Can the Energy Transition Be Smooth?

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Abstract

We analyse the transition of a decentralized economy whose energy supply switches progressively from non-renewable (NRE) to renewable energy (RE) sources. The two energies are perfect substitutes but RE production offers a lower Energy Return On Energy Invested (EROEI). The transition is characterized by a decreasing trend of the aggregate EROEI and by major changes both in the allocation of output between consumption and investment and in the allocation of capital between energy and final good productions. As a result, the energy transition may (and will usually) be characterized by a non-monotonic evolution of aggregate income and private consumption: after a peak and before the NRE exhaustion, the economy experiences a contraction. We analyze what affects 1) its magnitude and 2) the possibility of an ulterior recovery of income. Incidentally, a complementarity appears between a rapid development of RE production and the availability of NRE: the end of the NRE era puts a drag on the development of the RE production.

Keywords: energy transition, renewable energy, non-renewable energy, EROEI, growth.

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1 Introduction

Even if a transition from non-renewable to renewable energy sources seems inescapable in the long run, the primary energy supply to the World economy remains largely dominated by non-renewable energy sources and three fossil fuels (coal, oil, gas) still account for more than 80% of the total energy supply. According to the World Energy Outlook (2013) of the International Energy Agency (IEA), the share of renewables in worldwide primary energy use was only of 13% in 2011; it is expected to rise to 18% by 2035. Such a projection does not mean that renewable energy supply will not grow rapidly in absolute terms: it simply reflects that the IEA expects World economic growth to remain intensively dependant on non-renewable energy sources over the (at least two) decades to come. One may however question the possibility of maintaining much longer a growth process fed by an intensive use of non-renewables. Some authors like Meadows *et alii* (2004) even doubt that the transition towards renewable energy will allow our rich societies to maintain the level of material welfare (i.e. the income levels) reached during an era of (relatively) cheap access to non-renewable energy sources. This paper comes back on this issue.

A detour by the energy science literature highlights why energy transition may challenge economic prosperity. In order to survive or grow, "any being or system" needs energy and "must gain substantially more energy than it uses in obtaining that energy" (Hall *et alii*, 2009). The energy surplus generated by an energy production process can be measured by its EROEI ratio (EROEI for Energy Returned On Energy Invested): it is the ratio of the quantity of energy delivered by this process to the quantity of energy used by the process (see a.o. Cleveland (2008), Hall *et alii*, op citum). This use of energy can be direct (e.g. the coal burned in a boiler) or indirect (e.g. the energy necessary for building the boiler). High quality resources have a high EROEI ratio: relatively to their energy density, they require little energy to be discovered, extracted, processed and delivered to the point of use. If the EROEI concept can be used at a very disaggregated level to describe the efficiency of a given process (e.g. an oil refinery), it is also meaningful at a more global level to describe the efficiency of the energy supply in a society as a whole.

The development of contemporary industrial societies over the last two hundred years has been heavily dependent on fossil fuels that offered a very high EROEI. This large energy surplus allowed our economies to allocate most of their labour and man-made ressources (capital) to other activities than energy production. High EROEI resources have thereby contributed to economic growth: as e.g. Cleveland (2008) notes, "because the production of goods and services is a work process [in the physical sense], economies with access to higher EROEI fuel sources have greater potential for economic expansion and/or diversification." However, a declining trend of the global EROEI of our societies seems unavoidable. On the one hand, high quality ressources are progressively depleting and the exploitation of the residual resources is accompanied by a fall in their EROEI, either because their energy density is lower and/or because their processing gets -directly or indirectly- increasingly energy consuming. On the other hand, renewable energies seem characterized by much lower EROEI ratios than non renewable energies (see e.g. Cleveland (2004), Murphy and Hall, (2010)). Several authors suspect that this "declining EROEI will take a huge economic toll in the future" (Hall *et alii*, 2009, p.34). van den Bergh (2013, p.15) describes why in the following words: if renewable energy sources offer a lower EROEI, "an economy running entirely on renewable energy would then devote a disproportionally large share of activity and labor to provide intermediate services to the renewable energy sector: energy delivery, extraction of material resources, production of high-quality materials and equipment (solar PV panels, wind turbines), transport of materials and equipment, maintenance, and education of experts. Innovations in renewable energy technologies (whether wind, solar or biofuels) tend to increase the complexity and roundabout character of the supportive system, causing significant progress in associated EROEI values to be uncertain." One understands that if the energy transition forces an economy to devote much more inputs to energy production, it might experience a scenario à la Meadows (2004) where investments in the energy sector would crowd investments out of the final good sector and would so produce a severe decrease or even collapse of final production and consumption. In such a scenario, generations born at the dawn of -or during- this (temporary) collapse would thus suffer a severe loss of material well-being and, once the transition would be achieved, the forthcoming generations would not be guaranteed to enjoy a level of material well-being as high as their luckiest ancestors.

Papers dealing with the EROEI measure (like a.o. Hall et al. (op citum), King and Hall (2011), Murphy et Hall (2010), Heun et de Wit (2012)) do not develop agregate models able to generate and analyze such a scenario. The GEMBA model developed by Dale et al. (2012) is an exception which incorporates explicitly the EROEI measure into an aggregate model. However, it does not describe explicitly the economic agents' behaviours and is not closed in the sense that a component of aggregate demand remains exogenous. The economic growth literature, for its part, has totally ignored the EROEI concept until now. We propose here a first macroeconomic growth model which integrates the EROEI ratio and allows us to analyze the economic consequences of a progressively more intensive use of an energy source with a lower EROEI value. Thereby, our model contributes to bridging a gap between the macroeconomic growth- and energy science literatures, two literatures that are used to progress quite independantly. In accordance with the intuitions developped in the quotation of van den Bergh hereabove, we show that the transition towards a 100%-renewable energy era is characterized by a downward trend in the global EROEI ratio and deep changes both in the allocation of output between consumption and investement and in the allocation of capital between energy and final production sectors.

If they ignore the EROEI concept, several recent contributions to economic growth theory have nevertheless analyzed the issue of the energy transition. Tahvonen and Salo (2001) have proposed an optimal growth model able to explain the progressive rise and subsequent fall in the use of the non-renewable energy sources[1]: in a first phase of its development (a kind of preindustrial era where the capital stock is low), the economy only uses renewables; in a second phase where the capital stock is high enough and the demand for energy higher, the extraction of non-renewables becomes economically possible and the share of non-renewables in energy production increases progressively, before decreasing again as the marginal cost of extraction rises with the decline of the resource stock; in a last phase, the non-renewables extraction has become too costly and only renewables are used. Contrary to ours, this paper does not analyze the determinants of a possible fall in final production or consumption after a period of growth (albeit some figures of their articles show that such a negative adjustment may happen in the absence of technical progress). Nor is this issue dealt with in Tsur and Zemel (2005) or in Amigues *et alii* (2011). Tsur and Zemel characterize the dynamics of an optimal growth model with R&D investments which reduce the cost of use of backstop technologies. Amigues *et alii* study the optimal use of a polluting non renewable energy and a clean renewable one in the presence of a ceiling on the athmospheric carbon concentration. The possibility of a downward adjustment in output and consumption during the energy transition is analyzed in Growiec and Schumacher (2008) and Jouvet and Schumacher (2012). Growiec and Schumacher analyze the consequences of an imperfect substituability between non-renewables and renewables in a growth model without man-made capital. In Jouvet and Schumacher, non-renewables and renewables are perfect substitutes but are never used simultaneously and the energetic transition takes the extreme form of a one-period switch from a 100% non-renewable use to a 100% renewable use in energy production. If a downward adjustment of output takes place in their model, it occurs once the non-renewable ressource is exhausted and turns out to be particularly disastrous: indeed, unless a learning-by-doing effect reduces sufficiently the cost of renewable energy production, it will mean a neverending collapse of the economy towards a zero output equilibrium. In our model where the energy transition takes the form of a progressive rise in the share of renewable energy, a downward adjustment of output will be shown to start strictly before the depletion of the non-renewable energy source and will take a less extreme form.

We must outline two methodological differences with respect to the above quoted growth models. First, our model respects the ecological economics postulates in the line of the macro models of a.o. Krysiak (2006), Fagnart and Germain (2011) and Germain (2012). In particular, we consider that the technological progress is bounded by physical limits: it can reduce the energy intensiveness of human productions up to some point but the energy content of any such production is bounded

[1] This possibility of a non-monotonic evolution in the nonrenewable use contrasts with the outcomes of the traditional models of Dasgupta and Heal (1974) and Stiglitz (1974) in which it is necessarily monotonically decreasing.

from below by a strictly positive value. Consequently, perpetual economic growth (measured by the quantity of goods produced) is impossible in our framework, contrary to what may occur in the above mentionned models. A second methodological difference is the fact that we do not analyze the behaviour of a hypothetical central planner as in the optimal growth tradition. Our aim is to identify the private agents' behaviours and/or technological/environmental conditions that favor or not a smooth energy transition.

The structure of the article is the following. Section 2 presents the model of a decentralised economy. Section 3 describes the stationnary state of the economy where only renewable energy is used and it identifies the determinants of the long run EROEI and its relationship with the long run saving rate. In section 4, we study the dynamic behaviour of the model. Because our interest is mainly prospective and not historical, we concentrate on trajectories of the economy with two phases: during the first one, both renewable and non-renewable energies are worked simultaneously, during the second one, the economy relies only on renewable energy. We first present a reference scenario based on a parameter calibration which aims at reproducing several key macro ratios of our contemporary economies (share of renewable energy in current energy production, saving rate,...). The energy transition in this baseline scenario is characterized by a peak of GDP and consumption during the first phase, followed by the fall and next by the convergence of the economy to its stationnary state during the second phase. We then present a sensitivity analysis showing how some key parameters of the economy (the non-renewable resource stock, the discount factor of private agents, the potential of technological progress,...) affect the possibility of a non monotonic trajectory of the economy and the magnitude of the decrease (temporary or not) of GDP or consumption in such a case. Section 5 summarizes our main results and proposes several possible extensions.

