Net Energy Ratio, EROEI and the Macroeconomy

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Abstract

In an input-output model of a two-sector economy (energy and manufacturing), we analyse the macroeconomic implications of the quality of secondary energy production. We measure it by the Net Energy Ratio (NER), i.e. the fraction of produced energy available for net final production. NER is shown to be related to the EROEI concept encountered in energy science and to affect a) the energy intensiveness of final output, b) the capital requirements of the two sectors of the economy and the average productivity of capital, c) the rate of capital accumulation and the growth rate of the economy at given saving rate. As a consequence, an energy transition characterized by a decreasing NER would exert a drag on economic growth.

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1 Introduction

As for any being or system, the existence and development of our societies heavily rely on their ability "to gain substantially more energy than [they] use in obtaining that energy" (Hall *et alii*, 2009, p.25). If several sources of primary energy (i.e. coal, shale gas or solar energy) remain obviously abundant, the extent to which they can contribute to economic prosperity crucially depends on the ease with which man can transform these primary energy sources into a form of secondary energy useful to the economy. All primary energy sources do not offer the same quality in this respect. In order to assess the quality of an energy source, energy scientists have recently favoured[1] the concept of Energy Return On Energy Invested (EROEI in short). The EROEI of an energy production process[2] is the ratio of the quantity of energy it delivers to the quantity of energy used directly or indirectly by the process. As Cleveland (2008) notes, economies with access to higher EROEI fuel sources (i.e. to energy sources of higher quality) can allocate relatively more of their labour and man-made ressources (capital) to other activities than energy production; they so have greater potential for economic expansion and/or diversification.

To the eyes of many energy scientists and energy economists, a declining trend of the global EROEI of energy production seems hardly avoidable. On the one hand, non-renewable energy resources of high quality are progressively depleting and the exploitation of the residual resources is accompanied by a fall in their EROEI, either because their energy density is lower and/or because their processing gets -directly or indirectly- increasingly energy consuming. On the other hand, renewable energies might offer lower EROEI ratios than conventional fossil fuels (see e.g. Cleveland (2004), Murphy and Hall, (2010)). Several authors (e.g. Hall et alii (2009)) suspect that a declining EROEI would have negative implications on the prosperity prospects of our societies.

If widely acknowledged in energy economics, the importance of the quality of energy (or of its EROEI) is little studied in macroeconomics[3]. Bridging a gap between these two strands of literature, we analyse the macroeconomic implications of the quality of energy in an input-output model of a two-sector economy (energy and manufacturing). By describing the intra- and inter- sectoral dependancies, the input-output framework makes explicit that a part of the produced energy is absorbed in the intermediary consumption flows and that only a fraction of the produced energy is available for final production. We call this fraction the *Net Energy Ratio* (or NER is short). In an economy that has access to energy resources of high quality, energy production requires relatively little intermediary consumption of energy (either directly or indirectly) and NER is high. We show that NER is a key determinant a) of the energy

[1] See e.g. Hall *et alii* op citum, the papers in the special issue of Sustainability edited by Hall and Hansen (2011) or Yaritani and Matsushima (2014),...).

[2] Each time we use the terms "energy production" in this paper, we mean the transformation of a primary energy source into a useful form of secondary energy.

[3] An exception is Fagnart and Germain (2014).

intensiveness of final output and b) of the capital requirements of the economy. NER is also a driver of capital accumulation and of economic growth.

Section 2 presents the Input-Output framework and introduces the concept of NER. Section 3 shows that it is closely related to a concept of enlarged EROEI and affects the energy intensiveness of final production. Section 4 analyses the impact of NER on the capital requirements of the two sectors of the economy and on the average productivity of capital. Section 5 highlights its impact on capital accumulation and economic growth. Section 6 summarizes our results.

2 Input-Output Description of the Economy

We consider a continuous time Input-Output model of a closed-economy consisting of two production sectors: an energy sector (named sector e) and a sector manufacturing other goods and services (named sector y). Sector e transforms primary energy into secondary energy, i.e. a form of energy that can be used in production activities. Variable E denotes sector e output and is measured in units of energy (u.e. in short). Sector y produces intermediary and final products. Variable Q denotes its total output and is measured in units of goods and services (u.g. in short). Both sectors use outputs e and y as intermediary inputs and variable x_{ij} represents the quantity of output i ($i = e, y$) delivered to sector j ($j = e, y$) for intermediary consumption. Sector y also serves final demand and variable Y denotes the quantity of product y sold to final users. The two following equations describe the balance between resources and uses in each sector at time t[4]:

$$E = x_{ee} + x_{ey} \qquad (1)$$

$$Q = x_{ye} + x_{yy} + Y. \qquad (2)$$

Y consists of final (private and public) consumption C and gross investment I_j by sectors $j = e, y$: $Y = C + I_e + I_y$ is the gross domestic product.

