambda} (r - p_h)\, f(r)\, dr \\ &\quad + \left[1 - (1-\lambda)\, F(p_h) - \lambda F(p_h + c/\lambda)\right] \frac{\partial p_h}{\partial\lambda}. \end{aligned} \tag{6}$$

We now show that equation (6) is negative at $\lambda = 1$. From expression (5), we notice that $\lim_{\lambda\to 1}(\partial p_h/\partial\lambda) = p_h|_{\lambda=1} + c = \bar{r} + c$, as long as $f(p)$ and $\partial f(.)/\partial p$ are finite. But then, the second and third terms of the right hand side of equation (6) are equal to zero at $\lambda = 1$, and we obtain

$$\left.\frac{\partial\triangle(\lambda, c)}{\partial\lambda}\right|_{\lambda=1} = -\int_{p_0}^{\bar{r}} (r - p_0)\, f(r)\, dr < 0,$$

which completes the proof.

References

Acquisti, A., Taylor, C., and Wagman, L. (2016). The Economics of Privacy. *Journal of Economic Literature*, 54(2):442–92.

Bergemann, D. and Bonatti, A. (2015). Selling Cookies. *American Economic Journal: Microeconomics*, 7(3):259–94.

Conitzer, V., Taylor, C. R., and Wagman, L. (2012). Hide and Seek: Costly Consumer Privacy in a Market with Repeat Purchases. *Marketing Science*, 31(2):277–292.

Johnson, J. P. (2013). Targeted advertising and advertising avoidance. *RAND Journal of Economics*, 44(1):128–144.

Montes, R., Sand-Zantman, W., and Valletti, T. (2015). The value of personal information in markets with endogenous privacy. IDEI Working Papers 851, Institut d'conomie Industrielle (IDEI), Toulouse.

Taylor, C. and Wagman, L. (2014). Consumer privacy in oligopolistic markets: Winners, losers, and welfare. *International Journal of Industrial Organization*, 34(C):80–84.