Platform competition and vertical differentiation

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September 24, 2012

Abstract

We model platform competition in a market where products are characterized by cross network externalities. Consumers differ in their valuation of these externalities. Although the exogenous setup is entirely symmetric, we show that platform competition induces a vertical differentiation structure that allows for the co-existence of asymmetric platforms in equilibrium. We establish this result for the case of active and passive beliefs and show that in our setup, the case of active beliefs is formally equivalent to a model of Cournot competition.

Keywords: network externalities, two-sided market, cournot, vertical differentiation
JEL Classification: L13

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1 Introduction

In markets where products are subject to network externalities, the number of product's users determines, at least partially, the perceived quality of the product. As a result, consumers' willingness to pay for a product, say A, whose number of consumers exceeds that of a competing product B, is larger than that of B. Relying on the product differentiation literature, A and B should be considered as vertically differentiated products. By definition, vertical differentiation prevails if consumers would buy the same product should all of them be sold at equal prices. According to this definition, the presence of product's network externalities naturally induces vertical differentiation between competing products according to the size of their respective network of consumers. The recent literature on two-sided markets (see Rochet and Tirole (2006)) precisely builds on network externalities. The general idea is quite simple: a *product* is best viewed as a platform on which different groups of users *meet* or *trade*, ... The externalities that benefit to one group typically originate in the number of participants from the other group: network externalities cross from one side to the other. The example of credit cards is particularly adequate and simple to illustrate this situation. Consider two credit cards each issued by a different company.[1] As such, the pieces of plastic have no intrinsic value: their utility is derived only from the possibility of using them as a means of payment. Consequently, the larger the number of merchants accepting a specific credit card, the larger the utility of his/her owner and the higher its "quality", compared with the quality of the credit card issued by the rival company. Conversely, the larger the number of consumers holding a credit card, the higher the utility of the merchant accepting it and therefore the higher the willingness to pay for registering to the platform. Vertical differentiation seems thus endemic to the presence of consumption network externalities, which in turn suggests that models of vertical differentiation, as originally developed in Gabszewicz and Thisse (1979) or Shaked and Sutton (1972), could prove useful in modelling price competition in markets with network externalities. This class of models takes in particular as its primitive the fact that consumers are heterogeneous in their willingness to pay for quality and studies the extent to which the presence, and characteristics, of this heterogeneity affect equilibrium outcomes. This theoretical framework has been used to study competition in industries with simple, one-sided externalities (see in particular Bental and Spiegel (1995) or Baake and Boom(1999)). However, the two-sided market literature seems to have neglected it. Most applied papers indeed either assume that consumers are homogenoeus within groups or, if heterogeneous within group, that the dimension along which they are is orthogonal to the network effect (see Armstrong (2006) and Armstrong and Wright (200X) for representative models of this class).

As a first contribution, the present paper applies the (now) canonical model of Mussa and Rosen (1979) to analyze price competiton in a two-sided market with cross membership externalities. We start by illustrating the above concepts and assumptions in the framework of a two-sided market with a monopoly platform, then we turn to duopoly set-up, *in order to establish how the model of vertical product differentiation can be helpful in understanding the endogenous determination of network participation and, thus, the "quality" of the products exchanged on both sides.* We model a two-sided platform market with membership externalities, inspired by the Mussa and Rosen (1978) model of vertical product differentiation. In this set-up, the willingness to participate is exclusively related to the number of participants in the other side (no stand-alone value for the platforms). Furthermore, participants are heterogeneous in their

[1] We neglect in this example the complex intermediation structure that actually organizes the credit cards industry in order to focus on the casual exprerience end-users experiment with it

valuation of other side's participation size. Prices set by the platforms operators directly determine platforms' quality through the participation it elicits. In a duopoly context, asymmetric platforms then emerge as an equilibrium outcome.