Let $a_{ij}(\in [0, 1])$ be the technical coefficient associated to x_{ij}, i.e.,

$$a_{ie} =_{def} \frac{x_{ie}}{E}, \quad \text{for} \quad i = e, y \qquad (3)$$

$$a_{iy} =_{def} \frac{x_{iy}}{Q}, \quad \text{for} \quad i = e, y. \qquad (4)$$

Both productions require physical capital. Let K_j be the productive capital stock of sector j and b_j the technical coefficient of capital (or capital output

[4]Each variable in these equations is a function of time (e.g. E must be read as $E(t)$) but the functional argument (t) is omitted for notational convenience.

ratio) in this sector, i.e.

$$b_e =_{def} \frac{K_e}{E}, \tag{5}$$

$$b_y =_{def} \frac{K_y}{Q}. \tag{6}$$

Note that equations (3) to (6) are pure definitions which do not rely on any particular technological assumption.

Assuming that capital is sector specific but homogeneous at the sector level, the accumulation equation of sector j writes as:

$$\dot{K}_j = I_j - \delta_j K_j \tag{7}$$

where $\delta_j (\in]0,1])$ is the depreciation rate in sector j. The time derivative $\dot{K}_j$ is net investment in sector j, i.e. gross investment I_j net of depreciation δK_j.

Using (7), (2) may be rewritten as

$$\begin{aligned} Q &= x_{ye} + x_{yy} + C + \dot{K}_e + \delta_e K_e + \dot{K}_y + \delta_y K_y \\ &= [a_{ye} + \delta_e b_e]\, E + [a_{yy} + \delta_y b_y]\, Q + \underbrace{C + \dot{K}_e + \dot{K}_y}_{=_{def} Y_n}, \end{aligned} \tag{8}$$

where the last equality follows from the definitions of a_{ij} (eq. 3-4) and b_j (eq. 5-6). Y_n is net domestic product. The first (resp. second) term between brackets at the right-hand-side of (8) gives the total number of u.g. absorbed by the production process of 1 u.e. (resp. 1 u.g.), fixed capital consumption included. We name $\widetilde{a}_{ye}$ (resp. $\widetilde{a}_{yy}$) these adjusted technical coefficients:

$$\widetilde{a}_{ye} =_{def} a_{ye} + \delta_e b_e \tag{9}$$

$$\widetilde{a}_{yy} =_{def} a_{yy} + \delta_y b_y. \tag{10}$$

Using (8), (9) and (10), the matrix representation of the I-O system (1-2) writes as follows

$$\begin{bmatrix} E \\ Q \end{bmatrix} = \begin{bmatrix} a_{ee} & a_{ey} \\ \tilde{a}_{ye} & \tilde{a}_{yy} \end{bmatrix} \begin{bmatrix} E \\ Q \end{bmatrix} + \begin{bmatrix} 0 \\ Y_n \end{bmatrix}$$

or, equivalently,

$$\mathbf{A} \cdot \begin{bmatrix} E \\ Q \end{bmatrix} = \begin{bmatrix} 0 \\ Y_n \end{bmatrix} \qquad \text{with} \qquad \mathbf{A} = \begin{bmatrix} 1 - a_{ee} & -a_{ey} \\ -\tilde{a}_{ye} & 1 - \tilde{a}_{yy} \end{bmatrix}. \tag{11}$$

System (11) may be used to express the link between total output Q (resp. net final output Y_n) and the total quantity of energy its production requires (E), taking into account the intra- and inter-sectoral flows described by $\mathbf{A}$:

$$Q = \frac{1 - a_{ee}}{a_{ey}} E \tag{12}$$

and

$$Y_n = \frac{|\mathbf{A}|E}{a_{ey}}, \tag{13}$$

where $|\mathbf{A}| < 1$ is the determinant of $\mathbf{A}$, i.e. $|\mathbf{A}| = [1 - \tilde{a}_{yy}]\,[1 - a_{ee}] - a_{ey}\tilde{a}_{ye}$.

