Bargaining and Delay in Trading Networks [†]

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Abstract

We study a model in which heterogenous agents first form a trading network where link formation is costless. Then, a seller and a buyer are randomly selected among the agents to bargain through a chain of intermediaries. We determine both the trading path and the allocation of the surplus among the seller, the buyer and the intermediaries at equilibrium. We show that a trading network is pairwise stable if and only if it is a core periphery network where the core consists of all weak (or impatient) agents who are linked to each other and the periphery consists of all strong (or patient) agents who have a single link towards a weak agent. Once agents do not know the impatience of the other agents, each bilateral bargaining session may involve delay, but not perpetual disagreement, in equilibrium. When an agent chooses another agent on a path from the buyer to the seller to negotiate bilaterally a partial agreement, her choice now depends both on the type of this other agent and on how much time the succeeding agents on the path will need to reach their partial agreements. We provide sufficient conditions such that core periphery networks are pairwise stable in presence of private information.

Keywords: Bargaining; Trading networks; Private information.

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1 Introduction

We are interested in markets where trades between a buyer and a seller can occur through intermediaries and where each agent can be some day the buyer, some other day the seller, and the day after acting as an intermediary.[1] In such cases it is natural to model the market using a network where only pairs of connected agents may engage in trade. Examples of such trading networks are over-the-counter financial markets, housing markets, markets for antiques, energy markets, among others. Which trading networks are likely to emerge when agents can be either patient or impatient and the division of the surplus between the seller, the buyer and the intermediaries is determined through a series of bilateral bargaining sessions? How private information affects the trading path, the division of the surplus and the stability of trading networks?

To answer these questions we study a model where agents having different discount rates first form a trading network. Second, a seller and a buyer are randomly selected among the agents. The seller owns an indivisible good and the buyer has a valuation normalized to one for the good. The buyer can obtain the good from the seller if and only if they are connected to each other. Agents on a given path between the seller and the buyer can act as intermediaries if trade occurs along this path. Third, the trading path and the allocation of the surplus among the seller, the buyer and the intermediaries are determined as follows. The buyer first chooses one of her predecessors, say the first intermediary, on a path from the buyer to the seller to negotiate bilaterally a partial agreement. Each bilateral negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. Once a partial agreement is reached, the buyer exists the game and the first intermediary chooses one of her predecessors, say the second intermediary, on a path from the first intermediary to the seller. Once a partial agreement is reached between the first intermediary and the second intermediary, the first intermediary exists the game; and so on until a partial agreement is reached between the last intermediary and the seller. Each agent receives her share of the surplus once all partial agreements have been reached.

Suppose that the population of agents is partitioned in two types of agents: weak agents (or impatient agents) and strong agents (or patient agents). Our main result is that a trading network is pairwise stable[2] if and only if it is a core periphery network where the core consists of all weak agents who are linked to each other and

[1] See Goyal (2007), Jackson (2008), Easley and Kleinberg (2010) for a comprehensive introduction to the theory of social and economic networks.

[2] A trading network is pairwise stable if no agent benefits from severing one of her links and no other two agents strictly benefit from adding a link between them.

the periphery consists of all strong agents who have a single link towards a weak agent.[3] Intuitively, agents have first incentives to create and share some surplus, and so any pairwise stable trading network consists of only one component connecting all agents. Agents have also incentives to occupy a position in the trading network to extract more rents from intermediation. In addition, agents have incentives to negotiate a partial agreement with a weaker agent to exit with a larger share of the surplus. Hence, in any pairwise stable trading network each agent is linked to a weak agent. Each weak agent will also try to circumvent intermediaries to obtain more of the surplus for her. It follows that in any pairwise stable trading network all weak agents are linked to each other. Each strong agent will then destroy links to other strong agents because those links are never used or are harmful (more intermediaries lie on the trading path when she is the seller). Finally, in any pairwise stable trading network each strong player is linked to exactly one weak player to avoid sharing the surplus with more intermediaries when she is the seller. Thus, core periphery networks are the unique pairwise stable trading architectures. When we have more than two types of agents and agents can be ranked in terms of their discount rates, a star network with the weakest agent being the center is pairwise stable. Once agents become homogenous, there is a unique pairwise stable architecture, namely the complete network.

Finite series of bilateral bargaining sessions with complete information predict efficient outcomes of the bargaining process. In particular, each partial agreement is always reached immediately, so that delay cannot occur in equilibrium. This is not the case once we introduce private information into bargaining, in which the first rounds of negotiation are used for information transmission between the two parties.

Once agents do not know the discount rate of the other agents, each bilateral bargaining session may involve delay, but not perpetual disagreement, in equilibrium. In fact, delay can occur even when the game is close to one of complete information. We find that the maximum delay time in reaching an agreement can be substantial and is increasing with the amount of private information. Hence, when an agent chooses another agent on a path from the buyer to the seller to negotiate bilaterally a partial agreement, her choice now depend both on the type of this other agent and on how much time the succeeding agents will need to reach their partial agreements. We provide sufficient conditions such that core periphery networks are still pairwise

[3] Craig and von Peter (2009) have provided empirical evidence for a core-periphery structure in the German banking system. Interbank markets are tiered rather than flat, in the sense that most banks do not lend to each other directly but through money center banks acting as intermediaries.

stable when agents can be either strong or weak and do have private information about their discount rate. Core periphery networks are likely to be pairwise stable if all agents do not have too much private information and weak agents are quite more impatient than strong agents. Otherwise, agents may prefer to add links for reducing the length of trading paths and so avoiding costly delays in reaching a global agreement.

Most of the literature on decentralized trade in networks has focused on the exchange of goods in exogenously given networks with random matching, complete information and no intermediation. See, among others, Abreu and Manea (2012), Calvo-Armengol (2003), Corominas-Bosch (2004), Manea (2011) and Polanski (2007).[4] There are a number of papers where asymmetric information is present and/or link formation is endogenous. Kranton and Minehart (2001) have studied why buyers and sellers form bilateral links on which they mutually agree when buyers' valuations are private information and no intermediation takes place.[5] Elliott (2012) has examined the formation of buyer-seller networks when buyers and sellers need to make relationship specific investment to enable trade and when gains from trade are heterogeneous. Condorelli and Galeotti (2012a) have considered a model of sequential bargaining for a simple good when agents are located in a given network, agents' valuations are private information (high or low monetary valuation), and resale can take place. In each period the owner of the good makes a take-it-or-leave-it offer to one of the agents she is linked to. This agent either accepts or rejects the offer. When an agent becomes a new owner, she can either consume the good or resale it to another agent she is linked to in the next period. Once the good is consumed the game ends.

Recently, Condorelli and Galeotti (2012b) have investigated the effects of a class of trading protocols where the trade surplus is shared entirely between the initial owner of the good and the final buyer (i.e. intermediation rents are absent) on the architecture and efficiency properties of endogenously formed trading networks. Agents form costly links and a single good is randomly assigned to one of them. Then, valuations for the good are independently drawn and trade takes place. When the trading outcome is efficient and gives no intermediation rents, equilibrium and

[4] Blume, Easley, Kleinberg and Tardos (2009) have analyzed a complete information model where buyers and sellers are connected through intermediaries who strategically choose bid and ask prices to offer to the sellers and buyers they are connected to, and where the exogenously given network structure determines the amount of competition among intermediaries.

[5] Wang and Watts (2006) have considered the case when sellers can produce goods of a different quality. Mauleon, Sempere-Monerris and Vannetelbosch (2011) have studied the endogenous formation of networks between manufacturers of differentiated goods and multi-product retailers.

efficient networks are either minimally connected (when the link cost is small) or empty (when the link cost is large). However, a tension between equilibrium and efficient networks emerges when the cost of forming a link is intermediate.

The papers most closely related to our work are Babus (2012), Goyal and Vega-Redondo (2007). Babus (2012) has studied over-the-counter markets where bargaining along a given trading path consists of a finite series of bilateral bargaining sessions with a common discount factor.[6] If links are costly and agents are far-sighted, then a star network that connects all agents is an absorbing state of a dynamic network formation process. In Goyal and Vega-Redondo (2007), agents are homogenous and the surplus is shared equally among the buyer, the seller and the essential intermediaries. An intermediary is essential if she lies on all paths between the seller and the buyer.[7] If the formation of links is costly, a star network where a single agent acts as an intermediary for all transactions and enjoys significantly higher payoffs is the unique non-empty equilibrium architecture.