2 The model

We consider an economy with three competitive markets: final good, capital and energy. The final good is used for consumption and investment. Its production requires physical capital and energy, either renewable or non-renewable. Capital is accumulated by households and rent to firms.

The energy supply is a mix of non-renewable and renewable energy. Both are assumed to be perfect subtitutes and thus sold at the same price. Non-renewable energy (NRE hereafter) is supplied by households who run a NRE stock without extraction costs. NRE is assumed to be *final* and can be used for productive purposes without transformation. Renewable energy (RE hereafter) is supplied by competitive firms, which operate a free primary RE flux (for example solar energy) under decreasing returns-to-scale. This primary flux is assumed exogenous, constant and very large. The demand for final energy (either non-renewable or renewable) only comes from final firms.

Technical progress increases the energy efficiency of the economy through time. It is however bounded by physical constraints so that the ratio between energy input and final output cannot tend towards zero, even asymptotically.

2.1 Households

We consider a representative agent with a very long time horizon. She consumes the final goods, accumulates physical capital and operates a NRE stock. During a given period t, she chooses a consumption level C_t, makes an investment decision K_{t+1}; as long as the resource stock is positive, she also decides the quantity E_t of NRE to sell on the energy market.

The household receives the whole macroeconomic income under the form of capital rent, resource income and profits Ω_t. With the final good as numéraire, the household's budget contraint of period t writes as

$$C_t + \frac{K_{t+1}}{\varphi_t} = v_t K_t + q_t E_t + \Omega_t \tag{1}$$

where K_t is the capital stock accumulated in $t-1$, v_t is the rental price of capital, q_t is the energy price. K_{t+1}/φ_t is the investment level in period t, $\varphi_t > 0$ reflecting the productivity of the transformation of investment goods into productive capital. For analytical tractability, we assume a unitary depreciation rate, which implies that the length of a time period corresponds to the average life-time of productive capital.

Let $\mathcal{S}$ be the NRE endowment of the economy (i.e. the initial NRE stock). The NRE extraction cost is nil. The household knows $\mathcal{S}$ and operates the NRE stock under the following constraint:

$$\sum_{t=1}^{T_e} E_t = \mathcal{S}, \tag{2}$$

where T_e is the time length of the exploitation of the NRE stock; it is a decision variable of the household. The "no extraction cost" assumption (as in e.g. Jouvet and Schumacher (2012)) is made for the sake of simplicity and is not key since we do not aim at modelling the emergence of NRE [2]. The absence of extraction costs does not mean that NRE is free since extraction has an opportunity cost which justifies a positive market price q_t. This is enough for our setting.

The household's preferences are represented by an isoelastic utility function; the household's discount factor is $0 < \beta < 1$. The intertemporal decision problem writes :

$$\max_{\substack{\{C_t, K_{t+1}\}_{1\leq t\leq T} \\ T_e, \{E_t\}_{1\leq t\leq T_e}}} U = \sum_{t=1}^{T} \beta^t \frac{C_t^{1-\alpha} - 1}{1-\alpha}, \alpha > 0 \tag{3}$$

under the constraints (1)-(2) and with initial K_1 and Ω_1 given. T is the exogenous time horizon (possibly infinite).

We solve this problem in two steps. First, taking T_e as given, we determine the optimality conditions for the other decision variables: these variables are so expressed as functions of T_e. We next determine the optimal value of T_e. At T_e given, the first order conditions for an interior solution lead to (see Appendix 7.1 for details) :

$$\left[\frac{C_{t+1}}{C_t}\right]^\alpha = \beta \varphi_t v_{t+1},\ t \in \{1, ..., T-1\} \tag{4}$$

$$\frac{q_{t+1}}{q_t} = \varphi_t v_{t+1},\ t \in \{1, ..., T_e - 1\} \tag{5}$$

$$K_{T+1} = 0. \tag{6}$$

Equation (4) describes the well-known consumption smoothing behaviour ($\varphi_t v_{t+1}$ corresponding to 1+ the real interest rate in the present model). Equation (5) is the Hotelling rule governing the extraction of the resource: as long as NRE is available, postponing its extraction must offer the same return as the alternative asset (i.e. the return on investment in productive capital). The last equation is the terminal condition: the agent will not invest in the final period as there is a one-period time to build the capital stock.

In a second step, we determine T_e. Let $\{C_t^*\}_{1\leq t\leq T}, \{K_{t+1}^*\}_{1\leq t\leq T}$ and $\{E_t^*\}_{1\leq t\leq T_e}$ be the vectors that are solutions to the utility maximization problem at given T_e. These vectors depend

[2] In Tahvonen and Salo (2001), the NRE extraction is costly and the extraction cost of the first NRE unit is larger than the operating cost of the first RE unit. Hence, the trajectory of their model is characterized by 3 phases: a first one where only RE is used, a second one where the two energy sources are exploited, a last phase where NRE is economically depleted (i.e. too costly to extract) and only RE is used. In our model where NRE extraction is costless, the first phase is impossible and the third one starts after the physical exhaustion of the resource.

on T_e. The optimal T_e is the solution to the following problem:

$$\max_{T_e \in \{1,...,T\}} U(T_e) = \sum_{t=1}^{T} \beta^t \frac{[C_t^*(T_e)]^{1-\alpha} - 1}{1-\alpha}. \tag{7}$$

Because T_e is discrete, it cannot be determined by using differential calculus and numerical methods are necessary.

2.2 The renewable energy sector

The economy enjoys a constant flow $\mathcal{F}$ of renewable primary energy (say for example the radiant energy of the sun). We consider a perfectly competitive RE sector with a number N of identical price-taking producers. They use capital to capture and transform the primary energy. We assume that flow $\mathcal{F}$ is very large and does not constraint the firms' activity level.

In each period t ($t = 1, ..., T$), the representative energy producer chooses a RE supply f_t and a capital stock g_t by solving

$$\max_{f_t, g_t} q_t f_t - v_t g_t \tag{8}$$

under the constraint that

$$f_t = \frac{g_t^\gamma}{b_t}, \qquad \text{with } 0 < \gamma < 1. \tag{9}$$

B_t is a positive exogenous function, constant or decreasing over time and bounded from below by a strictly positive bound: this positive lowerbound means that the production of a given flow f_t will always require a non infinitesimal quantity of capital. The energy supply f_t must be understood as the production *net* of the energetic consumption of the producer. As returns-to-scale are decreasing ($\gamma < 1$), the capital intensiveness of RE production is increasing in the production level[3].

Let F_t be the total RE production at the sector level (i.e. $F_t = N f_t$) and G_t be the sector consumption of capital (i.e. $G_t = N g_t$). Firms' decisions lead to the following relationships at the *sectoral* level (see Appendix 7.2 for details):

$$G_t = \left[\frac{\gamma}{B_t}\frac{q_t}{v_t}\right]^{\frac{1}{1-\gamma}} \qquad \text{where} \quad B_t =_{def} \frac{b_t}{N^{1-\gamma}},\ t \in \{1, ..., T\} \tag{10}$$

$$F_t = \frac{G_t^\gamma}{B_t},\ t \in \{1, ..., T\}. \tag{11}$$

G_t and F_t are increasing in the energy price and decreasing in the rental price of capital and the technological coefficient B_t.

2.3 The final good sector

Final good production Y_t requires capital and energy. The production technology is of the *Leontief* type with constant returns-to-scale: in t,

$$Y_t = A_t X_t = \zeta H_t,\ t \in \{1, ..., T\} \tag{12}$$

where X_t is the energy consumption and $A_t > 0$ the energy productivity: $A_t > 0$ is increasing through time as a result of technological progress. However, as the physical laws constrain the productivity of energy, A_t is bounded from above by a strictly positive constant A_*. This assumption

[3]Operating solar or wind energy requires an access to different sites where this energy can be captured. Some sites are less favourable than others and need a higher windmill or more solar pannels to obtain the same quantity of energy than the best sites. Firms experience decreasing returns-to-scale as increasing production requires to operate sites of decreasing performance (as in a ricardian resource model). See also Fagnart-Germain (2011) for a more formal justification of the assumption of decreasing returns-to-scale in the use of a renewable resource.

means that it will never be possible to produce a given quantity of final good with an infinitely small quantity of energy, even asympotically. H_t is the capital stock allocated to final production. The output/capital ratio of the final sector is equal to ζ and is constant.

Because technological progress is necessarily bounded, modelling its trajectory (from A_1 to A_*) as an exogenous process is not a crucial shortcut with respect to an alternative modelling where the trajectory of A_t between A_1 and A_* would be the result of some endogenous activity. We could introduce a source of endogenous technological progress, e.g. a learning by doing effect as in Tahvonen-Salo (op. citum) and Jouvet-Schumacher (op citum). But this would complicate the model without really changing its properties from a qualitative point of view (in particular the long run would be the same as $A_t \to A_*$)[4].

We assume perfect competition in the final good sector. With the final good price as numéraire, the zero profit condition writes $Y_t = q_t X_t + v_t H_t$. Given (12), it leads to the following condition :

$$1 = \frac{q_t}{A_t} + \frac{v_t}{\zeta},\ t \in \{1, ..., T\}. \tag{13}$$

The real unit cost of energy q_t/A_t and the real unit cost of capital in final production v_t/ζ add up to 1 (the real output price).

2.4 Market equilibrium conditions

The equilibrium conditions of the different markets of the economy write as follows:

- On the final good market, output is allocated either to consumption or to investment (which determines the capital stock of the following period):

$$Y_t = \begin{cases} C_t + \frac{K_{t+1}}{\varphi_t}, \forall\ t \in \{1, ..., T-1\} \\ C_t, \quad \text{for } t = T. \end{cases}$$

- On the capital market, the capital stock supplied by households is demanded either by RE or final good producers:

$$K_t = G_t + H_t,\ \forall t \in \{1, ..., T\}.$$

- On the energy market, the demand of final good producers matches the supply of RE and NRE:

$$X_t = \begin{cases} E_t + F_t,\ \forall t \in \{1, ..., T_e\} \\ \\ F_t, \quad \forall t \in \{T_{e+1}, ..., T\} \end{cases}$$

2.5 The dynamic system

The trajectory of the economy consists of two phases : (i) a first phase (Phase 1 hereafter) lasts the first T_e periods and is characterized by the coexistence of renewable and non-renewable energy; (ii) a second phase (Phase 2 hereafter) lasts from $T_e + 1$ until the end of the horizon, and is characterized by the absence of any NRE use ($E_t = 0,\ \forall t > T_e$).