The term $|\mathbf{A}|E$ gives the *net energy* flow available for net final production: it is the part of the secondary energy flow E that is not consumed by the intra- and inter-industry intermediary flows and by fixed capital consumption. Determinant $|\mathbf{A}|$ corresponds to the fraction of the energy flow E available for net final production; it is named *Net Energy Ratio* (or NER) in the sequel. Equation (13) gives the final output level that can be produced from the net energy flow $|\mathbf{A}|E$, given the energy requirement a_{ey} of a unit of final output. We come back more extensively on this relationship in subsection 1.2.3.

Note that the concept of net energy used is not the same as in energy science where it is defined as the difference between the quantity of energy delivered by an energy system and the one used in the delivery process (see e.g. Cleveland (2004), Bardi *et alii* (2011)). This definition disregards the use that is made of the delivered energy. In our macroeconomic approach, net energy refers to the fraction of the delivered energy that is available for final production. NER is a ratio (i.e. a unitless number) contrary to net energy which is a difference between two energy quantities and thus expressed in units of energy. As we show hereafter, NER is however closely related to the concept of Energy Returned On Energy Invested (EROEI).

3 Net Energy Ratio and EROEI

EROEI is the ratio of the quantity of energy delivered by an energy production process to the quantity of energy consumed directly or indirectly by the process. Its value means that allocating 1 u.e. to this energy production process makes [EROEI$-$1] u.e. available for other uses in the economy. In the present model, EROEI is the ratio of the quantity of energy delivered by sector e to the quantity of energy used by e directly (the energy required to the working of K_e) or indirectly (the energy required to produce either the intermediary goods consumed by sector e or the capital goods it uses).

3.1 EROEI at the Level of the Energy Sector

Producing 1 u.e. requires 1) a direct consumption of a_{ee} u.e. and 2) an indirect energy consumption equal to the energy required to produce the quantity of good y used as input in sector e: fixed capital consumption included, producing 1 u.e. requires $\widetilde{a}_{ye}$ u.g. As each u.g. requires a_{ey} u.e., $\widetilde{a}_{ye}$ u.g. have an energy content of $a_{ey}\widetilde{a}_{ye}$ u.e. In total, the production of 1 u.e. requires $a_{ee} + a_{ey}\widetilde{a}_{ye}$

u.e. The EROEI at the energy sector level is the ratio between the two, i.e.:

$$\varepsilon_s = \frac{1}{a_{ee} + a_{ey}\widetilde{a}_{ye}}. \tag{14}$$

3.2 Enlarged EROEI

The expression of ε_s understates the indirect energy content of secondary energy production: it disregards that the delivery of $\tilde{a}_{ye}$ u.g. to sector e has required sector y to produce $\widetilde{a}_{ye}/[1-\tilde{a}_{yy}]$ u.g. Accordingly, the indirect energy content of the production of 1 u.e. is $a_{ey}\widetilde{a}_{ye}/[1-\tilde{a}_{yy}] > a_{ey}\widetilde{a}_{ye}$ u.e. The corresponding EROEI thus writes as:

$$\varepsilon_m = \frac{1}{a_{ee} + a_{ey}\dfrac{\widetilde{a}_{ye}}{1-\widetilde{a}_{yy}}} < \varepsilon_s \tag{15}$$

Note that the correction introduced in (15) does not only reduce the EROEI value measured at a macroeconomic level. In several energy sources were distinguished, it would also imply that the energy source that offers the best ε_s measured at the energy subsector level is not necessarily the one that offers the best enlarged eroei ε_m. See Appendix for a more detailed discussion.

3.3 Net Energy Ratio, Enlarged EROEI and Feasibility

Using (15), $|\mathbf{A}|$ may be rewritten as the products of two terms:

$$|\mathbf{A}| = \left[1 - \frac{1}{\varepsilon_m}\right][1-\widetilde{a}_{yy}]. \tag{16}$$

This decomposition outlines two sources of energy leakages between secondary energy production and its use in final production. First, leakages appear in energy production itself since it consumes energy directly (via a_{ee}) or indirectly (via the energy content of $\widetilde{a}_{ye}$): these leakages are measured by the inverse of the enlarged EROEI ε_m. A second source of leakages (captured by coefficient $\widetilde{a}_{yy}$) appears in net final production as it requires a self-consumption of product y. Note that at given $\tilde{a}_{yy}$, $|\mathbf{A}|$ always evolves in the same direction as ε_m: at given $\tilde{a}_{yy}$, statements like "energy production offers a higher NER" or "energy production offers a higher EROEI" are thus equivalent.