We go further their analysis by considering heterogeneous agents and allowing them to hold private information about their bargaining strength. In addition, we endogenize the trading path and we find that a core-periphery architecture emerges even when link formation is costless.[8]

The paper is organized as follows. In Section 2 we introduce trading networks. In Section 3 we consider the series of bilateral bargaining sessions with complete information and we determine both the equilibrium trading path and the equilibrium shares of the surplus to be divided. In Section 4 we characterize the pairwise stable trading networks. In Section 5 we consider the bargaining with private information. In Section 6 we conclude.

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[6] Gofman (2011) has studied a reduced-form model of bargaining in over-the-counter markets which are modeled as trading networks. Gale and Kariv (2009) have done an experimental study of trading networks where each trader can only exchange assets with a limited number of other traders and intermediation is used to transfer the assets between initial and final owners.

[7] This way of dividing the surplus implicitly assumes that bargaining is multilateral rather than consisting of a series of bilateral bargaining sessions. In addition, some intermediaries will get no surplus because they are not essential even though they may become essential once trade and exchange reach some intermediary on a path between the seller and the buyer.

[8] Core periphery networks also emerge in other models of network formation. For instance, Hojman and Szeidl (2008) have studied a model of communication network formation where connections do not require mutual consent and where the benefits from connections exhibit decreasing returns and decay with network distance. A star network where the center maintains no links and earns a high payoff and all other agents maintain a single link to the center and earn lower payoffs is the unique equilibrium architecture.

2 Trading Networks

Players are the nodes and links indicate bilateral relationships between players. Let N denote the set of n players or nodes, $N = \{1, 2, \ldots, n\}$. Let L denote the set of all possible links, $L = \{(i,j) \mid i \neq j \in N\}$. An undirected network $g \subseteq L$ is a set of links such that $(j,i) \in g$ whenever $(i,j) \in g$.[9] We use the shorthand $ij \in g$ instead of $(i,j) \in g$ to indicate that i and j are linked under the network g. Let G be the set of all possible networks on N. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network that results from deleting link ij from an existing network g is denoted $g - ij$. Let $N(g) = \{i \mid \exists\, j$ such that $ij \in g\}$ be the set of players who have at least one link in the network g. A path in a network g between i and j is a sequence of players $i_1, i_2, \ldots, i_{K-1}, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \ldots, K-1\}$ with $i_1 = i$ and $i_K = j$, and such that each player in the sequence $i_1, ..., i_K$ is distinct. We say that player i is connected in g to j if there is a path between i and j in g. A subnetwork $h \subseteq g$ is a component of g, if for all $i \in N(h)$ and $j \in N(h) \setminus \{i\}$, there exists a path in h connecting i and j, and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. We denote by $C(g)$ the set of components of g.

Players participate in the market and can be active either as a seller or as a buyer or as an intermediary. A pair of players is randomly selected. The probability that the pair (s, b) is selected, where s is the seller and b is the buyer, is $1/\left(n\left(n-1\right)\right)$. The seller owns an indivisible good and the buyer has a valuation $v = 1$ for the good. The buyer can obtain the good from the seller if and only if the two are connected. In other words, the buyer and the seller can trade the good if and only if they belong to the same component. Players on a path between the seller and the buyer can act as intermediaries if trade occurs along the path. When there is no path between the randomly selected pair, no surplus will be realized and both players receive 0. One central question is how the surplus is shared between the buyer, the seller and the intermediaries when trade is feasible.

3 Bargaining with Complete Information

Suppose that (s, b) is a pair randomly matched with s being the seller and b being the buyer and the sequence $(i_1, i_2, ..., i_k)$ are the intermediaries that facilitate the transaction in this order. The sequence $(s, i_1, i_2, ..., i_k, b)$ forms a path connecting

[9] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, $\#$ will refer to the notion of cardinality.

seller s to buyer b. Players along this path negotiate how to split the surplus via successive bilateral bargaining sessions.[10] Without loss of generality, we assume that players bargain in the following order: (i_k, b), (i_{k-1}, i_k), (i_{k-2}, i_{k-1}), ..., (i_1, i_2), (s, i_1) when s is the seller and b is the buyer.[11] In each bilateral bargaining session (i, j), the negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model where players make alternate offers, with i making offers in even-numbered periods and j making offers in odd-numbered periods. The length of each period is Δ. The negotiation starts in period 0 and ends when one of the players accepts an offer and leads to a partial agreement. A partial agreement specifies the share of the surplus for j to exit the game. No limit is placed on the time that may be expended in bargaining and perpetual disagreement is a possible outcome. Players have time preferences with constant discount rates, $r_1 > 0$, $r_2 > 0$, ... $r_n > 0$. After having reached a partial agreement with player j, player i negotiates with her predecessor her share of the remaining surplus to exit the game; and so forth until all $(k + 1)$ bargaining sessions end in a partial agreement. An outcome consists of $(k+1)$ partial agreements that specify player i's share of the surplus, $0 \leq x_i \leq 1$, for $i \in \{s, i_1, i_2, ..., i_k, b\}$, such that $x_s + x_{i_1} + ...x_{i_k} + x_b = 1$. Each player only receives her share once all $(k + 1)$ partial agreements have been reached. Under complete information it does not matter in our model whether a player can exit and obtains her share immediately or only at the end of the process. Players anticipate that an agreement will be reached immediately in all subsequent bilateral negotiations.

As the interval between offers and counteroffers shortens and shrinks to zero, there is a unique limiting subgame perfect equilibrium (SPE) outcome given by (see Appendix A for details):

[10] Multi-agent bilateral bargaining models consist of a potentially infinite series of bilateral bargaining sessions. In each session, two players bargain for a partial agreement that specifies who exits and who moves on to the next session (if there is any) via the alternating-proposal framework of Rubinstein (1982). See e.g. Suh and Wen (2009). Here, the bargaining consists of a finite series of bilateral negotiations along the trading path and the identity of the player who can exit in each bilateral sessions is determined by the trading path, two features which are more realistic for the markets we are interested in.

[11] Remember that each player has the same probability of being the seller or the buyer. In addition, all the results we obtain are robust to the alternative order (i_1, s), (i_2, i_1), ..., (i_{k-1}, i_{k-2}), (i_k, i_{k-1}), (b, i_k), where first the seller negotiates with an intermediary i_1.

$$
\begin{aligned}
x_b^* &= \frac{r_{i_k}}{r_{i_k}+r_b}, \\
x_{i_k}^* &= \frac{r_{i_{k-1}}}{r_{i_{k-1}}+r_{i_k}}\left(1-\frac{r_{i_k}}{r_{i_k}+r_b}\right), \\
x_{i_{k-1}}^* &= \frac{r_{i_{k-2}}}{r_{i_{k-2}}+r_{i_{k-1}}}\left(1-\frac{r_{i_{k-1}}}{r_{i_{k-1}}+r_{i_k}}\right)\left(1-\frac{r_{i_k}}{r_{i_k}+r_b}\right), \\
&\vdots \\
x_{i_1}^* &= \frac{r_s}{r_s+r_{i_1}}\left(1-\frac{r_{i_1}}{r_{i_1}+r_{i_2}}\right)\left(1-\frac{r_{i_2}}{r_{i_2}+r_{i_3}}\right)...\left(1-\frac{r_{i_{k-1}}}{r_{i_{k-1}}+r_{i_k}}\right)\left(1-\frac{r_{i_k}}{r_{i_k}+r_b}\right), \\
x_s^* &= \left(1-\frac{r_s}{r_s+r_{i_1}}\right)\left(1-\frac{r_{i_1}}{r_{i_1}+r_{i_2}}\right)\left(1-\frac{r_{i_2}}{r_{i_2}+r_{i_3}}\right)...\left(1-\frac{r_{i_{k-1}}}{r_{i_{k-1}}+r_{i_k}}\right)\left(1-\frac{r_{i_k}}{r_{i_k}+r_b}\right),
\end{aligned} \tag{1}
$$

where s is the seller, b is the buyer and the sequence $(i_1, i_2, ..., i_k)$ are the intermediaries that facilitate the transaction in this order. All $(k+1)$ partial agreements are reached immediately so that delay cannot occur in equilibrium.