As widely recognised today, a key difficulty in modelling platform competition lies in the fact that, given the fees selected by the platforms, consumers on both sides of the market have to coordinate their decisions among themselves. This follows from the fact that the "quality" perceived by each consumer (the number of participants in the other side of the market) is essentially a matter of beliefs and, accordingly, can only take the form of an expectation. The definition of equilibrium thus requires a consumers' coordination device to be at work, in order to balance the effective number of participants in one side of the market to the number obtained by aggregating, on the other side, the expectations (beliefs) of individuals about this number. As a result, equilibrium outcomes tend to depend on the particular assumptions one makes about beliefs' formation. Caillaud and Jullien (2002) and Hagiu (2006) very nicely illustrate the role of coordination rules in determining equilibrium outcomes.

Two polar cases can be identified, namely, the assumption of passive beliefs and that of active beliefs. *Passive* beliefs prevail when the platform sets its optimal strategy, conditional of consumers' beliefs, i.e. assuming that these beliefs are not affected by the observed prices.[2] The definition of equilibrium then only requires that, given the prices set by the platforms, the expectations of the agents on one side of the market about the resulting number of participants on the other side coincide with their *effective* number. This assumption corresponds to an assumption of self-fulfilling expectations. *Active* beliefs prevail when all the agents are assumed to incorporate, when deciding whether they participate or not, all of the feedback effects that characterize the two sided market. Active beliefs thus require full knowledge of all prices set by the platforms in both sides, as well as of the preferences of all agents in both sides. They also require that potential participants are endowed with sufficiently sophisticated reasoning skills to allow them to calculate the consequences of a price change along the whole chain of its effects through both sides of the market. In a recent paper, White and Weyl (2012) propose an alternative approach to solve the problem of two-sided competition, avoiding to rely on a specific consumers' coordination device. They propose a novel static solution concept that applies to platform competition: the *insulated equilibrium* concept. In this approach, platforms use pricing schemes not only for extracting revenue from the consumers, but also for coordinating them when choosing to participate or not. Although their approach proves to be very general and powerful, the intuition underlying their model is remarkably simple. These authors start from the fact that what really matters for the platform is the number of consumers willing to participate on each side of the market, i.e. the size of the networks. Since the willingness to participate on one side depends on the expectations about the participation size of the other, White and Weyl (2012) allow platforms to offer tariffs to potential participants of each side that simultaneously insure them against participation failures originating in the other side of the market. They solve thereby the coordination problem of the participants and ensure an optimal participation from their own viewpoint.

A second contribution of this paper is to directly compare equilibrium outcomes under passive or active beliefs. Unsurprisingly, active beliefs command for a wider participation of end-users. More interestingly, we show that, in our duopoly context, assuming active beliefs formally amounts to assume that platforms compete *a la Cournot*. More precisely, it will be shown that this approach amounts to consider the Cournot model obtained when inverting the demand functions obtained in the model with price strategies (Bertrand model) and passive beliefs. The

[2]After prices are set by the platform, end-user do not play a coordination game.

equilibrium of a Cournot model leaves a priori open the question of how consumers' coordinate their decisions at the market clearing prices. In our set-up however, it coincides with the equilibrium prices in the corresponding Bertand model with active beliefs. In this respect, the present paper can also be viewed as an application of White and Weyl (2012) to the case of a duopoly platform market with membership externalities.

In the next section, we present the model set-up. Then we characterize the optimal strategy of a monopolist under either passive and active beliefs and show how White and Weyl's insulated tariffs correspond to the case of active beliefs. The next section extends the analysis to a duopoly setup and establishes the existence of equilibria displaying asymmetric platforms.