Equations (13) and (16) lead straightforwardly to the following proposition which establishes that the energy intensiveness of final production has three determinants, i.e. the energy efficiency in final production $1/a_{ey}$ and the two energy leakages outlined hereabove:

Proposition 1 *Net final output as a function of net energy*

1. *The quantity of secondary energy necessary to produce a given level of net final output is smaller (resp. bigger) in an economy where NER (or the enlarged EROEI) is higher (resp. lower). Equivalently, the energy intensiveness of net final production, E/Y^n, is decreasing in EROEI:*

$$\frac{E}{Y^n} = \frac{a_{ey}}{1-\widetilde{a}_{yy}}\left[1-\frac{1}{\varepsilon_m}\right]^{-1}. \tag{17}$$

2. *An enlarged EROEI strictly larger than 1 is a necessary condition to a strictly positive level of net final output: $Y_n > 0$ requires that $\varepsilon_m > 1$.*

Given (11), the feasibility condition $Y_n > 0$ writes as $|\mathbf{A}| > 0$, i.e. $a_{ey}\tilde{a}_{ye} < [1-\tilde{a}_{yy}]\,[1-a_{ee}]$. The decomposition (16) shows that $|\mathbf{A}| > 0$ is equivalent to $\varepsilon_m > 1$, which also means that the EROEI measured at the level of the energy sector, ε_s, must be sufficiently above one. Indeed, one can rewrite $|\mathbf{A}| = 1-\widetilde{a}_{yy}-a_{ee}+\widetilde{a}_{yy}a_{ee}-a_{ey}\widetilde{a}_{ye}$, i.e. $|\mathbf{A}| = 1-\widetilde{a}_{yy}-{}^{1}\!/\!_{\varepsilon_s}+a_{ee}\widetilde{a}_{yy}$. Hence, $|\mathbf{A}| > 0$ is equivalent to

$$\varepsilon_s > \frac{1}{1-\widetilde{a}_{yy}\,[1-a_{ee}]} > \varepsilon_m > 1. \tag{18}$$

4 NER & Capital Requirement of the Economy

NER affects the capital requirement of each sector and thereby the capital intensiveness of final production:

Proposition 2 *In an economy where NER is lower, the production of a given level of net final ouput requires more capital in* <u>*both*</u> *sectors e and y:*

$$K_e = \frac{a_{ey}}{|\mathbf{A}|}b_e Y^n \tag{19}$$

$$K_y = \frac{1-a_{ee}}{|\mathbf{A}|}b_y Y^n. \tag{20}$$

Proof. (19) follows from (13) and (5); (20) follows from (13), (12) and (6). ■

Proposition 2 can be understood intuitively as follows. When $|\mathbf{A}|$ is lower, more secondary energy E is necessary to produce a given level of final output (cfr. Proposition 1). In order to produce more E, sector e needs 1) more capital K_e and 2) more intermediary consumption of good y. As a result, sector y must itself produce more units of Q for each unit of net final good: this means that sector y also needs more capital K_y. Consequently, a lower NER increases the capital requirements of *the two sectors.*

Equations (19) and (20) imply the following relationship between the total capital stock and net final production:

$$K =_{def} K_e + K_y = \left[\frac{a_{ey}}{|\mathbf{A}|}b_e + \frac{1-a_{ee}}{|\mathbf{A}|}b_y\right]Y_n, \tag{21}$$

or, equivalently,

$$Y_n = \Phi K \qquad \text{with} \qquad \Phi = \frac{|\mathbf{A}|}{a_{ey}b_e + [1-a_{ee}]b_y}. \tag{22}$$

Φ is thus the average/aggregate productivity of capital. It is important to note that all the determinants of NER affect Φ and NER in the same direction. As a consequence:

Proposition 3 *The average productivity of capital at the aggregate level is increasing in NER.*

In more details, Φ and NER are both higher when energy efficiency in sector y is better, i.e. when a_{ey} is lower; they are lower when sectors e and y are more capitalistic and make a more intensive intermediary use of their own or each other production (i.e. when b_e, b_y, a_{ye}, a_{yy} and a_{ee} are bigger[5]).