Since for a given network g there may exist more than one path connecting s and b, we need to determine which sequence $(i_1, i_2, ..., i_k)$ of intermediaries between s and b is going to emerge at equilibrium. The game proceeds as follows. Buyer b first chooses one of her predecessors, say intermediary i_k, on a path from b to s to negotiate bilaterally a partial agreement. Each bilateral negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. Once a partial agreement is reached, b exists the game and i_k chooses one of her predecessors, say intermediary i_{k-1}, on a path from i_k to s such that b does not lie on the path. Once a partial agreement is reached between i_k and i_{k-1}, i_k exists the game. Then, i_{k-1} chooses one her predecessors, say intermediary i_{k-2}, on a path from i_{k-1} to s such that i_k and b do not lie on the path. Once a partial agreement is reached between i_{k-1} and i_{k-2}, i_{k-1} exists the game; and so on until a partial agreement is reached between i_1 and s. In case a player is indifferent between two or more predecessors, we assume that she will choose to negotiate with the predecessor leading to the shortest path between the seller and herself, anticipating perfectly the behavior of the other players.[12]

[12] In case there are more than one predecessor leading to the shortest path between the seller and herself, then she will choose them with equal probability. One motivation for choosing the shortest path to break ties may be the existence of a small risk of breakdown for links.

Proposition 1. *The path* $(s, i_1, i_2, ..., i_k, b)$ *in* g *is an equilibrium trading path if and only if*

(**i**) *for each player* $j \neq s$ *in* $(s, i_1, i_2, ..., i_k, b)$ *the discount rate of her predecessor in* $(s, i_1, i_2, ..., i_k, b)$ *is greater or equal than the discount rate of her predecessor in any other paths in* g *between* j *and* s *such that her successors in* $(s, i_1, i_2, ..., i_k, b)$ *do not lie on those paths, and*

(**ii**) *there is no strictly shorter path in* g *connecting* s *and* b *than* $(s, i_1, i_2, ..., i_k, b)$ *that satisfies* (**i**).

From (1) we observe that the SPE share of each player only depends on (i) her own discount rate, (ii) the discount rate of her predecessor, and (iii) the discount rates of the players who have already exited the game with a partial agreement. Therefore, when choosing her predecessor on a path from the buyer to the seller for a bilateral negotiation, each player chooses her most impatient predecessor (i.e. the one with the highest discount rate). The trading network depicted in Figure 1 illustrates the proposition. Suppose that $r_4 > r_5$ and $r_3 > r_1$. Then, $(1, 3, 2, 4, 6, 7)$ is the unique equilibrium trading path. Suppose now that $r_4 = r_5$ and $r_3 > r_1 = r_2$. Then, $(1, 3, 5, 6, 7)$ is the unique equilibrium trading path. Player 6 is indifferent between players 4 and 5. Player 6 chooses 5 as her predecessor, because of the shortest path assumption for breaking ties, anticipating perfectly that player 3 will choose to negotiate with player 1. The trading path $(1, 3, 5, 6, 7)$ involves five players. If player 6 had chosen 4 as her predecessor then the trading path would have been $(1, 3, 2, 4, 6, 7)$ and would have involved six players.

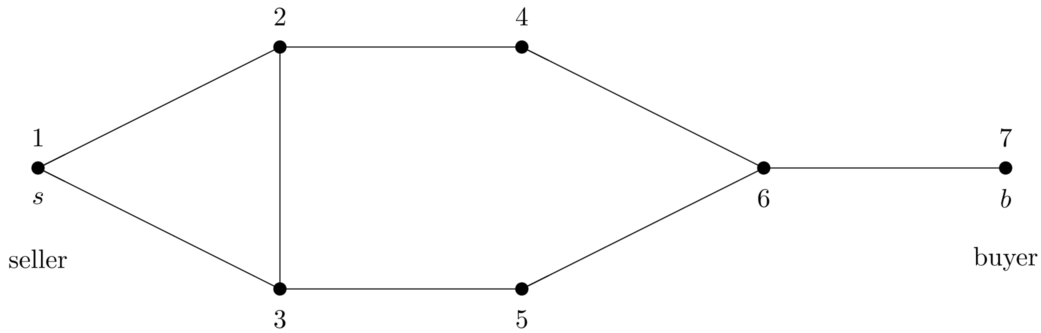

Figure 1: Equilibrium trading paths

4 Stable Trading Networks

4.1 Pairwise stability

Players form a trading network before knowing which pair of players will be randomly selected to become the seller and the buyer. Let $u_i(g,(s,b))$ be player i's SPE payoff (or share) in the trading network g with player s being the seller and player b being the buyer, and let $U_i(g)$ be player i's SPE expected payoff in the trading network g before knowing which pair of players will be randomly selected. For instance, suppose that the trading network is a star network $\{12,13\}$ where player 1 is the center. Player 1's expected payoff will be equal to

$$\begin{aligned} U_1(\{12,13\}) &= \frac{1}{6}u_1(\{12,13\},(2,1)) + \frac{1}{6}u_1(\{12,13\},(3,1)) + \frac{1}{6}u_1(\{12,13\},(1,2)) \\ &+ \frac{1}{6}u_1(\{12,13\},(1,3)) + \frac{1}{6}u_1(\{12,13\},(2,3)) + \frac{1}{6}u_1(\{12,13\},(3,2)). \end{aligned}$$

That is,

$$\begin{aligned} U_1(\{12,13\}) &= \frac{1}{6}\frac{r_2}{r_2+r_1} + \frac{1}{6}\frac{r_3}{r_3+r_1} + \frac{1}{6}\left(1-\frac{r_1}{r_1+r_2}\right) + \frac{1}{6}\left(1-\frac{r_1}{r_1+r_3}\right) \\ &+ \frac{1}{6}\frac{r_2}{r_2+r_1}\left(1-\frac{r_1}{r_1+r_3}\right) + \frac{1}{6}\frac{r_3}{r_3+r_1}\left(1-\frac{r_1}{r_1+r_2}\right). \end{aligned}$$

As our interest is in understanding which networks are likely to arise in trading networks when bargaining is with complete information and players are heterogeneous, we need to define a notion which captures the stability of a network. We use a strict version of Jackson and Wolinsky's (1996) notion of pairwise stability. A network is pairwise stable if no player does not lose from severing one of her links and no other two players strictly benefit from adding a link between them.[13]

Definition 1. A network g is pairwise stable if

(**i**) for all $ij \in g$, $U_i(g) > U_i(g-ij)$ and $U_j(g) > U_j(g-ij)$, and

(**ii**) for all $ij \notin g$, if $U_i(g) < U_i(g+ij)$ then $U_j(g) \geq U_j(g+ij)$.

Our first result is that, in the absence of costs for forming links, any pairwise stable trading network will consist of only one component connecting all players in N.

[13] Players are not farsighted in the sense that they do not forecast how others might react to their actions. Dutta, Ghosal and Ray (2005), Herings, Mauleon and Vannetelbosch (2009) and Page and Wooders (2009) have recently developed notions to predict which networks are likely to be formed among farsighted players.

Proposition 2. *A network g such that $\#C(g) > 1$ or $N(g) \subsetneq N$ is never pairwise stable.*

Suppose that, contrary to what is asserted, a pairwise stable network is split into two or more components. Consider two players, i and j, in different components C_i and C_j, respectively. First, we observe that, adding the link ij between player i and player j will not modify player i's payoff from trades when the seller and the buyer belong to the same component C_i (C_j). Indeed, the new link ij cannot be part of a trading path among players in C_i (C_j). Second, we have that, in the absence of the link ij, no trade is feasible when the seller (buyer) belongs to C_i and the buyer (seller) belongs to C_j. Once player i is linked to player j, player i will lie on all equilibrium trading paths when the seller (buyer) belongs to C_i and the buyer (seller) belongs to C_j. Hence, before knowing which pair of players will be randomly selected to become the seller and the buyer, player i and player j strictly benefit from adding the link ij. Notice that the same reasoning holds if player i or player j are isolated players. It then follows that networks that can be pairwise stable consist of only one component connecting all players.

4.2 Strong and weak players

Suppose that we have two types of players: weak players (impatient) and strong players (patient). Let $W = \{1, 2, ..., m\}$ be the set of weak players and r_W be the discount rate of weak players. Let $S = \{m + 1, m + 2, ..., n\}$ be the set of strong players and r_S be the discount rate of strong players. Obviously, $r_W > r_S$. Let g^T be the collection of all subsets of $T \subseteq N$ with cardinality 2. Then, g^W is the complete network among the weak players. The degree of i is the number of players that i is linked to. That is, $d_i(g) = \#\{j \mid ij \in g\}$. Which trading networks are pairwise stable when there are two types players?