2 The Setup

The basic ingredients of the model are drawn from the standard literature on vertical differentiation. The specification of preferences we retain here are those of Mussa and Rosen (1978). There are three types of agents:

- Platforms: they are denoted by i and sell product $i = 1, 2$. Product i is best viewed as a device that allows information exchange between agents. For the sake of illustration, we shall refer here to the exhibition centers' metaphor. Then one can think of product i as a commercial fair organized at an exhibition center i. Platforms are the organizers of the fairs in exhibition centers. They sell their product in two markets: the visitors' market and the exhibitors' market. The access permit paid by the visitors, as well as the rental fee paid to the platforms by exhibitors, allow visitors and exhibitors to meet and exchange information.

- Visitors: they are denoted by their type θ. Types are uniformly distributed in the $[0, 1]$ interval. The total number of visitors is normalized to 1. They possibly buy product $i = 1, 2$ according to a utility function $U_i = \theta x_i^e - p_i$, with x_i^e denoting the expectation visitors have about the number of exhibitors at platform i. Holding no access permit yields a utility level normalized to 0.[3]

- Exhibitors: they are denoted by their type γ. Types are uniformly distributed in the $[0, 1]$ interval. Their total number is normalized to 1. When they exhibit in center i, $i = 1, 2$, their utility is measured by $U_i' = \gamma v_i^e - \pi_i$, with v_i^e denoting the expectation they form about the number of visitors in center i. Refraining from exhibiting in any exhibition center yields a utility level normalized to 0.

Notice that our present setup is best viewed as a model where two vertically differentiated markets operate in parallel with the key feature that quality in one of the two markets is determined by the outcomes in the other market. Given some pair of prices, agents' participation on each side determines the perceived quality for the other side. Accordingly, when committing to network sizes, a platform jointly determines quality in each of the two sides.

We consider a stage-game where in the first stage platforms commit to uniform unit prices or some more general form of tariffs; in the second-stage, visitors and exhibitors decide of their participation. Our general equilibrium concept is subgame perfection.

[3]Multi-homing behaviour is rules out in the present model. See Gabszewicz and Wauthy (2004) for a comparable model with multi-homing.

3 The monopoly platform

Under the above set-up, demand from visitors can be expressed as a function of the fee, p_i, platform i charges to customers, and of the number ν_i of exhibitors expected to be present on the platform. Similarly, demand from exhibitors can be expressed as a function of the fee π_i opposed by the platform to exhibitors, as well as of the number x_i of visitors expected to participate. Now consider the simpler case in which there exists a *single* platform operating simultaneously on both sides of the market. In this case, the single (monopoly) platform selects the fee p (resp.π), to oppose to the potential visitors (resp. exhibitors). Given these fees, the utility obtained by a type-θ visitor (resp. a type-γ exhibitor) obtains as $U = \theta x^e - p$ (resp. $U' = \gamma v^e - \pi$). Solving these two equations in x^e and v^e, allows to identify the "marginal" exhibitor type γ (resp. visitor θ) who is the lowest type to buy at price π on the exhibitors' market (resp. price p on the visitors' market), given expectations x^e and v^e, namely,

$$\gamma = \frac{\pi}{\nu^e}; \theta = \frac{p}{x^e}.$$

Conditional on expectations, effective demands on both sides of the market then obtain as

$$v = 1 - \frac{p}{x^e} \quad (1)$$

$$x = 1 - \frac{\pi}{\nu^e} \quad (2)$$

3.1 Passive beliefs

Under passive beliefs, agents in both sides are partially myopic because, even if they understand that demand in their own market depends on its price and on the expectation about the number of active agents in the other market, they neglect the fact that it also depends on the price set on the other side. A natural way to justify this behavior is to assume that the agents of one side know the price on their own side, but ignore the price quoted in the other one. Given expectations x^e and ν^e, the objective of the platform is to maximize the function

$$\max_{p,\pi} p(1 - \frac{p}{x^e}) + \pi(1 - \frac{\pi}{\nu^e}).$$

The optimal strategies obtain as $p^*(x^e, v^e) = \frac{x^e}{2}$ and $\pi^*(x^e, v^e) = \frac{v^e}{2}$. Substituting these prices in the demand functions (1) and (2), the network sizes corresponding to these prices are equal to $\frac{1}{2}$. Requiring expectations to be fullfilled at equilibrium, we finally obtain

Proposition 1 *Under passive beliefs, the optimal strategy of the monopoly platform consists in setting both prices p^* and π^* equal to $\frac{1}{4}$.*

In this passive beliefs' equilibrium, the monopolist's payoff is equal to $\frac{1}{4}$. As for the participation of visitors and exhibitors, half of them (those with high $\theta-$ and $\nu-$ values) are embarked at equilibrium.