5 Net Energy, Capital Accumulation and Growth

In a dynamic perspective, (19-20) (and thus (22)) imply that economic growth will require more or less investment according to the NER of the economy. In this section, we analyse how the NER affects economic growth and the allocation of final output (between final consumption and investment) required to sustain a given growth level.

Let s be the saving rate measured with respect to net final output Y_n

$$s =_{def} \frac{I}{Y_n} = \frac{I_e + I_y}{Y_n} \tag{23}$$

Using (7) and assuming -for analytical simplicity- the same depreciation rate δ in the two sectors, the time derivative of $K = K_e + K_y$ may be written as

$$\begin{aligned}\dot{K} = \sum_{j=e,y} I_j - \delta \sum_{j=e,y} K_j &= \frac{\sum_j I_j}{Y_n} Y_n - \delta K \\ &= sY_n - \delta K. \end{aligned} \tag{24}$$

[5]Note that a_{ee} lowers NER but also the denominator of Φ. It is however easy to show that the effect of a_{ee} on NER is dominant and that Φ is decreasing in a_{ee}.

The growth rate of K writes as

$$\frac{\dot{K}}{K} = \Phi s - \delta, \tag{25}$$

which leads to the following proposition:

Proposition 4 *NER, saving and capital accumulation.*

> *At given saving rate, capital accumulation is faster in an economy where NER (or EROEI) is higher.*
>
> *More generally said, the saving rate required to reach a given growth rate of capital accumulation is inversely related to NER.*

The intuition underlying Proposition 4 is as follows: an economy endowed with a given stock of capital produces more units of net final output when NER and thus Φ are higher; at given saving rate, net investment and thereby the growth rate of capital are thus higher; alternatively, the saving rate necessary to reach a given rate of capital accumulation is decreasing in NER.

From now on, we assume that energy and capital are complementary production factors. More specifically we assume that sectors e and y use Leontieff technologies with constant returns-to-scale: the technical coefficients a_{ij} $(i = e, y)$ and b_j are now technological variables independant of the activity level of sector j. Under such an assumption, the output of each sector is a linear function of its capital stock and the aggregate production function of our two-sector economy $Y_n = \Phi K$ looks like an AK technology. With respect to the canonical AK model of the endogenous growth literature, one of the specificy of our representation is to make the productivity of capital endogenous: it depends in particular on NER, as outlined in section 4.

Given (25) , one can decompose the growth rate of net final output $Y_n = \Phi K$ as:

$$\frac{\dot{Y}_n}{Y_n} = [\Phi s - \delta] + \frac{\dot{\Phi}}{\Phi}. \tag{26}$$

At given saving rate, an increasing $|\mathbf{A}|$ (and thus Φ) has two impacts on the growth trajectory of an economy: an instantaneous productivity effect captured by the last term in (26) and a dynamic effect captured by the term between brackets in (26). This second effect follows from the fact that a positive $\dot{\Phi}$ raises the level of Φ through time and thus enhances capital accumulation at given saving rate. With a more positive $|\dot{\mathbf{A}}|$ (or $\dot{\Phi}$), the economy grows faster not only because of stronger "productivity gains" but also because capital accumulation proceeds on a steeper path. Said the other way around, these two effects reduce the saving rate that the economy needs to achieve a given final output growth rate[6].

[6]Note that if Φ was constant through time ($\dot{\Phi} = 0$), capital accumulation would be the only growth engine of final output. What has been written about capital accumulation in Proposition 3 would thus also hold for economic growth.

Consider now an energy transition characterized by a more costly access to non renewable resources and/or by an increased use of renewable energy sources. If this transition means that energy production consumes directly or indirectly the output of sector y more intensively, it will be characterized by a lower $\dot{\Phi}$ and the argument of the previous paragraph implies the following proposition.

Proposition 5 *Energy transition and economic growth*

1. *An energy transition characterized by a lower $\dot{\Phi}$ exerts a double drag on the growth trajectory of the economy: average productivity of capital grows more slowly; capital accumulation proceeds at a lower speed at unchanged saving rate.*
2. *A higher saving rate is necessary to mitigate the growth impact of such an energy transition.*

As a comment on Point 1, let us stress that the growth impact of the transition could be particularly severe if (part of) the transition was characterized by a negative $\dot{\Phi}$. A technical progress which improves energy efficiency is indeed not enough to imply that Φ grows through time: it might decrease if NER was sufficiently decreasing along the transition, which might occur either because of a sufficiently more costly access to non-renewable resources and/or because of an increased use of renewable energy sources offering a significantly lower NER than fossil fuels.