Our main result is that pairwise stable networks are core periphery networks where the core only consists of weak players who are linked to each other and the periphery only consists of strong players who are linked to one weak player.[14] Figure 2 illustrates a core periphery network where $W = \{1, 2, 3\}$ and $S = \{4, 5, ..., 11\}$.

[14]Notice that rich (patient) people have fewer friends than poor (impatient) people. An explanation provided by Granovetter (1983) is that individuals develop strong social ties to those similar to themselves and since there are fewer individuals in the upper strata of society, those at the top have fewer close friends.

Proposition 3. *Suppose that* $r_i = r_W > 0$ *for* $i \in W = \{1, ..., m\}$ *and* $r_j = r_S$ *for* $j \in S = \{m+1, ..., n\}$ *and* $r_S < r_W$. *A network* g *is pairwise stable if and only if*

(i) $g^W \subseteq g$;

(ii) $d_i(g) = 1$ *for all* $i \in S$;

(iii) $\#C(g) = 1$.

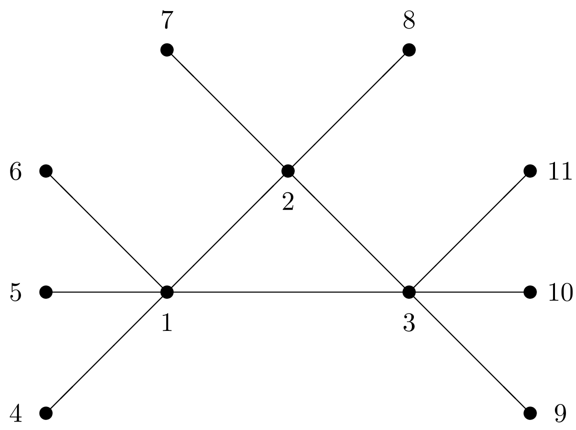

Figure 2: A core periphery network.

Proposition 1 tells us that a trading network g is pairwise stable if and only if (i) all weak players are linked to each other, (ii) each strong player has exactly one link (and so, each strong player is only linked to one weak player), and (iii) g consists of only one component connecting all players in N. Part (iii) follows from Proposition 2. The proof of part (i) and part (ii) of Proposition 3 proceeds in five steps. First, we show that networks that can be pairwise stable are such that each strong player is linked to at least one weak player.

Lemma 1. *Suppose that* $r_i = r_W > 0$ *for* $i \in W = \{1, ..., m\}$ *and* $r_j = r_S$ *for* $j \in S = \{m+1, ..., n\}$ *and* $r_S < r_W$. *A network* g *cannot be pairwise stable if there is some strong player that is not linked to at least one weak player.*

All the proofs not in the main text can be found in Appendix B. Suppose that g consists of one component connecting all players and that there is some strong player $i \in S$ that is not linked to at least one weak player $j \in W$ in g. The strong player i has incentives to add the link ij because, as a buyer or intermediary or seller, she will get a larger share of the surplus when bargaining with the weak player j rather than having to bargain with another strong player. Moreover, she will endorse more often the role of intermediary in $g + ij$. Precisely, in $g + ij$ player i is winning when she is matched as a buyer to the weak player j or to one of the strong players on the geodesic between i and j or to any other player l such that player j lies on a path between i and l in g.[15] Indeed, player i will choose to negotiate with the weak player j to obtain a larger share than the one she would get when bargaining with strong players. Player j is indifferent between g and $g + ij$ when he is the buyer. In $g + ij$ player i is winning when she is matched as a seller to a player l such that player j was lying on the trading path in g since the trading path in $g + ij$ will be shorter and j will end the sequence of bilateral bargaining sessions negotiating with i. In addition, in $g + ij$ player i is winning when she is matched as a seller to a player l such that player j was not lying on the trading path in g and the length of the geodesic between l and j is shorter than the length of the geodesic between l and i. When j is the seller, he is either better off or equal off depending if the length of the equilibrium trading path becomes shorter or not in $g + ij$. When i was an intermediary in g for some match then she is still an intermediary for the same match in $g+ij$ and she is either better off or equal off. Finally, it may happen that i was not an intermediary in g for some match and now becomes in $g + ij$ an intermediary for the same match. Similarly, for player j. Thus, both players i and j have incentives to add the link ij.

Lemma 2. *Suppose that* $r_i = r_W > 0$ *for* $i \in W = \{1, ..., m\}$ *and* $r_j = r_S$ *for* $j \in S = \{m + 1, ..., n\}$ *and* $r_S < r_W$. *A network* g *cannot be pairwise stable if* $g^W \nsubseteq g$.

Lemma 2 follows from two observations. Firstly, two weak players $i, j \in W$ having a common weak player $l \in W$ as neighbor (i.e. $il, jl \in g$ but $ij \notin g$) have incentives to link to each other in g to form $g + ij$. Both i and j never make losses by adding the link ij. When i is the buyer, her payoff does not change since she is already linked to another weak player l (that is linked to j) with whom she can negotiate first. When i is the seller or an intermediary, her payoff increases for all trades such that player j is either the buyer or a preceding intermediary in g since the new equilibrium trading path in $g+ij$ will be shorter than the one in g avoiding one intermediary, namely player l. Similarly for player j. Next, we proceed from $g+ij$ by adding a link between any two weak players having a common weak player as neighbor until we cannot add such links and we end up with the new network g' where the set of weak players can be partitioned into coalitions such that all weak players within each coalition are linked to each other and no weak player from a given coalition is linked to a weak player from another coalition. Secondly, two weak

[15] The distance between two nodes is the length of (number of links in) the shortest path or geodesic between them.

players i and j of different coalitions have incentives to add the link ij to form the network $g' + ij$. When i is the seller she is winning for all trades where j or one of his coalition partner is the buyer or an intermediary since the new equilibrium trading path in $g' + ij$ will be shorter than the one in g' avoiding one strong intermediary. When i is the buyer she is indifferent. When i is an intermediary in g' she is also an intermediary in $g' + ij$ and she is either equal off or better off. Similarly for player j. Next, we repeat the process until we end up with a network where all weak players are linked to each other and all strong players have exactly the same links as in g.

Lemma 3. *Suppose that $r_i = r_W > 0$ for $i \in W = \{1, ..., m\}$ and $r_j = r_S$ for $j \in S = \{m + 1, ..., n\}$ and $r_S < r_W$. A network g cannot be pairwise stable if there is some link between two strong players that are linked to the same weak player.*

Here, the main point is that, when in g there is a link between two strong players $i, k \in S$ that are linked to the same weak player $j \in W$, either the link ik is never used or one of the strong players is better off in $g - ik$. For instance, suppose that i is only linked to one weak player j. If k is only linked to i and j then the link ik will never be used. If k is only linked to i and j and to another weak player then player i has incentives to delete the link ik because when the match is (i, j) player j will choose to negotiate first with the other weak player instead of negotiating directly with i.

Lemma 4. *Suppose that $r_i = r_W > 0$ for $i \in W = \{1, ..., m\}$ and $r_j = r_S$ for $j \in S = \{m + 1, ..., n\}$ and $r_S < r_W$. A network g cannot be pairwise stable if there is some link between two strong players that are not linked to the same weak player.*

Suppose that in g there is a link between two strong players that are not linked to the same weak player: $ik \in g$, $jl \in g$ and $ij \in g$ with $i, j \in S$ and $k, l \in W$. Depending on the other links in g, either player i (or j) has incentives to delete the link ij or player i (or j) has incentives to add a link with another weak player ($\neq k, l$). For instance, if i and j do not have other links then i has incentives to delete the link ij. By deleting ij she is only loosing the payoff she obtains as an intermediary for the match (j, l). This loss is largely compensated by the gains she makes by shortening the trading path for the match (i, k) in $g - ij$. If j is linked to another weak player (say $m \in W$) then i would have even more incentives to delete ij since otherwise she would earn less from the match (i, k) and she would get nothing from the matches (j, l) and (j, m).

Lemma 5. *Suppose that $r_i = r_W > 0$ for $i \in W = \{1, ..., m\}$ and $r_j = r_S$ for $j \in S = \{m + 1, ..., n\}$ and $r_S < r_W$. A network g cannot be pairwise stable if some strong player is linked to more than one weak player.*

We already know that the candidates for being pairwise stable are networks g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^W \subseteq g$, (iii) $ij \notin g$ if $i \in S$ and $j \in S$. Suppose that in g player $i \in S$ is linked to two weak players $k, l \in W$. Clearly, player i is indifferent when she is the buyer and she is never an intermediary. When she is matched to a weak player ($\neq k$) or to a strong player that is not linked to player k she is better off by deleting the link ik since the equilibrium trading path is shortened of one link. Hence, we obtain our main result that a network g is pairwise stable if and only if $g^W \subseteq g$, $d_i(g) = 1$ for all $i \in S$, and $\#C(g) = 1$.