2.5.1 The model equations

The macroeconomic equilibrium can be summarized by the following equations describing the agents' behaviours, the production technologies, the NRE stock constraint and the equilibrum conditions:

[4] Note that in Tahvonen and Salo (2001) and Jouvet and Schumacher (2012) technological progress is assumed unlimited and thus makes perpetual economic growth possible, which is not the case here.

$$\left[\frac{C_{t+1}}{C_t}\right]^\alpha = \beta\varphi_t v_{t+1},\ t \in \{1, ..., T-1\} \tag{14}$$

$$Y_t = C_t + \frac{K_{t+1}}{\varphi_t},\ t \in \{1, ..., T-1\} \text{ and } Y_T = C_T \tag{15}$$

$$X_t = E_t + F_t,\ t \in \{1, ..., T_e\} \text{ and } X_t = F_t,\ t \in \{T_e+1, ..., T\} \tag{16}$$

$$K_t = G_t + H_t,\ t \in \{1, ..., T\} \tag{17}$$

$$F_t = \frac{G_t^\gamma}{B_t},\ t \in \{1, ..., T\} \tag{18}$$

$$G_t = \left[\frac{\gamma}{B_t}\frac{q_t}{v_t}\right]^{\frac{1}{1-\gamma}},\ t \in \{1, ..., T\} \tag{19}$$

$$Y_t = A_t X_t = \zeta H_t,\ t \in \{1, ..., T\} \tag{20}$$

$$1 = \frac{q_t}{A_t} + \frac{v_t}{\zeta},\ t \in \{1, ..., T\} \tag{21}$$

$$\frac{q_{t+1}}{q_t} = \varphi_t v_{t+1},\ t \in \{1, ..., T_e-1\} \tag{22}$$

$$\sum_{t=1}^{T_e} E_t = \mathcal{S}. \tag{23}$$

To these equations, one must add a(n implicit) condition on T_e following from (7). $K_1, \mathcal{S}, \alpha, \beta, \gamma, \varphi_t, B_t, A_t, \zeta$ are exogenous functions or parameters[5].

The saving rate of the economy is given by:

$$s_t = \frac{K_{t+1}}{\varphi_t Y_t} \tag{24}$$

and, using (17), it is useful to decompose it the following way:

$$s_t = s_t^h + s_t^g \qquad \text{with} \quad s_t^h = \frac{H_{t+1}}{\varphi_t Y_t} \quad \text{and} \quad s_t^g = \frac{G_{t+1}}{\varphi_t Y_t}, \tag{25}$$

s_t^h (resp. s_t^g) is the fraction of period t output invested in the final production sector (resp. in the RE sector) in $t+1$.

2.5.2 Evolution of the EROEI ratio during the transition

Recall that the Energy Returned on Energy Invested ratio (EROEI) is "the ratio of gross energy delivered by an energy production process to input energy required to obtain that gross energy" (Heun and de Wit, 2012, p.147). In our model, the EROEI linked to the NRE production is infinite since there is no extraction cost so that the supply of a certain NRE flow does not consume any energy. On the contrary, the EROEI linked to the RE production is finite since the (net) RE production F_t requires a capital stock level G_t, the building of which has consumed energy. G_t has required an investment G_t/φ_{t-1} during $t-1$. This investment corresponds to a fraction of $t-1$ output given by $\frac{G_t}{K_t}\frac{K_t}{\varphi_{t-1}Y_{t-1}}Y_{t-1}$, i.e. using (25), $s_{t-1}^g Y_{t-1}$. If X_{t-1} has been necessary to produce Y_{t-1}, the quantity of energy absorbed by the building of G_t is $s_{t-1}^g X_{t-1}$. The EROEI for RE production in period t is thus equal to:

$$\varepsilon_t^{RE} = \frac{F_t}{s_{t-1}^g X_{t-1}}.$$

[5] One can verify that there are as many equations as unknowns. Indeed, the unknowns are $C_t, Y_t, X_t, F_t, G_t, H_t, v_t, q_t,\ \forall t \in \{1, ..., T\},\ K_t,\ \forall t \in \{2, ..., T\},\ E_t,\ t \in \{1, ..., T_e\}$ and T_e. This amounts to $8T + (T-1) + T_e + 1 = 9T + T_e$ unknowns. On the other hand, the system (14)-(23) is formed by $8T + (T-1) + (T_e - 1) + 1 + 1 = 9T + T_e$ equations.

A larger s^g_{t-1} implies that a larger fraction of the energy used to produce Y_{t-1} has been absorbed in the RE production process of period t: the EROEI is accordingly lower *ceteris paribus*.

Globally (i.e. considering both types of energies), producing $X_t = E_t + F_t$ has indirectly consumed a quantity of energy given by $(0+)s^g_{t-1}X_{t-1}$. Thus at the macroeconomic level, the EROEI of period t is equal to[6] :

$$\varepsilon_t = \frac{1}{s^g_{t-1}}\frac{X_t}{X_{t-1}} = \frac{1}{s^g_{t-1}}\frac{A_{t-1}}{A_t}\frac{Y_t}{Y_{t-1}}. \tag{26}$$

When rewritten as $X_t/X_{t-1} = \varepsilon_t s^g_{t-1}$, (26) defines the set of combinations of ε_t and s^g_{t-1} that allow the economy to achieve a given growth of energy consumption (and thereby a given growth of output at a given level of energy efficiency gains A_t/A_{t-1}). In the positive orthant of the space $(\varepsilon_t, s^g_{t-1})$, these combinations form a hyperbola and define an iso-growth curve comparable to an isoquant in production theory: in order to reach a given rate of energy (or output) growth, the fraction of final output that must be allocated to the energy production process is inversely related to the EROEI level of this process. Hence, if the energy transition is accompanied by a fall in EROEI, maintaining a given energy (or output growth) will require a compensatory increase in the share of saving that must be invested in energy production.

3 Stationnary state

In this section, we assume that $T \to +\infty$ so that the exogenous parameter functions φ_t, B_t, A_t are equal to their respective asymptotic values φ, B_*, A_*. The NRE is exhausted and production activities only rely on RE.

Given our technological assumptions, the stationnary state is characterized by constant values $C_*, Y_*, X_*, F_*, G_*, H_*, K_*, v_*, q_*$ of variables $C_t, Y_t, X_t, F_t, G_t, H_t, K_t, v_t, q_t$. The system (14)-(23) then becomes (see Appendix 7.3 for details) :

$$v_* = \frac{1}{\varphi\beta} \tag{27}$$

$$q_* = A_*\left[1 - \frac{1}{\zeta\varphi\beta}\right] \tag{28}$$

$$F_* = \left[\frac{\gamma A_*}{B_*^{\frac{1}{\gamma}}}\left[\beta\varphi - \frac{1}{\zeta}\right]\right]^{\frac{\gamma}{1-\gamma}} \tag{29}$$

$$Y_* = A_* F_* \tag{30}$$

$$X_* = F_* \tag{31}$$

$$H_* = \frac{Y_*}{\zeta} \tag{32}$$

$$G_* = [B_* F_*]^{\frac{1}{\gamma}} = \gamma\left[\beta\varphi - \frac{1}{\zeta}\right]Y_* \tag{33}$$

$$K_* = \left[\gamma\beta\varphi + \frac{1-\gamma}{\zeta}\right]Y_* \tag{34}$$

$$C_* = \left[1 - \left[\beta\gamma + \frac{1-\gamma}{\zeta\varphi}\right]\right]Y_* \tag{35}$$

[6] One verifies that the EROEI is a ratio without unity.

The stationary expressions of s_t and g_t are given by

$$s_* = \gamma\beta + \frac{1-\gamma}{\zeta\varphi} \quad (36)$$

$$= \underbrace{\frac{1}{\zeta\varphi}}_{s_*^h} + \underbrace{\gamma\left[\beta - \frac{1}{\zeta\varphi}\right]}_{s_*^g} \quad (37)$$

(36) follows straightforwardly from the stationnary state expression of (24) and (34); (37) is a mere rewriting of (36) that gives the stationnary state expression of the decomposition (25). The saving rate is increasing in β and in the returns-to-scale in RE production γ: the higher γ, the higher the marginal productivity of capital in RE production, the stronger the incentive to invest in the RE sector and the incentive to save. On the contrary, the saving rate is decreasing in ζ and φ: a higher ζ or φ makes investment more productive and saving (per unit of output) less necessary.

Lemma 1 *Feasibility condition: The stationnary state is feasible only if households are sufficiently long-termist and if investment goods are productive enough, i.e. if*

$$\beta > \frac{1}{\zeta\varphi} \qquad \textit{and} \qquad \zeta\varphi > 1.$$

Proof:

To be meaningful, the stationnary state must verify $q_* > 0$. (28) thus implies the inequality:

$$\zeta\varphi\beta > 1, \quad (38)$$

which is equivalent to the condition on β in Lemma 1. As (28) shows, (38) simply means that the real unit cost of energy, q^*/A^* (which is also the share of energy in added value), must be smaller than 1. Alternatively, (27) leads to $\frac{v_*}{\zeta} = \frac{1}{\zeta\beta\varphi} < 1$: the real unit cost of capital in final production must be smaller than 1. Thus (38) ensures that each factor share in added value is lower than 1.