3.2 Active beliefs

An alternative route to solve the monopoly platform problem consists in keeping the fullfilled expectations assumption while endowing also potential participants with more sophisticated reasoning skills and better information. Suppose to this end that all agents on all sides observe the prices set by the platform on both sides and have perfect information on the preferences of every agent. Then, when they have to decide whether to participate or not, agents can incorporate all this information to determine the exact feedback effects of the platform's price decision: in other words, they form active beliefs. Notice that this assumption is by far more demanding in a context of multi-sided markets than in the traditional one sided market context with network effects, where the knowledge of price and own group preferences only is required. If we nevertheless endorse this assumption, we can develop the following theory to solve the monopoly platform problem.

Suppose the monopoly sets a pair of prices (p, π). Endowed with active beliefs, agents correctly anticipate that, corresponding to these prices, there exists a unique pair of network sizes $(v,\ x)$ solving the system (1)-(2). Rearranging these expressions we get

$$\begin{aligned} \pi &= v(1-x); \\ p &= x(1-v). \end{aligned}$$

The optimal monopoly solution must satisfy these two conditions simultaneously. Notice that these formulae completely capture the network externalities that link the two sides of the market: a higher v allows the platform to benefit from a higher value of p on the x - side of the market. These equations could be understood as a kind of cournotian inverse demands system: they define indeed, for all possible participation levels on the two sides, the highest prices that are compatible with those levels. In this sense they define market clearing prices. Now the objective of the platform writes as

$$\max_{v,x} v\left[x(1-v)\right] + \pi\left[v(1-x)\right].$$

First order conditions yield the system

$$\begin{aligned} x(2-x-2v) &= 0; \qquad (3) \\ v(2-v-2x) &= 0. \qquad (4) \end{aligned}$$

This system has two solutions: $x = v = 0$ and $x^* = v^* = \frac{2}{3}$, sustained by prices $p^* = v^* = \frac{2}{9}$. Summing up, we have established the following

Proposition 2 *Under active beliefs, the optimal solution for the monopoly platform consists in charging the pair of prices* $p^* = v^* = \frac{2}{9}$.

It is immediate to see that participation is larger under active beliefs than under passive beliefs. Needless to say, the assumption on agents' information necessary to reach this optimum is by far too demanding and, as a consequence, makes it quite implausible[4]. This is why we turn to the approach proposed by White and Weyl (2012), who show how platforms can solve by themselves the coordination problem faced by end-users, provided they are allowed to rely on more flexible pricing strategies.

[4]One can hardly imagine that potential exhibitors take into consideration the subscription fee paid to the platform by the visitors when deciding to participate or not!

3.3 Network size-secured tariffs

One way to get rid of specific assumptions about beliefs consists in assuming that the platform proposes tariffs to potential participants that insure them against defection from the other side. The platform will therefore offer tariffs that depend on effective participation and choose tariff levels in such a way that the desired participation level is guaranteed because the tariff is such that the marginal type on one side exactly defines the desired participation level on that side, independently of the expected participation on the other side. How to proceed? Suppose the platform wants to guarantee a level of demand equal to n_x on the market of exhibitors and to n_v on the market of visitors. To realize this program, it is sufficient to propose tariffs t and τ such that, at these tariffs, we obtain $1-z=n_v$ and $1-y=n_x$, with the visitor (resp. exhibitor) with type z (resp.y) being indifferent between participating or not. For this, we need