Suppose now that we allow group of players to modify their links. Link addition is bilateral, link deletion is unilateral, and multiple link changes can take place at a time. A network g' is obtainable from g via deviations by group $Q \subseteq N$ if (i) $ij \in g'$ and $ij \notin g$ implies $\{i,j\} \subseteq Q$, and (ii) $ij \in g$ and $ij \notin g'$ implies $\{i,j\} \cap Q \neq \varnothing$. A network g is strongly stable if (i) for all $ij \in g$, $U_i(g) > U_i(g - ij)$ and $U_j(g) > U_j(g - ij)$, and (ii) for all $Q \subseteq N$, g' that is obtainable from g via deviations by Q, there exists $i \in Q$ such that $U_i(g') \leq U_i(g)$.[16] Strong stability is a refinement of pairwise stability. We have that a network g is strongly stable if and only if (i) $g^W \subseteq g$, (ii) $d_i(g) = 1$ for all $i \in S$, (iii) $d_i(g) = n - 1$ for some $i \in W$, and (iii) $\#C(g) = 1$. Thus, strongly stable networks are core periphery networks where the core consists of weak players who are linked to each other and the periphery consists of strong players who are only linked to the same weak player. Those core periphery networks give to the strong players (and to player $i \in W$ for which $d_i(g) = n - 1$) their best payoffs among pairwise stable networks. Notice that in those core periphery networks the payoff of a strong player may be greater or smaller than the payoff of the weak player $i \in W$ for which $d_i(g) = n - 1$ depending on the discount rates, the number of weak players (m) and the number of strong players ($n - m$). See Appendix C for the details.

4.3 Ranked players

Suppose now that players can be ranked in terms of their discount rates with player 1 being the most impatient player (weakest player): $r_1 > r_2 > ... > r_{n-1} > r_n$. Clearly, a star network with the weakest player being the center is pairwise stable.

[16] This definition of strong stability reverts to Dutta and Mutuswami (1997) definition of strong stability if we do not require that no player does not lose from severing one of her links. Strong stability of Dutta and Mutuswami considers a deviation to be valid only if all members of a deviating coalition are strictly better, while the definition of Jackson and van den Nouweland (2005) is slightly stronger by allowing for a deviation to be valid if some members are strictly better and others are weakly better.

Proposition 4. *Suppose that $r_1 > r_2 > ... > r_{n-1} > r_n$. The network g such that*

(i) $d_1(g) = n - 1$;

(ii) $d_i(g) = 1$ *for all* $i \in \{2, 3, ..., n\}$;

(iii) $\#C(g) = 1$.

is pairwise stable.

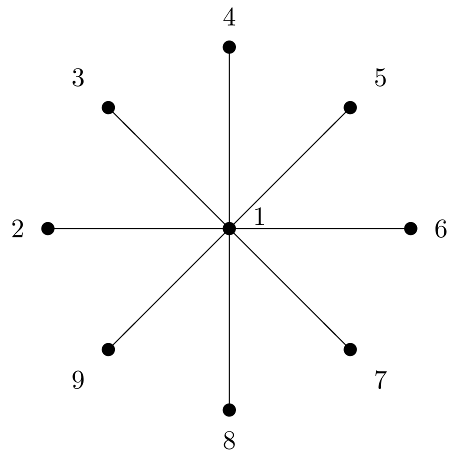

Figure 3: A star network with the weakest player being the center.

However, a star network with the weakest player being the center is not the unique pairwise stable network when players can be ranked based on their impatience. For instance, take $N = \{1, 2, 3\}$ and $r_1 > r_2 > r_3$. The star network $\{12, 23\}$ is pairwise stable if $3r_2(r_2 + r_3)^{-1} > r_1(r_1 + r_3)^{-1}(2r_1 + 3r_2)(r_1 + r_2)^{-1}$. The last condition is likely to hold the more impatient player 2 is (that is, when r_2 is close to r_1).

In addition, suppose that there are more than one player of each type and that we have more than two types of players. That is, $r_i = r_1 > 0$ for $i \in W = \{1, 2, ..., m\}$, $r_i = r_2 > 0$ for $i \in \{m + 1, ..., l\}$, $r_i = r_3 > 0$ for $i \in \{l + 1, ..., k\}$, ... with $r_1 > r_2 > r_3 > ....$ Then, the network g such that $g^W \subseteq g$, $d_i(g) = 1$ for $i \in \{m + 1, ..., n\}$ and $\#C(g) = 1$. So, core periphery networks where the core only consists of the weakest players $i \in W$ who are linked to each other and the periphery consists of all other players who are linked to one player in W are pairwise stable.

4.4 Homogenous players

Suppose now that players are homogenous in terms of their discount rates: $r_1 = r_2 = ... = r_n$. From Proposition 2 and Lemma 2 we have that there is a unique pairwise stable architecture, namely the complete network.

Corollary 1. *Suppose that $r_i = r$ for all $i \in N$. The complete network g^N is the unique pairwise stable network.*

Babus (2012) has shown that, if the formation of links is costly and players are farsighted, then a star network that connects all players is an absorbing state of Dutta, Ghosal and Ray (2005) dynamic network formation process. Intuitively, players have incentives to reduce the number of intermediaries to obtain a larger share of the surplus. In addition, decreasing monitoring costs over time provide incentives to interact frequently with the same partners.[17] Farsighted players can rely on their successors in the network formation process to converge towards a star network. Both the order of play in which players decide about their links and the amount of intermediation rents each player extracts in the initial network matters for determining who will become the center of the star.

5 Bargaining with Private Information

Under complete information, agreement is reached immediately in each bilateral bargaining session. This is not true if we introduce incomplete information into the bargaining. In this case, the early rounds of negotiation are used for information transmission.

5.1 Maximal delay in reaching an agreement

We now suppose that players have private information. Players do not know the impatience (or discount rate) of the other players. It is common knowledge that player i's discount rate lies in the range $[r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$, where $0 < r_i^{\mathrm{P}} \leq r_i^{\mathrm{I}}$ and $i \in N$. The superscripts "I" and "P" identify the most impatient and most patient types, respectively. The types are independently drawn from the set $[r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$ according to the probability distribution p_i, $i \in N$. Watson (1998) has characterized the set of perfect Bayesian equilibrium (PBE) payoffs which may arise in Rubinstein's alternating-offer

[17] Players incur monitoring costs for each transaction along the trading path. Decreasing monitoring costs over time reflect that players need to do less effort to get information about players with whom they interacted already.

bargaining game and constructed bounds (which are met) on the agreements that may be made. The bounds and the PBE payoffs set are determined by the range of incomplete information and are easy to compute because they correspond to the SPE payoffs of two bargaining games with complete information. These two games are defined by matching one player's most impatient type with the opponent's most patient type. Each bilateral bargaining session may involve delay, but not perpetual disagreement, in equilibrium.[18] In fact, delay is positively related to the distance between the discount rates of the most and least patient types of the players. If the range of types is reduced, then this leads to a smaller range of possible payoffs and less delay. Delay can occur even when the game is close to one of complete information (as the type distributions converge to point mass distributions).

We propose to analyze the maximum delay time in reaching an agreement. Only on average is this measure a good proxy for actual delay.[19] In each bilateral bargaining session (i, j), the maximum real time player j would spend bargaining is the time $D(i, j)$ such that player j is indifferent between getting her lower bound PBE payoff at time 0 and getting her upper bound PBE payoff at time $D(i, j)$. In Appendix D we derive the expression for the maximum delay in equilibrium which shows that an agreement is reached in finite time and that delay time equals zero as incomplete information vanishes (in that r_i^{P} and r_j^{P} converge to r_i^{I} and r_j^{I}, respectively).