As $\beta < 1$, (38) also requires that $\zeta\varphi > 1$. ■

Intuitively, the existence of a stationary state requires a sufficiently high saving/investment rate (production requires capital and energy, which requires itself capital) and so a sufficiently low discount rate (or sufficiently high β). Moreover, investment must be productive enough: one unit of investment good creates φ units of capital stock, with which $\zeta\varphi$ units of output can be produced. If $\zeta\varphi < 1$, the economy would be unable to maintain its capital stock at a constant level (it would unavoidably decrease through time), even under the extreme assumption that output is exclusively allocated to investment. Note finally that $\zeta\varphi > 1$ ensures that $s_* < 1$.

The following lemma is rather obvious but useful for the interpretation of the numerical experiments that will follow.

Lemma 2 *Stationary level of the economy:*

1. *The stationary levels of RE production F_* and final output Y_* are increasing functions of* [7]
   - *the productivity of energy A^* and capital ζ in final production;*
   - *the returns-to-scale γ and the productivity factor of capital $1/B_*$ in the RE sector;*

[7] Recall that we assumed that the RE flow $\mathcal{F}$ is so large that it is not binding. Should it be binding, the steady state values of RE and output would simply be $F_* = \mathcal{F}$ and $Y_* = A_*\mathcal{F}$.

- *the productivity of investment goods φ in the formation of the capital stock;*
- *the long-termism of private agents β.*

The same is true for the stationnary levels of capital stocks G_*, H_*, K_*. C_* also depends positively on A_*, ζ, $1/B_*$ and φ but ambiguously on β and γ.

2. *The higher $\zeta\varphi\beta$, the higher q_*: the real energy price q_* (or the real unit energy cost q_*/A_*) is higher in an economy where private agents are more long-termist and where investment goods are more productive.*

Proof:

The part of point 1 that concerns F_* follows straightforwardly from (29). Equations (33)-(35) show that the quantity variables C_*, Y_*, G_*, H_*, K_* are linear functions of the energy consumed F_* and allow ones to establish the rest of point 1 very easily. The ambiguity of the impact of β and γ on C_* is the consequence of their positive impact on both Y_* and s_*: $(1 - s_*)Y_*$ may thus increase or decrease.

The second point follows from (28). ■

Point 1 is very obvious and does not need any particular comment. Point 2 may be intuitively understood as a consequence of Point 1: a higher β, φ or ζ stimulates the production of final output and thereby the demand for -and real price of- energy.

Proposition 1 *About the long run EROEI and its economic implications*

1. *The long run EROEI is given by:*

$$\varepsilon_* = \left[\beta\gamma - \frac{\gamma}{\zeta\varphi}\right]^{-1}. \quad (39)$$

2. *An economy with a lower EROEI must save a larger share of final output and allocate a larger share of capital to energy production. Formally, s_*^g and s_* are inversely related to ε_*:*

$$s_*^g = \frac{1}{\varepsilon_*} \quad (40)$$

$$s_* = \frac{1}{\zeta\varphi} + \frac{1}{\varepsilon_*}. \quad (41)$$

3. *The real unit cost of enery is inversely related to the EROEI:*

$$\frac{q_*}{A_*} = \frac{1}{\beta\gamma\varepsilon_*}. \quad (42)$$

4. *The stationnary state is feasible only if the EROEI is sufficiengly larger than 1:*

$$\varepsilon_* > \left[1 - \frac{1}{\zeta\varphi}\right]^{-1} > 1. \quad (43)$$

Proof:

The stationary expression of (26) writes as

$$\varepsilon_* = \frac{1}{s_*^g} > 1. \quad (44)$$

(37) and (44) leads to (39). Note that (38) ensures that ε_* is necessarily strictly positive. (41) follows straightforwardly. Expression (42) is obtained by rewriting (28) so as to make appear the expression of ε_* given by (39). Inequality (43) is the mirror image of the fact that s_* must be smaller than 1: (41) implies indeed that $1/\varepsilon_*$ must be smaller than $1 - 1/(\zeta\varphi)$, which requires a not too small value of ε_*.

■

Note that:

1. Equation (39) outlines that the global EROEI of the economy does not depend exclusively on technological variables (here γ, ζ, φ) but also on β. It also stresses that a partial equilibrium reasoning about the determinants of the EROEI may be misleading at a general equibrium level. Indeed, for a given production level F_*, a higher γ reduces the quantity of capital that must be used: it thus improves the energy return of RE production at given F_*; similarly, a higher ζ or a higher φ reduces the quantity of final output that must be invested in the RE sector at given F_*. A higher values of γ, ζ or φ thus increases the EROEI value at given F_*. However, F_* changes endogenously and a rebound effect in the production of F_* actually makes the global EROEI decreasing in γ, ζ and φ.

2. (42) implies, at given γ, a negative relationship between the long run EROEI and the real price of energy q_*: a larger value of β, φ or ζ lowers ε_* and raises q_*. This confirms, in a macroeconomic framework, the inverse relationship established between these two variables by King and Hall (2011) or by Heun and de Wit (2012) in a partial equilibrium analysis.

   We will explore in the dynamic analysis whether such a negative relationship between ε_* and q_* is also observed during the energetic transition.

3. (40) and (41) confirm that the EROEI value has important implications on both the allocation of final production and the sectoral structure of the economy. When the EROEI is lower, RE production is more capital intensive: as a result, a larger share of output must be allocated to capital accumulation and a larger share of capital must be allocated to energy production. This suggests that the move towards a 100% renewable energy era could be accompanied by a dramatic change in the allocation of final output. This issue will be further analyzed in the next section.

4. Obviously, an EROEI above 1 is a necessary condition for the long run survival of a(n economic) system. In our decentralized economy, it must be strictly above 1 as established by (43). This result echoes the literature in energy science which argues that an economy cannot survive with a global EROEI lower that 3 and that an EROEI close to 5 is even necessary “to maintain what we call civilisation” (Hall et al., 2009, p.45).

4 Dynamics

The energy transition raises obviously a crucial issue: will the economy be able to maintain in the RE era the income level reached during the NRE era? As already stressed, an energetic transition that would be accompanied by a significant fall in the EROEI might require an important reallocation of capital towards the energy sector, with a negative impact on the final sector production. After an initial phase of economic growth, the transition might thus be characterized by a downward adjustment of output and private consumption. We study hereafter this possibility and the elements that affect the magnitude of such a downward adjustement. We also analyze what influences the ability of the economy to recover ultimately the level of output reached during the era of the high EROEI resources.

Our study of the model dynamics must rely mainly on numerical simulations and we calibrate the model parameters to obtain a baseline scenario satisfying several “reasonable” assumptions and properties. As we have assumed a unitary depreciation rate of capital, the length of a time period is slightly more than a decade. Considering a period length of 12 years, we have chosen a reference value of $\beta = 0.8$, which corresponds to an annual discount rate of 1.9% (=$(0.8)^{-1/12} - 1$)).

As explained earlier, the potential of technical progress affecting the energy productivity in the final goods sector is supposed bounded by physical limits: we assume the following law of motion of A_t:

$$A_t = A_* + \frac{A_1 - A_*}{a^{t-1}}, \; t \geq 1 \tag{45}$$

where $0 < A_1 < A_*$ and $1 < a$. Parameter a determines the speed of convegence of A_t towards the asymptotic value A^*. With respect to the contemporary level of energy productivity (measured by A_1), we assume a gain of efficiency of 30% by 2050, and an asymptotic gain between 50% and 100% [8]. For the baseline scenario, we take the middle of the interval ($A_*/A_1 = 1.75$). Assuming periods of a time length of about 12 years, one can determine a [9].

We assume that φ_t, b_t are constant. These parameters and the remaining model parameters have been choosen so as to satisfy the feasibility constraints (38) and to obtain “reasonable” contemporary values (i.e. for the first period) of economic variables such as the saving rate (s_1 around 20-25%), the GDP growth rate or the share of NRE in total energy (around 90% at the world level according to various estimations)[10].

It goes without saying that the model is highly stylized and that the generated numerical values are not interesting *per se*. We will mainly pay attention to the general shape of the trajectories of the different variables.

4.1 The baseline scenario

The baseline scenario describes the transition dynamics of an economy with a rather low initial capital stock ($K^*/K_1 \approx 6$) and with a potential of technical progress of 75% ($A^*/A_1 = 1,75$) as explained just above. It is illustrated in Figures 1.a to 1.e hereafter. Phase 1 (during which NRE and RE sources are simultaneously used) lasts the seven first periods.

Figures 1.a shows the transition from a purely energetic point of view. NRE production E_t peaks at the very beginning of the trajectory ($t = 2$)[11] and then declines monotonically until exhaustion of the NRE stock. During this phase, RE production, F_t, increases monotonically and the share of NRE in total energy production declines monotonically. At the beginning of Phase 2, RE production declines for a while before increasing slowly again towards its asymptotic value.

Figure 1.b illustrates the non monotonic evolution of final output Y_t, consumption C_t and investment I_t. Thanks to capital accumulation and energetic efficiency gains, output and consumption

[8] The assumption of a gain of energy efficiency of 30% by 2050 relies on computations that Vermeulen (2013) realised from IEA data (IEA, 2010). It corresponds to the efficiency gains that would be ideally obtained if the use of the best available technologies known today was generalized at the level of the whole economy by 2050.

A quantification of the *global* potential of efficiency gains in the very long term is obviously difficult. The lower bound of the interval (50%) expresses a “conservative” vision that reflects the conclusions of numerous studies on future technological evolutions: they suggest that the main efficiency gains linked to technological innovations will be achieved by the next 40 to 50 years. On the contrary, the upper bound of 100% expresses a much more optimistic vision and supposes the emergence of radically new technologies.