$$t = zx^E,$$

where E stands for effective participation. Since $z=\frac{t}{x^E}$ and type z is indifferent between participating or not (see equation (1) above), all types larger than z enjoy a strictly positive surplus and types lower than z do not register. Similarly,

$$\tau = yv^E :$$

$y=\frac{\tau}{v^E}$ and y is indifferent between participating or not. Furthermore, given the targeted sizes, we must also have

$$x^E = 1-y$$

and

$$v^E = 1-z.$$

Combining the four above equations, the payoff function of the monopoly platform$\pi(\cdot)=tn_v+\tau n_x$ can be rewritten as

$$\begin{aligned} t(1-z)+\tau(1-y) &= \\ &= zx^E(1-z)+yv^E(1-y) \\ &= z(1-y)(1-z)+y(1-z)(1-y) \\ &= (1-y)(1-z)(z+y). \end{aligned}$$

Optimizing the profit with respect to z and y, we get

$$z^* = y^* = \frac{1}{3},$$

leading to equilibrium network sizes

$$n_v^* = n_x^* = 1-z^* = 1-y^* = \frac{2}{3}$$

and effective equilibrium tariffs levels, as paid by participants,

$$t^* = \tau^* = y^*v = z^*x^* = \frac{2}{9}.$$

Thus we conclude that

Proposition 3 *The network size- secured tariffs maximizing the profits of the monopoly platform coincide with the optimal monopoly solution under active beliefs.*

4 Duopoly Competition

4.1 Passive beliefs

Now suppose that *two* platforms operate on the two-sided market and let us start again with the expression of demand for participation from both sides, defined as a function of the expected participation on the other side. We assume first passive beliefs. Consider demands addressed to these two platforms by the exhibitors, with v_i^e denoting the expectation exhibitors have about the number of visitors at platform i, and π_i the price paid by the exhibitors to accept the card issued by platform i. These demands clearly depend on exhibitors' expectations (v_1^e, v_2^e) about the number of visitors using card 1 and 2, respectively. Assume $v_2^e > v_1^e$; then we get

$$D_1^x(\pi_1, \pi_2) = \frac{\pi_2 v_1^e - \pi_1 v_2^e}{v_1^e(v_2^e - v_1^e)}$$

$$D_2^x(\pi_1, \pi_2) = 1 - \frac{\pi_2 - \pi_1}{v_2^e - v_1^e}$$

These are the demand functions of a vertical differentiation model with quality products defined exogenously by $v_2^e > v_1^e$. A similar demand specification $D_i^v(p_1, p_2)$ can be defined for the visitors' market, given expectations $x_2^e > x_1^e$ visitors have about the number of exhibitors accepting the cards1 and 2, respectively. Conditional on expectations v_1^e, v_2^e, $v_2^e > v_1^e$, and $x_1^e, x_{2,}^e x_2^e > x_1^e$, the payoff functions of platform i is then derived as

$$p_i D_i^v(p_1, p_2) + \pi_i D_i^x(\pi_1, \pi_2), i = 1, 2.$$

Formally, we define a Nash equilibrium in the two-sided market duopoly as follows:[5] *A Nash Equilibrium is defined by two quadruples* (p_i^*, π_i^*) *and* (v_i^*, x_i^*) *with* $i = 1, 2$, *such that (i) given expectations* $(v_1^*, v_2^*, x_1^*, x_2^*)$, $\left(p_{i,}^* \pi_i^*\right)$ *is a best reply against* $(p_{j,}^* \pi_j^*)$, $i \neq j$, *and vice-versa ;(ii)* $D_i^v(p_1^*, p_2^*) = x_i^*$; $D_i^x(\pi_1^*, \pi_2^*) = v_i^*$, , $i = 1, 2$.