Proposition 5. *In each bilateral bargaining session (i, j), the maximum real delay time in reaching a partial agreement is given by*

$$D(i,j) = -\frac{1}{r_j^P} \cdot \log\left[\frac{r_i^P}{r_i^I} \cdot \frac{r_i^I + r_j^P}{r_i^P + r_j^I}\right].$$

In fact, $D(i, j)$ is the maximum real time player j would spend negotiating if she were of the most patient type. We have $\partial D(i, j)/\partial r_j^{\mathrm{P}} < 0$, $\partial D(i, j)/\partial r_j^{\mathrm{I}} > 0$, $\partial D(i, j)/\partial r_i^{\mathrm{P}} < 0$ and $\partial D(i, j)/\partial r_i^{\mathrm{I}} > 0$. Given the trading path $(s, i_1, i_2, ..., i_k, b)$, the maximum real delay time in reaching $k + 1$ partial agreements is equal to $D(s, i_1, i_2, ..., i_k, b) = D(s, i_1) + D(i_1, i_2) + ... + D(i_{k-1}, i_k) + D(i_k, b)$.

We now provide an example of the maximum delay. Suppose that (s, i_1, i_2, i_3, b) is the trading path and let $r_i^{\mathrm{P}} = r^{\mathrm{P}}$, $r_i^{\mathrm{I}} = r^{\mathrm{I}}$, $r^{\mathrm{I}} = 0.33 - r^{\mathrm{P}}$ with $r^{\mathrm{P}} \in [0.04, 0.17]$, $i \in \{s, i_1, i_2, ..., i_k, b\}$. Table 1 gives the integer part of the maximum delay. We can

[18] Watson (1998) has constructed equilibria with delay in which the types of each player behave identically (no information is revealed in equilibrium), players use pure strategies, and players make non-serious offers until some appointed date.

[19] It is not uncommon in the literature on bargaining to analyze the maximum delay before reaching an agreement. See, for instance, Cramton (1992) and Cai (2003).

interpret r as the annual discount rate and the numbers in Table 1 as the maximum number of days needed to reach an agreement.[20] We observe that the real delay time in reaching an agreement is not negligible (many bargaining rounds may be needed in equilibrium before an agreement is reached) and is increasing with the amount of private information $\left| r^{\mathrm{P}} - r^{\mathrm{I}} \right|$.

r^{P}	.17	.16	.15	.14	.13	.12	.11	.10	.09	.08	.07	.06	.05	.04
"partial delay"	0	1	2	3	4	5	7	9	12	15	20	26	36	51
"total delay"	0	4	8	12	16	20	28	36	48	60	80	104	144	204

Table 1: Maximum delay in reaching an agreement

5.2 Stability

When a player chooses one of her predecessors on a path from the buyer b to the seller s to negotiate bilaterally a partial agreement once there is private information about the impatience of the players, her choice still does not depend on how the successors are going to share the rest of the surplus but now depends on how much time the successors will need to reach their partial agreements.

Suppose that $N = \{1, 2, 3\}$, $g = \{12, 23, 13\}$ and that $r_2^{\mathrm{I}} = r_2^{\mathrm{P}} = r_3^{\mathrm{I}} = r_3^{\mathrm{P}} < r_1^{\mathrm{P}} < r_1^{\mathrm{I}}$. That is, it is common knowledge that player 1 is a weak player and players 2 and 3 are strong players. Suppose first that player 1 is the seller and player 3 is the buyer. Since $r_1^{\mathrm{P}}/(r_1^{\mathrm{P}} + r_3^{\mathrm{I}}) > r_2^{\mathrm{I}}/(r_2^{\mathrm{I}} + r_3^{\mathrm{P}}) = 1/2$, player 3 will choose to negotiate directly with player 1. Suppose now player 2 is the seller and player 3 is the buyer. Since $r_2^{\mathrm{I}}/(r_2^{\mathrm{I}} + r_3^{\mathrm{P}}) = 1/2 \gtrless r_1^{\mathrm{P}}/(r_1^{\mathrm{P}} + r_3^{\mathrm{I}}) \exp(-r_3^{\mathrm{I}} D(1, 2))$, it is not excluded that player 3 would choose to negotiate directly with player 2 instead of going through the weak player 1. Player 3 will choose to bargain with player 2 instead of player 1 if the expected delay for reaching an agreement in a negotiation between player 1 and player 2 is large enough. Hence, player 3 may now have incentives to be linked to both players 1 and 2 although it is commonly known that player 2 is stronger than player 1.

We now provide sufficient conditions such that core periphery networks are still pairwise stable when players have private information.

[20] The integer part of the maximum delays for $\Delta = 1/365$ are exactly the numbers in Table 1.

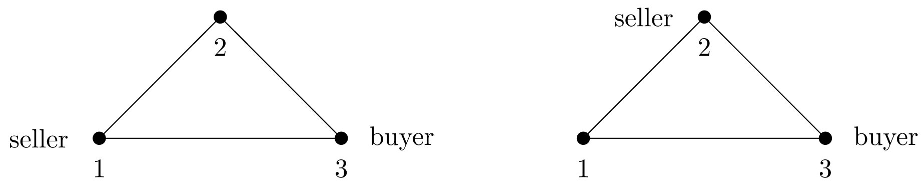

Figure 4: Trading paths with private information.

Proposition 6. *Suppose that* $r_i = r_W > 0$ *for* $i \in W = \{1, ..., m\}$ *and* $r_j \in [r_S^P, r_S^I]$ *for* $j \in S = \{m+1, ..., n\}$ $(0 < r_S^P \leq r_S^I)$ *and that it is common knowledge that any player* $i \in W$ *is less patient than any player* $j \in S : r_S^P < r_S^I < r_W$*. If*

$$(\mathbf{i})\ D(i,j) < \frac{-1}{r_S^I} \log\left(\frac{r_S^I}{r_W}\frac{r_W + r_S^I}{r_S^P + r_S^I}\right) \ \textit{and}\ (\mathbf{ii})\ D(j,i) < \frac{-1}{r_W} \log\left(\frac{2r_S^I}{r_W + r_S^I}\right),$$

then a network g *such that* $g^W \subseteq g$*,* $d_i(g) = 1$ *for all* $i \in S$ *and* $\#C(g) = 1$ *is pairwise stable.*

Condition (**i**) in Proposition 6 is a sufficient condition for player $j \in S$ for not adding a link to another player $k \in S$ in a core periphery network g. It implies that if $j \in S$ and $k \in S$ are matched then buyer j prefers to negotiate with the player $i \in W$ is linked to rather than building the link jk and negotiating directly with k. For condition (**i**) to hold we need that $r_W - r_S^{\mathrm{I}}$ is large enough (for the right-hand side of the inequality being positive) and $r_S^{\mathrm{I}} - r_S^{\mathrm{P}}$ is not too large (for $D(i,j)$ being small enough). Condition (**ii**) in Proposition 6 is a sufficient condition for player $j \in S$ for not adding a link to another weak player $l \in W$ $(l \neq i)$ in a core periphery network g. It implies that if $i \in W$ is an intermediary (or the buyer) in a match where $j \in S$ is the seller then i prefers to negotiate with player $l \in W$ rather than negotiating directly with j. For condition (**ii**) to hold we need that $r_S^{\mathrm{I}} - r_S^{\mathrm{P}}$ is not too large (for $D(j,i)$ being small enough) and $r_W - r_S^{\mathrm{I}}$ is large enough.

Proposition 7. *Suppose that* $r_i \in [r_W^P, r_W^I]$ *for* $i \in W = \{1, ..., m\}$ $(0 < r_W^P \leq r_W^I)$ *and* $r_j = r_S > 0$ *for* $j \in S = \{m+1, ..., n\}$ *and that it is common knowledge that any player* $i \in W$ *is less patient than any player* $j \in S : r_S < r_W^P < r_W^I$*. If*

$$(\mathbf{i})\ D(l,i) + D(i,j) < \frac{-1}{r_S} \log\left(\frac{r_W^P + r_S}{2r_W^P}\right) \ \textit{and}\ (\mathbf{ii})\ D(j,i) < \frac{-1}{r_W^I} \log\left(\frac{r_S}{r_W^P}\frac{r_W^P + r_W^I}{r_W^P + r_S}\right),$$

with $i, l \in W$ *and* $j \in S$*, then a network* g *such that* $g^W \subseteq g$*,* $d_i(g) = 1$ *for all* $i \in S$ *and* $\#C(g) = 1$ *is pairwise stable.*

Similarly to conditions (**i**) and (**ii**) in Proposition 6, both conditions (**i**) and (**ii**) in Proposition 7 are sufficient conditions for a strong player $j \in S$ for not adding a link to another strong player and to another weak player, respectively. For condition (**ii**) to hold we need that $r_W^{\mathrm{P}} - r_S$ is large enough (for the right-hand side of the inequality being positive) and $r_W^{\mathrm{I}} - r_W^{\mathrm{P}}$ is not too large (for $D(j,i)$ being small enough).