[9] If we assume that 2014 is the middle of period 1, 2026 the middle of period 2,..., 2050 is then the middle of period 4. and a can be calibrated as the solution of

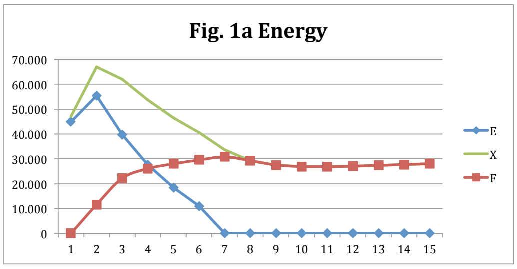
$$\frac{A_4}{A_*} = \frac{A_4}{A_1}\frac{A_1}{A_*} = 1 + \frac{1}{a^4}\{\frac{A_1}{A_*} - 1\}$$

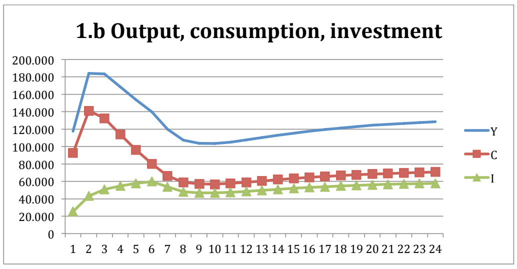
where $\frac{A_4}{A_1} = 1.3$ et $\frac{A_*}{A_1} \in [1.5, 2]$.

[10] The values of parameters and initial conditions characterising the baseline scenario are the following: $\beta = .8; \gamma = .5; \varphi = 2; \zeta = 5; b = 1; A_1 = 2.5; A_* = 1.75A_1; a = 1.15; \mathcal{S} = 20; K_1 = 2.4$. The simulations have been runned with the software MATLAB.

[11] The exact period of the peak depends on the initial conditions. For example, a lower initial capital stock can lead to a later peak of NRE.

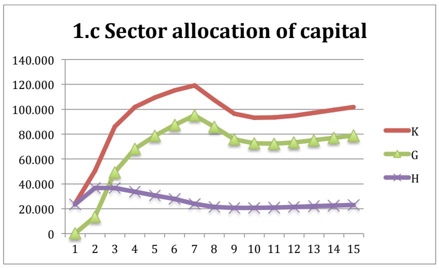
increase initially. But they reach a peak during Phase 1 and next decrease to a (local) minimum. During Phase 2, they increase again and converge to their asymptotic steady state values.

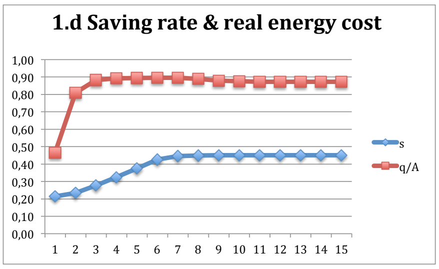




The analysis of the above figures puts forward four striking features of *the baseline scenario*.

1. During the NRE era, final output and private consumption (Fig. 1.b) overshoot the level that the economy is able to sustain in the long run.
2. The downward adjustment that follows the peak in output and private consumption starts well before the exhaustion of the non-renewable energy source (Fig. 1.a-b).
3. The energetic transition is accompanied by a strong increase in the saving rate (Fig. 1.d) and a very important switch in the allocation of capital between the energetic and final good sectors (Fig. 1.c)[12], a downward trend in the evolution of the EROEI (Fig. 1.f) and a progressive rise in the real energy price (Fig. 1.d).
4. The NRE availability allows for a rapid development of RE production (Fig. 1.a). But the end of the NRE era puts a drag on this development .





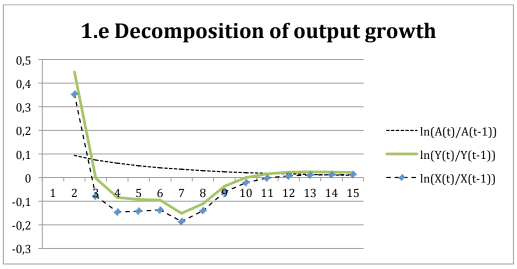
We first give an intuitive explanation of the mechanisms underlying these observations. We will next formulate a proposition about the properties of an overshooting like the one observed here.

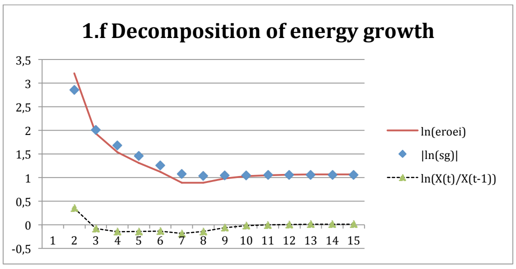
The downward adjustment in output and private consumption follows from the fact that final production and renewable energy production make a rival use of the capital stock. Output growth requires more capital and, in spite of the energetic efficiency gains, more energy. But energy production becomes itself more capital intensive as the share of renewable energy rises. Consequently, even though the capital intensiveness of the technology in the final good sector is constant, the

[12]The amplitude of these changes could however be due (at least partly) to some modelling choices (a.o the absence of extraction costs of the NRE source) and should further investigated in a more "realistic" model. For the sake of comparison, Giampietro et al. (1997) (cited by van den Bergh, 2013, p.22) calculate that for developed countries complete dependance on biofuels could imply that 20 to 40% of the working force would be employed in the energy sector.

capital intensiveness of the overall human activity increases since energy production requires progressively more capital: the lower the share of NRE in total energy consumption, the higher the aggregate capital stock required by the production of a given final output level. Rather early in the first phase of the baseline scenario, the increasing need of capital in the energetic sector diverts capital from the final production sector and does not allow the economy to maintain the level of its final output[13]. As shown in figure 1.b, private consumption decreases even more than output: the saving rate (figure 1.d) indeed rises in response to the increase in the capital demand that accompanies the development of RE production (figure 1.c). Up to the NRE exhaustion, investment (or equivalently here the capital stock) keeps rising so that RE production can expand rather rapidly (fig. 1.a). However, the rising scarcity of the NRE resource strengthens the tension between the rival uses of capital and the resulting contraction in final output eventually stops capital expansion. Capital even falls (temporarily) at the beginning of Phase 2 as shown in Figure 1.c, which leads to a temporary rise in the rental price of capital v and a temporary fall in the profit maximizing level of RE (Figure 1.a) [14].

Figures 1.e-f present a decomposition of the growth of output and energy. Fig1.e shows the growth factor of output, energy and energy efficiency in a log scale (such that the output growth curve corresponds to the vertical summation of the two other curves): in spite of the energy efficiency gains, the evolution of output growth appears as largely driven by the evolution of energy growth. Energy growth is itself decomposed in Fig. 1.f: using (26), the growth factor of energy can be written as the product of ε_t and s^g_{t-1} and the figure shows the evolution of the (absolute value of the) logarithm of these variables. The energy growth curve corresponds to the vertical difference between the eroei and s^g curves: energy growth is negative from periods 3 to 9 when the rise in s^g_{t-1} is not strong enough to offset the negative impact of the EROEI fall.



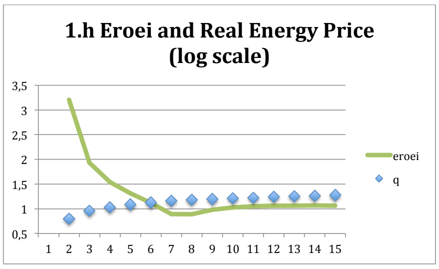


The quasi monotoneous decline of the EROEI is not surprising. The high initial values are linked to the infinite EROEI of NRE and its high share in total energy production at the beginning of the trajectory. The decline of the EROEI is achieved at the end of Phase 1 and illustrates the end of a "golden" era characterized by abundant and free energy. This result is in line with the literature on EROEI which expects a clear decline of this variable because RE technologies will not achieve as high EROEI ratios as those of fossil energies (Murphy and Hall, 2010). Note the negative link between the evolution of the price of energy q_t (Figure 1.h) and the shape of the EROEI ratio. The stationary state analysis and the simulation thus confirm in a macroecomic setting the inverse relationship between the energy price and the EROEI ratio established by King and Hall (2011) and Heun and de Wit (2012) in partial equilibrium settings.

In the baseline scenario, the decline of production and consumption observed in Phase 1 is quite sharp and one may even speak of a collapse of the economy. However, because of technical progress, an economic recovery occurs during Phase 2 where only RE is used. In comparison with the peak values of C, Y and K during Phase 1, the recovery is partial for consumption and output and complete for the capital stock. In the sensivity analysis that we will propose hereafter, we will

[13]This remains true if we assume that the technological progress A_t bears not only on energy but also on capital.

[14]Equation (21) implies that $q_t/v_t = A_t\,(1/v_t - 1/\zeta)$. A rise in v_t that is sufficiently strong to offset the impact of technological progress reduces q_t/v_t and the profit maximizing level of G_t (or F_t).



analyze what influences the possibility and the magnitude of a contraction of consumption and output. We can already establish two general properties:

Proposition 2 *1. A contraction of private consumption occurs during the energetic transition when the real unit cost of energy overcomes its stationnary state value, i.e. when*

$$\frac{q_t}{A_t} > \frac{q^*}{A^*} = 1 - \frac{1}{\beta\zeta\varphi}. \tag{46}$$

2. If a contraction of output happens in a phase of increasing capital, it necessarily starts before the completion of the energetic transition, i.e. at a time where energy production still relies on the non-renewable resource.

Proof:

1. One observe $C_t < C_{t-1}$ if the right hand side of (14) (with $\varphi_t = \varphi$) is smaller than 1, i.e. if

$$v_t < \frac{1}{\beta\varphi}. \tag{47}$$

This inequality and (21) imply that

$$\frac{q_t}{A_t} = 1 - \frac{v_t}{\zeta} > 1 - \frac{1}{\beta\zeta\varphi}.$$

2. In a transition where $K_t > K_{t-1}$, (20) implies that output starts falling ($Y_t < Y_{t-1}$) only if $H_t < H_{t-1}$ (less capital is allocated to final production). Given (17), this means that more capital is dedicated to RE production: $G_t = K_t - H_t > G_{t-1} = K_{t-1} - H_{t-1}$ (since K is bigger and H smaller in t than in $t-1$). Hence, more RE is produced: $F_t > F_{t-1}$. Since energy is more productive ($A_t > A_{t-1}$), a larger F_t can only be consistent with a smaller Y_t if total energy X_t decreases sufficiently, i.e. if NRE is still used and if its consumption decreases sufficiently to more than offset the positive impact of the increases in A_t and F_t.