This definition allows firms to deviate simultaneously in the two components of the strategies at their disposal. Obviously, it also implies that, at a Nash equilibrium of the two-sided market, each pair of prices (p_1^*, p_2^*) , (π_1^*, π_2^*) defines as well a price equilibrium in its respective market. Notice that, due to the assumption of passive beliefs, when the two pairs (p_1^*, p_2^*) and (π_1^*, π_2^*) define each a price equilibrium in the visitors' and exhibitors' market respectively, the pair of strategies (p_1^*, π_1^*) , (p_2^*, π_2^*) must also satisfy condition *(i)* in the above definition of equilibrium.

We now derive the price equilibrium on the exhibitors' market, conditional on expectations $v_1^e < v_2^e$:

$$\pi_2(v_1^e, v_2^e) = \frac{2v_2^e(v_2^e - v_1^e)}{4v_2^e - v_1^e}$$

$$\pi_1(v_1^e, v_2^e) = \frac{v_1^e(v_2^e - v_1^e)}{4v_2^e - v_1^e},$$

with corresponding demands:

$$D_2^x(v_1^e, v_2^e) = \frac{2v_2^e}{4v_2^e - v_1^e}$$

$$D_1^x(v_1^e, v_2^e) = \frac{v_2^e}{4v_2^e - v_1^e}.$$

[5] This definition essentially extends the definition of Katz and Shapiro (1984) to a context of multsided market.

Obviously, the symmetry of our model allows us to directly infer the price equilibrium, conditional on expectations, $x_2^e > x_1^e$, on the visitors' market. We obtain

$$D_2^v(x_1^e, x_2^e) = \frac{2x_2^e}{4x_2^e - x_1^e}$$

$$D_1^v(x_1^e, x_2^e) = \frac{x_2^e}{4x_2^e - x_1^e}.$$

Then it remains to solve the model for fullfilled expectations, i.e. condition *(ii)* in the above definition of a Nash equilibrium. This is done by solving the system

$$x_2 = \frac{2D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}$$

$$x_1 = \frac{D_2^v(x_1, x_2)}{4D_2^v(x_1, x_2) - D_1^v(x_1, x_2)}.$$

Straighforward computations yield $x_1^* = v_1^* = \frac{2}{7}$ and $x_2^* = v_2^* = \frac{4}{7}$, and corresponding prices $\pi_1^* = p_1^* = \frac{2}{49}$, $\pi_2^* = p_2^* = \frac{8}{49}$.

Proposition 4 *The presence of heterogeneity on both markets allows for an interior equilibrium where both platforms enjoy strictly positive networks and profits.The quadruples ($x_1^* = v_1^* = \frac{2}{7}$, $x_2^* = v_2^* = \frac{4}{7}$) and ($\pi_1^* = p_1^* = \frac{2}{49}$, $\pi_2^* = p_2^* = \frac{8}{49}$) define the unique (up to permutation) interior equilibrium.*

The last proposition clearly illustrates the links that relate markets with cross network externalities and vertically differentiated industries. When setting different prices, platforms actually attract different types of agents on both sides of the market and thereby fix the size of the networks. In equilibrium, the size of the network endogenously determines the willingness of the consumers to participate in one of the two platforms. When the population of agents on both sides of the market is heterogeneous in its willingness to pay for network sizes, asymmetric equilibria naturally emerge. In these equilibria, the two platforms are clearly ranked by size but nevertheless enjoy positive market shares and profits. On each side of the market, equilibrium outcomes resemble those obtained in standard models of vertical differentiation: one firm is perceived by all agents as better than the other but not all agents register to that firm because of the price differential. The relative sizes of the platform can thus be viewed as the qualities attached to these platforms, so that by playing on agents' heterogeneity, a dominated platform can survive by charging lower prices, without inducing the dominant platform to price aggressively and preempt the market. A key difference with standard models of vertical differentiation is obviously that realized qualities are not directly controlled by the firms but depend on their prices and on the network externalities that cross from one side of the market to the other.