So, once there is private information about the impatience of the players, a core periphery network is likely to be pairwise stable if all players do not have too much private information and weak players are quite more impatient than strong players. Otherwise, players may prefer to add links for reducing the length of trading paths and so avoiding longer costly delays in reaching a global agreement.

6 Conclusion

We have analyzed a model in which heterogenous players form a trading network and a seller and a buyer are randomly selected among the players to bargain through a chain of intermediaries. We have determined both the trading path and the allocation of the surplus among the seller, the buyer and the intermediaries at equilibrium. We have shown that a trading network is pairwise stable if and only if it is a core periphery network where the core consists of all weak (or impatient) players who are linked to each other and the periphery consists of all strong (or patient) players who have a single link towards a weak player. Once players do not know the impatience of the other players, each bilateral bargaining session may involve delay, but not perpetual disagreement, in equilibrium. When a player chooses another player on a path from the buyer to the seller to negotiate bilaterally a partial agreement, her choice now depends both on the type of this other player and on how much time the succeeding players on the path will need to reach their partial agreements. We have provided sufficient conditions such that core periphery networks are still pairwise stable.

Recently, Siedlarek (2012) has studied a stochastic model of bargaining and exchange with common discount factor and intermediation on an exogenously given network. There is one seller who holds the indivisible good and trade is only feasible if there exists one path in the network connecting the seller to the buyer. In each period, a stochastic process determines both a buyer, a trade route and an order of play for agents on this route. Agents that are on the route bargain according to the order of play. If one agent along the route rejects the proposed split of the surplus, bargaining terminates and the game moves to the next period where a new buyer, a new trade route and a new order of play is redrawn. The process goes on until an allocation of the surplus is accepted by all players.

In Siedlarek (2012) bargaining is multilateral instead of having a series of bilateral negotiations along the trade route. It is close as if in each period a coalition of agents is drawn and has to divide some surplus and if they do not reach an agreement, then a new coalition is drawn to bargain over the division of some surplus, and so forth until an agreement is reached. We rather adopt a finite series of bilateral negotiations along the trade route and we make endogenous the trade route, two features which are more realistic for the markets we are interested in.

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Appendix

A Bargaining with complete information

Take the path $(s, i_1, i_2, ..., i_k, b)$ that connects s to b. Players along this path negotiate how to split the surplus via successive bilateral bargaining sessions. Players bargain in the following order: (i_k, b), (i_{k-1}, i_k), (i_{k-2}, i_{k-1}), ..., (i_1, i_2), (s, i_1). In each bilateral bargaining session (i, j), the negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model where players make alternate offers, with i making offers in even-numbered periods and j making offers in odd-numbered periods. The length of each period is Δ. The negotiation starts in period 0 and ends when one of the players accepts an offer and leads to a partial agreement. A partial agreement specifies the share of the surplus for j to exit the game. Players have time preferences with constant discount factors, $1 > \delta_1 > 0$, $1 > \delta_2 > 0$, ... $1 > \delta_n > 0$.

Let y_t be the surplus left to be shared among the remaining players after t bilateral bargaining sessions. So, y_0 is the initial surplus to be shared and is equal to 1; y_1 is the surplus left to be shared after buyer b has taken her share; y_2 is the surplus left to be shared after intermediary i_k and buyer b have taken their shares; y_3 is the surplus left to be shared after intermediary i_{k-1}, intermediary i_k and buyer b have taken their shares; y_k is the surplus left to be shared after intermediary i_2, intermediary i_3, ... intermediary i_{k-1}, intermediary i_k and buyer b have taken their shares.

Consider first the bargaining session (s, i_1). The SPE partial agreement is $x_{i_1} = y_k \delta_{i_1} (1-\delta_s) / (1-\delta_s \delta_{i_1})$. Consider next the bargaining session (i_1, i_2). The SPE partial agreement is $x_{i_2} = y_{k-1} \delta_{i_2} (1-\delta_{i_1}) / (1-\delta_{i_1}\delta_{i_2})$; and so forth. Consider finally the bargaining session (i_k, b). The SPE partial agreement is $x_b = y_0 \delta_b (1-\delta_{i_k}) / (1-\delta_{i_k}\delta_b)$. Since $y_0 = 1$, buyer b will obtain at equilibrium

$$x_b^* = \frac{\delta_b(1-\delta_{i_k})}{1-\delta_{i_k}\delta_b}.$$

Since $y_1 = y_0 - x_b^*$, intermediate i_k will obtain at equilibrium

$$x_{i_k}^* = \frac{\delta_{i_k}(1-\delta_{i_{k-1}})}{1-\delta_{i_{k-1}}\delta_{i_k}} \frac{1-\delta_b}{1-\delta_{i_k}\delta_b}.$$

Since $y_2 = y_1 - x_{i_k}^*$, intermediate i_{k-1} will obtain at equilibrium

$$x_{i_{k-1}}^* = \frac{\delta_{i_{k-1}}(1-\delta_{i_{k-2}})}{1-\delta_{i_{k-2}}\delta_{i_{k-1}}} \frac{1-\delta_{i_k}}{1-\delta_{i_{k-1}}\delta_{i_k}} \frac{1-\delta_b}{1-\delta_{i_k}\delta_b};$$

and so on. Since $y_k = y_{k-1} - x_{i_2}^*$, intermediate i_1 will obtain at equilibrium

$$x_{i_1}^* = \frac{\delta_{i_1}(1-\delta_s)}{1-\delta_s\delta_{i_1}} \frac{1-\delta_{i_2}}{1-\delta_{i_1}\delta_{i_2}} \frac{1-\delta_{i_3}}{1-\delta_{i_2}\delta_{i_3}} \cdots \frac{1-\delta_{i_k}}{1-\delta_{i_{k-1}}\delta_{i_k}} \frac{1-\delta_b}{1-\delta_{i_k}\delta_b};$$

and seller s will obtain at equilibrium

$$x_s^* = \frac{1-\delta_{i_1}}{1-\delta_s\delta_{i_1}} \frac{1-\delta_{i_2}}{1-\delta_{i_1}\delta_{i_2}} \frac{1-\delta_{i_3}}{1-\delta_{i_2}\delta_{i_3}} \cdots \frac{1-\delta_{i_k}}{1-\delta_{i_{k-1}}\delta_{i_k}} \frac{1-\delta_b}{1-\delta_{i_k}\delta_b}.$$

It is customary to express the players' discount factors in terms of discount rates, $r_1 > 0$, $r_2 > 0$, ... $r_n > 0$, and the length of the bargaining period, Δ, according to the formula $\delta_i = \exp(-r_i\Delta)$. As Δ approaches zero, using l'Hopital's rule, the SPE outcomes $x_b^*, x_{i_k}^*, x_{i_{k-1}}^*, ..., x_{i_1}^*, x_s^*$ tend to the equilibrium outcomes given in (1).

B Stable trading networks

Proof of Lemma 1.

(i) First, we show that a strong player always wants to link to a weak player if in the current network there are at least two strong players as intermediaries on the geodesic between the strong player and the weak player. Remember that a geodesic between players i and j is a shortest path between these nodes; that is, a path with no more links than any other path between these nodes.

Suppose that $1 \in W$ and $\{2, 3, 4\} \subseteq S$. Take any network g_1 such that $\{12, 23, 34\} \subseteq g_1$, the path $1, 2, 3, 4$ is a geodesic between 1 and 4, and the distance between player 4 and any other weak player is greater than 3. We now show that 1 and 4 have incentives to add the link 14 to form $g_2 = g_1 + 14$. Player 4 is winning when she is matched as a buyer to the weak player 1 or to one of the strong players 2 and 3 or to any other player j such that in g_1 players 2 or 3 were on the equilibrium trading path between 4 and j. Notice that 4 is strictly winning because she is avoiding at least one of the two intermediaries 2 and 3 if not both depending to whom she is matched. If 4 is matched to a player j such that in g_1 players 2 or 3 are not on the equilibrium trading path between 4 and j, then 4 is indifferent between g_1 and g_2. So, when 4 is the buyer she is never loosing by adding the link 14 to g_1. When 4 is the seller, she is always better off when she is matched to a player j such that player 1 was lying on the trading path in g_1 since the trading path in g_2 will be shorter and 1 will end the sequence of bilateral bargaining sessions negotiating with 4. In addition, in g_2 player 4 is winning when she is matched as a seller to a player j such that player 1 was not lying on the trading path in g_1 and the length of the geodesic between 1 and j is shorter than the length of the geodesic between 4 and j. Otherwise, she is equal off. When 4 was an intermediary in g_1 for some match (i, j) then she is still an intermediary for the match (i, j) in g_2 and she is either better off or equal off. Finally, it may happen that 4 was not an intermediary in g_1 for some match (i, j) and now becomes in g_2 an intermediary for the match (i, j). Player 1 is indifferent between g_1 and g_2 when he is the buyer. When player 1 is the seller, he is either better off or equal off depending if the length of the equilibrium trading path becomes shorter or not in g_2. When 1 was an intermediary in g_1 for some match (i, j) then he is still an intermediary for the match (i, j) in g_2 and he is either better off or equal off. Finally, it may happen that 1 was not an intermediary in g_1 for some match (i, j) and now becomes in g_2 an intermediary for the match (i, j). Thus, we conclude that both players 1 and 4 have incentives to add the link 14.