■

As we mentioned in the introduction, the possibility of a contraction of final output and consumption is analyzed in Jouvet and Schumacher (2012). But in their model where RE and NRE are never used simultaneously, the beginning of a downward adjusment of output (if any) necessarily coincides with the time of exhaustion of the NRE resource[15]. Point 2 of Proposition 1 shows that it necessarily starts earlier in a world where the energetic transition takes the form of a progressive

[15] Furthermore, it implies a perpetual negative growth rate (and thus a progressive collapse of the economy towards a zero activity level) if the technological progress (under the form of a learning by doing effect that reduces cost of RE production) is not strong enough.

rise in the share of RE in total energy consumption. This puts forward that the reduction in the macroeconomic income level that might accompany the energetic transition is not necessarily a remote threat that would only materialize itself at the end of the fossil energy era.

It is worth outlining that all the above observations do not depend on the magnitude of the RE flux the economy enjoys. We have assumed it to be sufficiently large to never be binding. This is consistent with the results obtained by Dale et al. with the energy model GEMBA. As they emphasize, the "... availability of energy resources is not the limiting factor in energy production. Instead the limiting factor seems to be allocating resources to building the capital to extract energy from renewable resources. Within the GEMBA model, such allocation of industrial output into the energy sector stymies the re-investment of capital toward growing the rest of the economy which curtails growth in energy demand. The energy economy system comes into a state of dynamic balance between the energy sector and the rest of the economy. Supplying a greater amount of industrial output towards increasing energy supply entails a decline in the size of the rest of the economy and a subsequent decline in energy demand." (Dale et al., 2012).

4.2 Sensitivity analysis

4.2.1 The impact of the NRE stock

Since the NRE is exhausted in the long run, its initial stock has no impact on the steady state of the economy. But it can obviously affect the transition. If we refer to the dynastic interpretation of the model, there is no doubt that the NRE resource stock has a positive impact on the intertemporal utility level of the whole dynasty but one may question how it affects the material well-being of generations born during the possible consumption peak.

One can formulate two conjectures:

1. *The condition of a downward adjusment of consumption and output during the energetic transition is more easily met when the initial NRE stock is high.*

   Consider indeed the case of a zero initial NRE stock: $\mathcal{S} = 0$. In this case, an initially under-capitalized economy (i.e. an economy for which $K_1 < K^*$) is totally unlikely to overshoot its steady consumption and output levels during the transitory dynamics[16]. By continuity, this remains certainly true if the NRE stock is positive but small enough. Otherwise said, an overshooting of output and consumption can only occur in an economy with a sufficiently large NRE stock.

2. *If a contraction of consumption and output occurs during the energetic transition of an economy with an initial NRE stock $\mathcal{S}$, its magnitude is increasing in $\mathcal{S}$.*

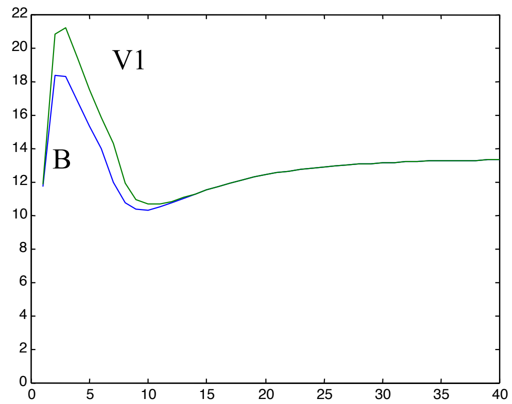
   Ceteris paribus, a larger resource stock implies an initially lower price of energy q_t (energy is more abundant in abolute terms). Accordingly, a given output level is compatible with a lower price of energy in the better endowed economy. Hence, the real unit cost of energy satisfies the condition of an overshoothing (46) at a higher output level -and possibly at a later period- than in the lesser endowed economy is reached. Since the NRE stock does not affect the steady state output level, a downward output adjustment will also be deeper.

The two conjectures have been confirmed in all our numerical experiments. To illustrate the second conjecture and to gain extra insights on the implications of a higher NRE stock, Figures 2.a-c show the transitory dynamics of an economy better endowed in NRE (indicated by V1 in the figures) than in the baseline scenario (indicated by B in the figures): the NRE stock is 25% higher ($S = 25$ in V1 instead of $S = 20$ in B). The two economies are initially identical on all the other dimensions.

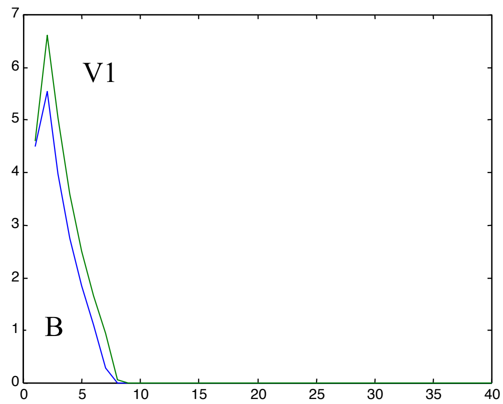
[16] This is in accordance with Germain (2012) who studies the transitory dynamics of an economy that only uses a renewable resource.

Figure 2. Variant 1: increase of *S* (B: baseline, V1: variant)

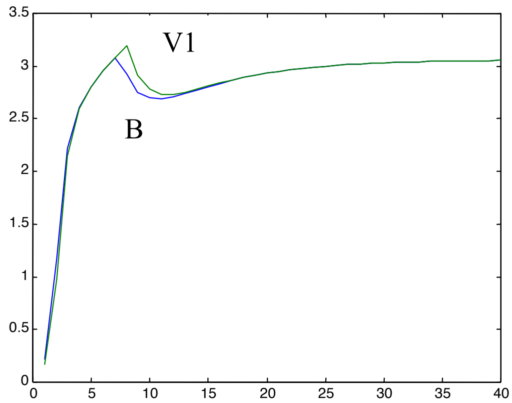
2.a : Y_t



2.b : E_t



2.c : F_t



With respect to the baseline scenario, the overshooting of output is amplified as expected (Fig. 2.a). For the generations born at (or close to) the peak, one may speak of a "malediction of abundance": with a higher initial NRE stock, they inherit a higher level of consumption but experience a more severe consumption loss later during the transition.

Figure 2.b shows that the NRE era is now one period longer and the extracted NRE flow is higher at each period (but it has the same profile as in the baseline scenario). As already observed in the baseline scenario, the fast development of the RE production is crucially linked to the availability of the NRE. As Phase 1 lasts longer, RE production expands more (Figure 2.c).

4.2.2 The role of the time discount rate

We examine now to what extent the observations of the baseline scenario are sensitive to the value of the time discount rate. The following proposition is rather straightforward.

Proposition 3 *1. The condition of a downward adjustment of private consumption and final output during the energetic transition is more easily met in an economy where agents are more short-termist (i.e. have a larger time discount rate or a lower β).*

2. *If a contraction of final output occurs during the energetic transition, its magnitude is increasing in the time discount rate (or decreasing in β).*

Proof:

1. From Proposition 1, we know that if an output fall occurs during the transition, it starts when NRE is still in use. Hence, (22) holds at that time. Given (47), (22) implies that
$$\frac{q_t}{q_{t-1}} = \varphi_{t-1} v_t < \frac{1}{\beta}. \quad (48)$$
This means that a downward output adjustment occurs during the transition when the growth factor of the real price of energy is not too high, i.e. lies in the interval $[1, \beta^{-1}]$. The lower β, the larger this interval. *Ceteris paribus*, a lower β thus make a downward adjustment more likely.

2. A lower β leads households to favour immediate consumption and to exploit the NRE more intensively so as to enjoy the income rent more rapidly. This allows a stronger output increase during the early periods of Phase 1 (*ceteris paribus*, a bigger E_t means bigger X_t and Y_t) and leads to higher peak values of Y_t and C_t. In the long run however, a lower β implies a lower saving rate (see (36)) and lower stationary levels of RE production and final output (see 29 and 30). *Ceteris paribus*, the gap between the peak in output during Phase 1 and its stationary state value in phase 2 is larger when β is lower.

■

The possible negative impact of the time discount rate on the feasibility of a non decreasing consumption path has already been evocated in Solow (1974) and analyzed in e.g. Jouvet and Schumacher (op citum). Point 1 of Proposition 3 confirms this result in our setting. Point 2 appears as a logical corrollary. If we refer again to a dynastic interpretation of our model, the generations born during (or close to) the peak of consumption suffer a weaker loss of their material well-being (as measured by C) when the first generation of the dynasty is less "impatient".

We propose hereafter a simulation of the case where $\beta = .9$ (instead of $\beta = .8$ as in the baseline scenario). Figures 3.a-d compare the trajectories of the higher-β economy (V2 in the figures) and the reference economy (B in the figures). This comparison is in accordance with Proposition 2. The NRE era is longer than in the reference scenario (10 periods instead of 7) and the peak of NRE production is lower (Figure 3.b). As consumers save initially more than in the baseline scenario, the trajectory of the capital stock remains initially very close in the two simulations (Figure 3.d). This allows a very similar evolution of RE production F_t during the common part of Phase 1 (Figure 3.c). As the NRE exhaustion is delayed in the high-β economy, the increase in RE production is extended, which confirms again that a rapid development of RE production depends heavily on the presence of NRE resource. Finally, even though the saving rate tends towards a higher asympotic value, it increases slowlier than in the baseline, in accordance with the fact that the energetic transition is smoother (see Figure 3.e).

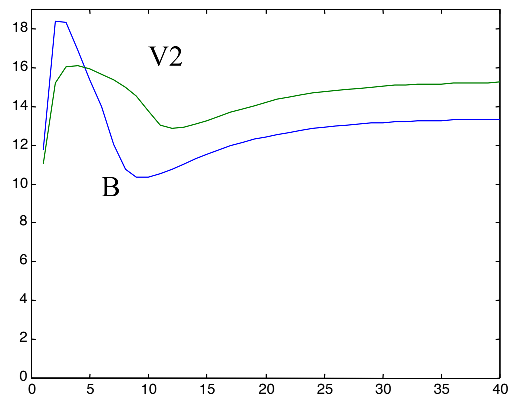
Combining Propositions 2 and 3, one understands that a totally smooth energetic transition (i.e. a transition without contraction of output) could occur in an economy which is not too much endowed in NRE and where agents are very cautious about long term issues (agents have a high β). In our numerical experiments, we have been able to generate a perfectly smooth energetic transition by simultaneously increasing β up to 0.99 and decreasing $\mathcal{S}$ to 8 (the rest of the calibration being unchanged).