4.2 Active beliefs and network size-secured tariffs

Let us then proceed to the analysis of optimal platforms' strategies under active beliefs. Using the specification of the consumers' preferences, we build the reduced form associated with an equilibrium configuration where $x_2 > x_1$ and $v_2 > v_1$ given some quadruple of prices. We have to satisfy simultaneously:

$$v_2 = \frac{x_2 - x_1 - p_2 + p_1}{x_2 - x_1} \qquad (5)$$

$$v_1 = \frac{x_2 p_1 - x_1 p_2}{x_1(x_2 - x_1)} \tag{6}$$

$$x_2 = \frac{v_2 - v_1 - \pi_2 + \pi_1}{v_2 - v_1} \tag{7}$$

$$x_1 = \frac{v_2 \pi_1 - v_1 \pi_2}{v_1(v_2 - v_1)}. \tag{8}$$

Taken side by side, these expressions can be inverted to express corresponding prices as a function of the expected participation size on the other side and participation levels on the current side. On the visitors' side for instance, one immediately gets:

$$p_2 = x_2(1 - v_2) - x_1 v_1 \tag{9}$$

$$p_1 = x_1(1 - v_2 - v_1). \tag{10}$$

These expressions are nothing else but the Cournotian system of inverse demands that would prevail in a vertically differentiated market where firm 1 and 2 would sell products of quality $x_1 < x_2$.

Comparable expressions obtain for the exhibitors' side, holding fixed the coarse allocation on the visitors' side:

$$\pi_2 = v_2(1 - x_2) - x_1 v_1 \tag{11}$$

$$\pi_1 = v_1(1 - x_2 - x_1) \tag{12}$$

If we assume that platforms are able to commit to network sizes, the above expression can be understood as the highest prices quadruple at which the committed sizes would realize provided end-users' behaviour is coordinated so as to take all of the feed-back loops into account. This case corresponds to the assumption of active beliefs. Formally speaking, we may then characterize optimal network sizes by solving the following Cournot game between platforms. We have

$$\Pi_1 = v_1 x_1(2 - v_1 - v_2 - x_1 - x_2) \tag{13}$$

$$\Pi_2 = x_2 v_2(2 - v_2 - x_2) - x_1 v_1(v_2 + x_2) \tag{14}$$

Maximizing over v_i, x_i, first order conditions are:

$$v_1 = \frac{2 - x_1 - x_2 - v_2}{2} \tag{15}$$

$$x_1 = \frac{2 - v_1 - v_2 - x_2}{2} \tag{16}$$

$$v_2 = \frac{2 - x_2}{2} - \frac{v_1 x_1}{2x_2} \tag{17}$$

$$x_2 = \frac{2 - v_2}{2} - \frac{v_1 x_1}{2v_2} \tag{18}$$

Solving the system of equations above we obtain two quadruples of interior solutions, but only one of them satisfies the required hierarchy $x_2 > x_1$ and $v_2 > v_1$. Namely:

$$x_1^* = v_1^* = \frac{2}{31}(6 - \sqrt{5}) \cong .242,$$

$$x_2^* = v_2^* = \frac{1}{31}(13 + 3\sqrt{5}) \cong .636.$$

Proposition 5 *Under active beliefs, there exists a duopoly equilibrium in network size strategies at which one platform dominates the other in the two markets, while both platforms enjoy positive profits. Equilibrium sizes are given by* $x_1^* = v_1^* = \frac{2}{31}(6-\sqrt{5})$ *and* $x_2^* = v_2^* = \frac{1}{31}(13+3\sqrt{5})$

Let us show now that the above characterization can be rationalized as the equilibrium of a game where firms rely of the insulated tariffs proposed by White and Weyl (2012). Let us think of building insulated tariffs starting from the demand functions. Suppose platform 2 wants to insure v participants against defection on the other side.