Point 2 stresses that in order to cope with the consequences of an energy transition, an economy could need a deep structural change in the allocation of final output between consumption and investment and, possibly, in the allocation of capital between energy and final production.

6 Conclusion

In an input-output model of a two-sector economy (energy and manufacturing), we have introduced the concept of Net Energy Ratio (NER), i.e. the fraction of secondary energy which remains available for net final production, given the energy absorbed in the intermediary consumption flows. NER is the determinant of the input-ouput matrix obtained when capital depreciation is considered as an intermediary consumption flow. NER has been shown to be related to the concept of EROEI encountered in energy science and to affect directly several macroeconomic variables. 1) A lower NER increases the energy intensiveness of final output by increasing the quantity of secondary energy necessary to the production of a given level of (net) final output. 2) A lower NER increases the capital/output ratio of the economy (or decreases the average productivity of capital) by raising the capital requirements of both the energy and manufacturing sectors. 3) A lower NER slows down capital accumulation and economic growth at given saving rate. As a consequence, an energy transition characterized by a decreasing NER exerts a drag on economic growth by slowing down -at given saving rate- both global productivity gains and capital accumulation. The mitigation of these effects would require a sufficient increase in the saving rate of the economy.

In the light of an input-output approach, the quality of the energy mix at the disposal of an economy thus appears to have key macroeconomic implications. Our formal results complement the arguments of those who, like Ayres *et alii* (2013), advocate for an explicit modelling of energy in macroeconomics, in particular in growth theory.

7 Bibliography

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Appendix: Comparison of EROEIs of Various Energy Sources

Let us consider n energy sources $i = 1, ..., n$ that are perfect substitutes in the final production process. Each energy subsector consumes part of its own output but none of the other energy subsectors. In terms of technical coefficients, the I-O table of the economy is as follows when $n = 2$ (ID stands for Intermediate Deliveries):

sector	total output	ID to e_1	ID to e_2	ID to y	Net final demand
e_1	E_1	a_{11}	0	a_{1y}	0
e_2	E_2	0	a_{22}	a_{2y}	0
y	Q	$\tilde{a}_{y1}$	$\tilde{a}_{y2}$	$\tilde{a}_{yy}$	Y^n

The balance between ressources and uses in each energy subsector writes as

$$E_i = a_{ii}E_i + a_{iy}Q, \quad \text{for} \quad i = 1, ..., n.$$

Summing these equations gives

$$\begin{aligned} E = \sum_i E_i &= \sum_i a_{ii}E_i + \left[\sum_i a_{iy}\right] Q \\ &= a_{ee}E + a_{ey}Q, \quad \text{with } a_{ee} = \sum_i a_{ii}\frac{E_i}{E} \text{ and } a_{ey} = \sum_i a_{iy}. \end{aligned}$$

If the n sources are perfect substitutes in the production of good y, one has indeed that $Q = [1 - a_{ee}]E/a_{ey}$, where $[1 - a_{ee}]E$ is the total quantity of energy delivered to sector y by the n energy subsectors.

The EROEI measures for energy subsector i are

$$\begin{aligned} \varepsilon_s^i &= \frac{1}{a_{ii} + a_{ey}\tilde{a}_{yi}} \\ \varepsilon_m^i &= \frac{1}{a_{ii} + a_{ey}\frac{\tilde{a}_{yi}}{1-\tilde{a}_{yy}}}. \end{aligned}$$

It is worth outlining that the ranking between the eroeis ε_s^i measured at the energy subsector level is not necessarily the same as the one between the enlarged ereois ε_m^i. For the example, let us say that $n = 2$ and that resource 1 offers a higher eroei than resource 2 at the energy producer level:

$$\varepsilon_s^1 > \varepsilon_s^2 \qquad \Leftrightarrow \qquad a_{11} + a_{ey}\tilde{a}_{y1} < a_{22} + a_{ey}\tilde{a}_{y2}.$$

If the second equality holds but if resource 1 does not dominate resource 2 in all dimensions (i.e. if $a_{11} < a_{22}$ but $a_{y1} > a_{y2}$ or if $a_{11} > a_{22}$ but $a_{y1} < a_{y2}$), nothing guarantees that $\varepsilon_m^1 > \varepsilon_m^2$ once $\tilde{a}_{yy} \neq 0$: in such a case indeed, the larger $\tilde{a}_{yy}$, the more likely the ranking inversion.