4.2.3 The roles of the potential of energy productivity gains and of the returns-to-scale in the RE sector

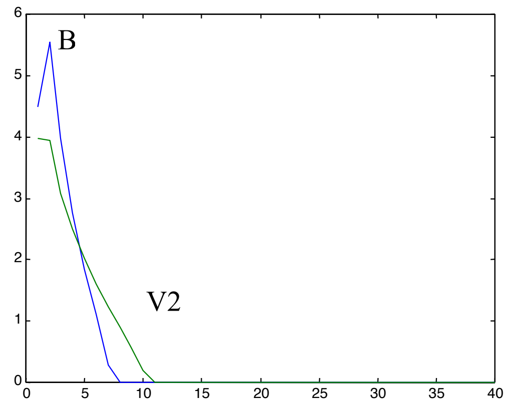
The technological assumptions of the model imply that long term economic growth is nil. As Lemma 2 has shown, the larger the potential of technological progress (here the potential of energy

Figure 3. Variant 2 : increase of β (B: baseline, V2: variant)

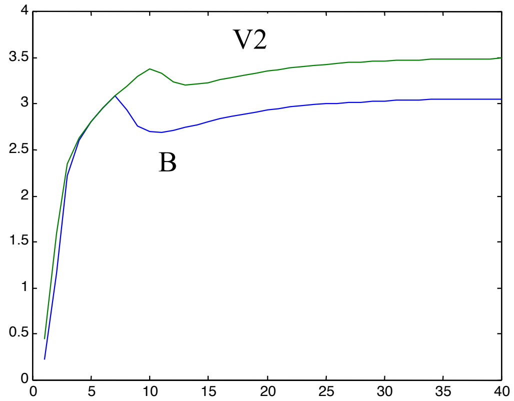
3.a : Y_t



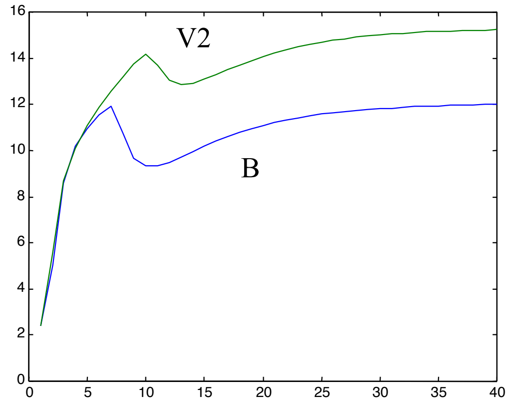
3.b : E_t



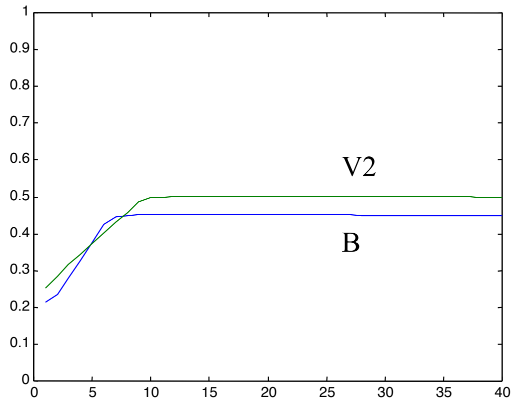
3.c : F_t



3.d : K_t



3.e : s_t

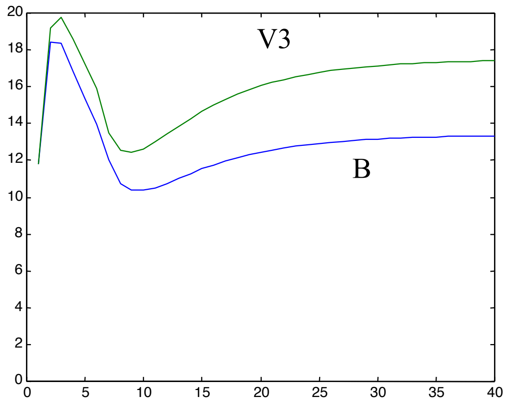


productivity gains A^*), the larger the steady output level; less decreasing returns-to-scale in the RE sector (a higher gamma) will also favour long term output.

If the energetic transition is characterized by a contraction of output, A^* and γ play a key role in the ability of the economy to recover long run levels of output and consumption at least equal to the highest level reached during the NRE era. We illustrate this issue numerically by analysing successively the impact of a larger A^* ($A_* = 2A_1$ instead of $A_* = 1.75A_1$ in the reference scenario) and a larger γ ($\gamma = .6$ instead of $\gamma = .5$). Figure 4 compares the trajectories of final output for the two values of A^*.

Figure 4. Variant 3 : increase of A* (B: baseline, V3: variant)

4.a : Y_t



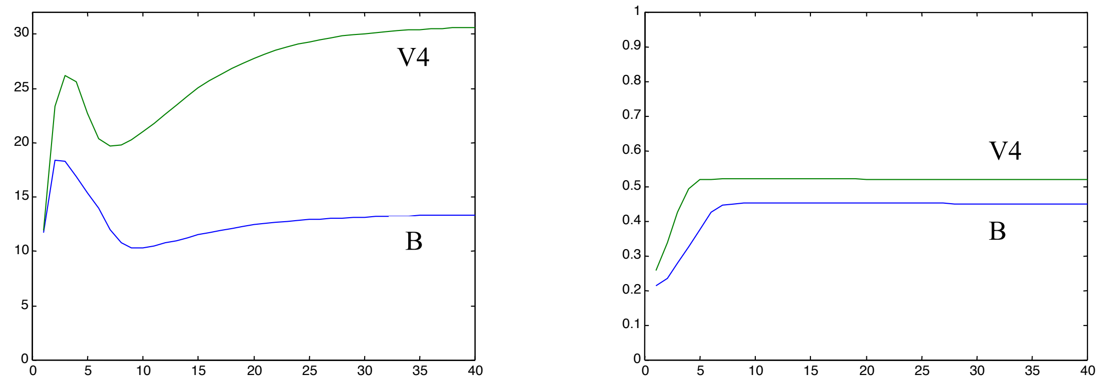
The law of motion of A_t we have assumed implies that a change in A^* has an impact on the whole trajectory of A. For the economy, an increase in A^* is thus not only a prospect of a higher energetic efficiency in the future but also an immediate increase in A_t. The trajectory of output V3 is thus unsurprisingly always above the reference trajectory B (the difference is very small at the very beginning as the productivity of capital in final production is unaffected). During the transition, output and consumption thus peak at a higher level but the consecutive contraction is relatively weaker than in the reference scenario: since energy productivity is already higher during the transition, the exhaustion of NRE is relatively less painful. Furthermore, a higher A^* allows a stronger recovery during Phase 2.

The impact of a higher γ is described by Figures 5.a-b which illustrate that the size of the returns-to-scale in the RE production is a variable that deeply affects the energetic transition and its aftermath.

A higher γ means a higher efficiency of capital in RE production, which is also an incentive to a less parcimonious use of NRE. Accordingly, the extraction of the NRE stock lasts only 5 periods (instead of 7), which contributes to a higher peak of output (Figure 5.a). As RE production is more efficient, the exhaustion of NRE is however less painful than in the reference scenario and the contraction of output and consumption is relatively weaker. In the long run, the recovery of GDP is *more than complete* (GDP is higher at its steady state than at its peak during Phase 1). It is not totally the case for private consumption since the higher efficiency of capital in RE production is an incentive to invest and save more (Figure 5.b).

Figure 5. Variant 4 : increase of γ (B: baseline, V4: variant)

5.a : Y_t 5.b : s_t



5 Conclusion

This paper has developed a model of a decentralised two-sector economy (energy and final good sectors), which initially enjoys two growth engines: capital increase and technological progress improving the energy efficiency. We have analyzed how the dynamics of this initially growing economy is affected by the energy transition from an era where the energy consumption relies mainly on non-renewable resources towards a 100%-renewable era. The first era is sooner or later characterized by a progressive rise of renewables in the energy mix of the economy. Since renewable energy production offers a lower EROEI than non-renewable resources, this progressive substitution is unavoidably accompanied by a decreasing global EROEI. We have analyzed what structural changes this evolution implies and whether it makes the transition painful in terms of economy activity and private consumption.

The structural changes can be linked to the negative relationship between the saving rate of the economy and its EROEI in the long run: a lower EROEI of the energy production process constrains the economy to allocate more capital to energy production and a higher fraction of final output to capital accumulation. If the economy enjoys initially a non-renewable energy with a high EROEI, the transition towards the 100%-renewable era may thus be characterized by major changes both in the allocation of capital between energy and final good productions and in the allocation of output between consumption and investment. All the numerical experiments carried on a calibrated version of the model have confirmed such evolutions.

But these experiments have also shown that in an economy where agents are "normally impatient", the energy transition might put a halt to the growth process fueled by the access to high EROEI resources and might even lead to a contraction of economic activity and private consumption. Indeed, the increasing capital requirements linked to the progressive substitution of high EROEI resources by lower EROEI ones may crowd capital out of final production sectors and push the economy into a contraction phase. The magnitude of this contraction has been shown to depend positively a.o. on the initial stock of the non-renewable energy source and the short-sightedness of private agents: a more abundant (and thus cheaper) energy (in the case of a higher initial stock) or a more intensive use of a given stock (in the case of more impatient agents) allows an initially stronger growth process and leads to higher levels of energy, output and consumption peaks. For the generations born during (or close to) the peak of consumption, the loss of material well-being (as measured by consumption) is thus stronger when the first generations of the dynasty enjoy a higher resource stock or are more short-sighted ("impatient").