Using (5) and (6) we may rewrite:

$$p_2 = v_2(x_2 - x_1) + p_1$$

$$p_1 = (v_1 x_1(x_2 - x_1) + x_1 p_2)\frac{1}{x_2}$$

Using (7) and (8) we may rewrite:

$$\pi_2 = x_2(v_2 - v_1) + \pi_1 v_1$$

$$\pi_1 = (x_1 v_1(v_2 - v_1) + v_1 \pi_2)\frac{1}{v_2}$$

Each of these expressions can be understood as follows: Viewed from side i, given the network allocation on side j and the other platform's price on side i, what is the price that would maintain participation to the platform unchanged on side i? These expressions can be transformed to define insulating tariffs, i.e. tariffs that insulate participants' decision on one side from participation decisions on the other side. The intutition underlying such tariffs is the following: the platform proposes a tariff that endows participants with a dominant strategy in terms of (non)participation decisions (i.e. the decision does not depend on other side's participation's decision) such that the desired level of participation is ensured.

Suppose we want to secure the presence of k visitors on platform 2 and l visitors on platform 1. We know that $k = 1 - \tilde{x}$ and $l = \tilde{x} - \overline{x}$ with $\tilde{x} = \frac{\pi_2 - \pi_1}{v_2 - v_1}$ and $\overline{x} = \frac{\pi_1}{v_1}$ must be statisfied. A pair of tariffs set at

$$\pi_2 = \pi_1 + (1-k)(v_2 - v_1)$$

$$\pi_1 = v_1 \overline{x}$$

has the property that it leaves participants with type $\tilde{x}$ and $\overline{x}$ exactly indifferent between participating to platform 2 and 1 for the first, and between participating to platform 1 or no to participate for the second, whatever the values of (v_1, v_2). These expressions can thus be viewed as insulated tariffs. We must therefore satisfy

$$\pi_2 = \overline{x} v_1 + \tilde{x}(v_2 - v_1)$$

$$\pi_1 = \frac{\pi_1}{v_1}.$$

Recall then that relying on participation decisions as a function of types we also have that

$$\overline{x} = 1 - x_1 - x_2$$

$$\tilde{x} = 1 - x_2.$$

Plugging these expressions into the previous ones, we end up with

$$\pi_2 = v_2(1 - x_2) - x_1 v_1 \tag{19}$$

$$\pi_1 = v_1(1 - x_2 - x_1). \tag{20}$$

These expressions are of course strictly identical to the inverse demand functions established in equations (11) and (12). As a result, insulated tariffs that would result from platforms' optimal behaviour would induce the participation levels and the asymmetric structure displayed in Proposition 5.

5 Final Remarks

In market with cross network externalities, it is often the case that participants within each group differ in their valuation of the externality. When this is the case, we have shown that the vertical differentiation setup that has become standard in the IO literature offers a very natural vehicle to model platform competition in two-sided markets. In a market with membership externalities, prices set by the firms elicit participation on either side and thereby simultaneously determine platform quality. Because participants are heterogeneous, asymmetric platforms, i.e. platforms with different sizes, co-exist in the market. It would be interesting to explore the applicability of this set-up to more general types of externalities and more general tariffs.

Because our theoretical model is simple, it is very easy to compare equilibrium outcomes depending on the way consumers coordinate their behaviour given prices. In particular, we have compared equilibrium under passive and active beliefs. The case of active beliefs endow participants with all the information and capabilities that allow them to internalize all the feedback loops. In other words, active beliefs ensure the largest possible participation, given prices. Interestingly enough, our results show that this is strictly equivalent to assume a form a Cournot competition whereby firms would commit to network sizes and leave to a Walrasian auctionneer the task of clearing the two-sides of the market. This result should be put in line with the approach proposed in White and Weyl (2012) where platforms are shown to be able to enforce such a coordination provided they rely on insulated tariffs. The links between their concept of insulated tariffs and Cournot competition should be explored in further details.

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