Moreover, a formal result establishes that such a contraction phase -if it occurs- necessarily begins (well) before the exhaustion of the non-renewable source. The reduction in the standard of living that might accompany the energy transition is thus not necessarily a remote threat that would only materialize itself at the very end of the fossil energy era.

The contraction phase may be followed by a phase of recovery, particularly if the potential of energy efficiency gains is still high at the beginning of the 100%-renewable era and/or if the returns-to-scale in renewable energy production are not too decreasing. However as the potential of energy efficiency gains is bounded by physical laws[17], the long run of the 100%-renewable era is characterized by a stationary output level. Accordingly, nothing guarantees that the recovery observed after the contraction will allow the economy to reach long run output and consumption levels as high as the peak levels observed during the non-renewable era.

Simulations also show the complementarity between a rapid development of the renewable energy production and the availability of non-renewable energy. The end of the non-renewable era puts indeed a drag on the development of the renewable energy production. Here again, as renewable energy and final productions make a rival use of capital, it is easier to expand the productive capacity of the renewable energy sector when final production can still rely on a high EROEI non-renewable resource.

Note finally that if an overshooting of final output and consumption is the most commonly observed trajectory in our numerical experiments, it is in no way a fatality. A smooth transition (i.e. a monotonous increase of GDP and consumption) is possible if the non-renewable resource stock is low enough and if agents are sufficiently long-termist. It would be interesting to analyze whether a government sufficiently more long termist than private agents could implement policy tools (e.g. a tax on the non-renewable energy use) favorable to a smooth transition.

At least three other research lines are worth mentioning. The model developed in this paper stands on different simplifying assumptions and it would be interesting to test the robustness of our results in a more general setting. The first research line would introduce extraction costs of the non-renewable energy. In such a setting, non-renewable energy extraction would require more and more capital and would exhibit a decreasing EROEI. The end of the extraction period would be determined by cost considerations and not by the physical exhaustion of the stock. A second research line would endogenise technical progress. With respect to the results of the paper, this would not change anything in the long run if technical progress remains bounded but could change the trajectory of the economy during its energy transition. Finally, we could extent our framework to pollution issues (linked to the use of non-renewable energy).

6 Bibliography

Amigues J.-P., M. Moreaux and K. Schubert (2011), “Optimal use of a polluting non-renewable resource generating both manageable and catastrophic damages”, *Annals of Economics and Statistics*, 103/104, 107-130.

Cleveland C. Ed.(2004) , *Encyclopedia of energy*, Elsevier, Dordrecht, the Nertherlands.

Cleveland C. (2008), “Energy Return on Investment EROI, Encyclopedia of the Earth”, http://eoarth.org/article/Energy_return_on_invesment

Dale M., S. Krumdieck and P. Bodger (2012), “Global energy modelling ? A biophysical approach (GEMBA) Part 1: An overview of biophysical economics”, *Ecological Economics*, 73, 152-157.

[17] The ratio between energy consumption and final output decreases through time but cannot tend towards zero even asymptotically.

Dale M., S. Krumdieck and P. Bodger (2012), "Global energy modelling ? A biophysical approach (GEMBA) Part 2: Methodology", *Ecological Economics*, 73, 158-167.

Dasgupta P. and Heal G., 1974, "The optimal depletion of exhaustible resources", *The Review of Economic Studies*, 41, 2-28.

Fagnart J.-F. and M. Germain (2011), "Quantitative versus qualitative growth with recyclable resource", *Ecological Economics*, 70(5), 929-941.

Germain M. (2012), "Equilibres et effondrement dans le cadre d'un cycle naturel", *Brussels Economic Review - Cahiers Economiques de Bruxelles*, 55(4), 427-455.

Giampietro M., S. Ulgiati and D. Pimentel (1997), "Feasibility of large-scale biofuel production: does an enlargement of scale change the picture ?", *Bioscience*, 47(9), 587600.

Growiec J. and I. Schumacher (2008), "On technical change in the elasticities of resource inputs"', *Resources Policy*, 33(4), 210-221.

Hall C., S. Balogh, D. Murphy (2009), "What is the minimum EROI that a sustainable society must have ?", *Energies*, 2, 25-47.

Heun M. and M. de Wit (2012), "Energy return on (energy) invested (EROI), oil prices, and energy transitions", *Energy Policy*, 40, 147-158.

IEA (2010), "Energy technology perspectives 2010 - scenarios & strategies to 2050"', International Energy Agency, OECD.

IEA (2013), "World energy outlook 2013", International Energy Agency, OECD.

Jouvet P.-A. and I. Schumacher (2011), "Learning-by-doing and the costs of a backstop for energy transition and sustainability", *Ecological Economics*, 73, 122-132.

King C. and C. Hall (2011), "Relating financial and energy return on investment", *Sustainability*, 3, 1810-1832.

Krysiak F. (2006), "Entropy, limits to growth, and the prospects for weak sustainability", *Ecological Economics*, 58(1), 182-191.

Meadows D., J. Randers and D. Meadows (2004), *Limits to growth. The 30-year update*, Chelsea Green Publishing Company.

Murphy D. and C. Hall (2010), "EROI or energy return on (energy) invested", *Annals of the New-York Academy of Sciences*, 1185, 102-118.

Solow R. (1974), "Intergenerational equity and exhaustible resources"', *The Review of Economic Studies*, 41, 2945.

Stiglitz (1974), "Growth with exhaustible natural resources : the competitive economy", *The Review of Economic Studies*, 41, 139-152.

Tahvonen O. and S. Salo (2001), "Economic growth and transitions between renewable and non-renewable energy resources", *European Economic Review*, 45(8), 1379-1398.

Tsur Y. and Zemel A. (2005), "Scarcity, growth and R&D", *Journal of Environmental Economics and Management*, 49(3), 484499.

van den Bergh J. (2013), "Environmental and climate innovation: Limitations, policies and prices", *Technological Forecasting & Social Change*, 80, 11-23.

Vermeulen O. (2013), "Impacts des limites de l'efficience énergétique sur la production économique", Ecole Polytechnique de Louvain, Université de Louvain.

7 Appendix

7.1 Households

In a first step, we suppose that T_e is given. After substituting C_t by its expression in (1), the Lagrangean of the decision problem writes:

$$\mathcal{L} = \sum_{t=1}^{T} \beta^t \frac{(v_t K_t + q_t E_t + \Omega_t - K_{t+1}/\varphi_t)^{1-\alpha} - 1}{1-\alpha} + \lambda \left[\mathcal{S} - \sum_{t=1}^{T_e} E_t \right]$$

where λ is a multiplier. The first order condition with respect to K_{t+1} gives:

$$-\frac{\beta^t}{\varphi_t C_t^\alpha} + \frac{\beta^{t+1}}{C_{t+1}^\alpha} v_{t+1} = 0, , \ t \in \{1, ..., T-1\},$$

with final condition $K_{T+1} = 0$. For any $t \in \{1, ..., T-1\}$, this condition is equivalent to (4).

The first order condition with respect to E_t writes

$$\frac{\beta^t}{C_t^\alpha} q_t - \lambda = 0 \Rightarrow C_t^\alpha = \frac{\beta^t q_t}{\lambda}.$$

Likewise, $C_{t+1}^\alpha = (\beta^{t+1} q_{t+1})/\lambda$. The ratio between the FOCs of period $t+1$ and t thus gives

$$\left[\frac{C_{t+1}}{C_t} \right]^\alpha = \beta \frac{q_{t+1}}{q_t}, \ t \in \{1, ..., T-1\}. \tag{49}$$

Combining (4) et (49), one obtains (5).

7.2 RE sector

After substituting f_t by its expression (9) into the objective (8), one obtains the following FOC for an interior maximum:

$$g_t = \left[\frac{\gamma}{b_t} \frac{q_t}{v_t} \right]^{\frac{1}{1-\gamma}}, t \in \{1, ..., T\}.$$

Hence, using (9),

$$f_t = b_t^{-\frac{1}{1-\gamma}} \left[\frac{\gamma q_t}{v_t}\right]^{\frac{\gamma}{1-\gamma}}$$

With N identical firms, the RE capital use at the sector level is

$$G_t = N g_t = \left[N^{1-\gamma} \frac{\gamma}{b_t} \frac{q_t}{v_t}\right]^{\frac{1}{1-\gamma}}. \tag{50}$$

Defining B_t as $B_t = b_t/N^{1-\gamma}$, one recasts (50) as (10). Similarly, the RE production at the sector level is given by

$$F_t = N f_t = [B_t]^{-\frac{1}{1-\gamma}} \left[\frac{\gamma q_t}{v_t}\right]^{\frac{\gamma}{1-\gamma}} = \frac{1}{B_t} \left[\frac{\gamma}{B_t} \frac{q_t}{v_t}\right]^{\frac{\gamma}{1-\gamma}}. \tag{51}$$

Using (50), the last equality gives $F_t = G_t^{\gamma}/B_t$ as in (11).

7.3 Stationary state

In a stationary state where $C_{t+1} = C_t = C_*$, (4) leads to (27). Given (21), it next implies (28).

Equations (18) and (19) imply $G_* = [B_* F_*]^{\frac{1}{\gamma}} = \frac{\gamma q_* F_*}{v_*}$ or, equivalently,

$$\frac{G_*}{F_*} = B_*^{\frac{1}{\gamma}} [F_*]^{\frac{1}{\gamma}-1} = \frac{\gamma q_*}{v_*}.$$

Using (27-28), this equality becomes (33) and (29) is easily obtained.

Equations (31)-(30) follow immediately from (16) and (20).

Equation (17) then implies $K_* = G_* + H_* = \gamma \left[\beta\varphi - \frac{1}{\zeta}\right] A_* F_* + \frac{A_* F_*}{\zeta}$ which leads to (34).

Finally, (15) leads to $C_* = A_* F_* - K_*/\varphi$, which can straightforwardly be rewritten as (35) by using (34).