(ii) Second, we show that a strong player having links with at least two other strong players that are linked to the same weak player has always incentives to link to the weak player.

Suppose that $1 \in W$ and $\{2,3,4\} \subseteq S$. Take any network g_1 such that $\{12,13,24,34\} \subseteq g_1$, the paths $1,2,4$ and $1,3,4$ are geodesics between 1 and 4, and the distance between player 4 and any other weak player is greater or equal than 2. We now show that 1 and 4 have incentives to add the link 14 to form $g_2 = g_1 + 14$. Player 4 is winning when she is matched as a buyer to the weak player 1 or to one of the strong players 2 and 3 or to any other player j such that there is a path between players 1 and j and player 4 does not lie on the path; otherwise, player 4 is equal off. Player 4 is winning when she is matched as a seller to a player j such that player 1 is on the equilibrium trading path in g_1; otherwise, he is equal off. Player 4 is winning when she is an intermediary on trades that are passing through the weak player 1 in g_1; otherwise, player 4 is equal off. Player 1 is equal off when he is the buyer, but he is better off or equal off when he is the seller or an intermediary. Thus, we have that both players 1 and 4 have incentives to add the link 14.

(iii) Third, suppose that $1 \in W$ and $\{2,3\} \subseteq S$. In any network g_1 such that $\{12,23\} \subseteq g_1$, the path $1,2,3$ is a geodesic between 1 and 3 in g_1, $d_2(g_1) = 2$, and the distance between the strong player 3 and any other weak player is greater or equal than 2, players 1 and 3 have incentives to add the link 13 to form $g_2 = g_1 + 13$.

(iv) Fourth, suppose that $1 \in W$ and $\{2,3,4\} \subseteq S$. In any network g_1 such that $\{12,23,24\} \subseteq g_1$, the path $1,2,3$ is a geodesic between 1 and 3, the path $1,2,4$ is a geodesic between 1 and 4, and the distance between the strong player 3 (4) and any other weak player is greater or equal than 2, the strong players 3 and 4 have first incentives to add the link 34 to form $g_2 = g_1 + 34$. Once the link 34 is formed, the strong player 4 has now incentives to link to the weak player 1 to form the network $g_2 + 14$. Player 1 is equal off when he is the buyer, but he is better off or equal off when he is the seller or an intermediary. Thus, player 1 agrees to add the link 14.

From (i)-(iv) we conclude that a network g cannot be pairwise stable if there is some strong player that is not linked to at least one weak player. □

Proof of Lemma 2.

Consider any network g such that $\#C(g) = 1$, $N(g) = N$ and each strong player is linked to at least one weak player.

(i) First, we will show that two weak players $i, j \in W$ having a common weak player $l \in W$ as neighbor (i.e. $il, jl \in g$ but $ij \notin g$) have incentives to link to each other in g to form $g + ij$. When i is the buyer, her payoff does not change by adding the link ij since she is already linked to another weak player l (that is linked to j) with whom she can negotiate first. When i is the seller, her payoff does not change by adding the link ij for all trades such that player j is not the buyer nor an intermediary in g since the equilibrium trading path in $g+ij$ will be the same as the one in g. When i is the seller, she is winning by adding the link ij for all trades such that player j is either the buyer or an intermediary in g since the new equilibrium trading path in $g + ij$ will be shorter than the one in g avoiding one intermediary, namely player l. When i is an intermediary, she is winning by adding the link ij for all trades such that player j is either the buyer or a preceding intermediary in g since the new equilibrium trading path in $g + ij$ will be shorter than the one in g avoiding one intermediary, namely player l. Finally, when i is an intermediary, her payoff does not change by adding the link ij for all trades such that player j is not on the equilibrium trading path in g or is not a preceding intermediary in g. Similarly for player j. Hence, players i and j have incentives to add the link ij.

(ii) Next, we proceed from g by adding a link between any two weak players having a common weak player as neighbor until we cannot add such links and we end up with the new network $g' = g_W \cup g_S$ where

$$g_W = \{ij \in g^N \mid \text{there is a path between } i \text{ and } j \text{ in } g \setminus g_S\}$$

and $g_S = \{ij \in g \mid i \in S \text{ or } j \in S\}$. Let $\Pi(g_W)$ be the partition of W induced by g_W. That is, $P \in \Pi(g_W)$ if and only if either there exists $h \in C(g_W)$ such that $P = N(h)$ or there exists $i \notin N(g_W)$ such that $P = \{i\}$. The set of weak players is partitioned into coalitions such that all weak players within each coalition are linked to each other and no weak player from a given coalition is linked to a weak player from another coalition. We want now to prove that, in g', two weak players i and j of different coalitions P_i and P_j in $\Pi(g_W)$ ($i \in P_i$ and $j \in P_j$) of fully connected players that are not linked to any strong player on the path between these two coalitions P_1 and P_2 have incentives to add the link ij to form the network $g' + ij$. When i is the seller she is winning for all trades where j or one of his coalition partner in P_j is the buyer or an intermediary since the new equilibrium trading path in $g' + ij$ will be shorter than the one in g' avoiding one strong intermediary; otherwise she is indifferent. When i is the buyer she is indifferent between $g' + ij$ and g'. When i is an intermediary in g' she is also an intermediary in $g' + ij$ and she is either equal off or better off (when the new equilibrium trading path in $g' + ij$ is shorter than the one in g' and avoids one strong preceding intermediary). Similarly for player j. In addition, in g', two weak players i and j of different coalitions P_i and P_j in $\Pi(g_W)$ ($i \in P_i$ and $j \in P_j$) of fully connected players that are linked to a strong player on the path between these two coalitions P_i and P_j have also incentives to add the link

ij to form the network $g' + ij$. When i is the buyer or the seller she is either better off or equal off between $g' + ij$ and g' depending to whom she is matched. When i is an intermediary in g' she is also an intermediary in $g' + ij$ and she is either equal off or better off or worse off. However, the losses she makes as an intermediary in some matches are easily compensated by the gains she makes as an intermediary in other matches. Precisely, player i can only make losses when she is an intermediary in matches between two strong players who are linked to weak players both in P_i and in P_j, and these losses are compensated by the gains she makes when she is an intermediary in matches between those two strong players (as sellers) and weak players (as buyers) from P_j.

(iii) Next, we repeat the process of step (ii) until we end up with network $g^W \cup g_S$ where all weak players are linked to each other and all strong players have exactly the same links as in g. □

Proof of Lemma 3.

From Proposition 2, Lemma 1 and Lemma 2 we know that the candidates for being pairwise stable are networks g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^W \subseteq g$, (iii) for each $i \in S$ there is $j \in W$ such that $ij \in g$. We now show that g cannot be pairwise stable if there is some link between two strong players that are linked to the same weak player. Five cases have to be considered.

(a) In g the strong player i is only linked to one weak player j. Suppose that we add the link ik to g to form $g + ik$ where $i, k \in S$. (a.1) If k is only linked to i and j then the link ik will never be used. (a.2) If k is only linked to i and j and to another weak player then player i has incentives to delete the link ik because when the match is (i, j) player j will choose to negotiate first with the other weak player instead of negotiating directly with i. (a.3) If k is only linked to i and j and to another strong player that is only linked to j then the link ik will never be used. (a.4) If k is only linked to i and j and to another strong player that is linked to another weak player ($\neq j$) then player i has incentives to delete the link ik.

(b) In g the strong player i is only linked to weak players $j, k \in W$ (at least two). Suppose that we add the link il to g to form $g + il$ where $i, l \in S$. (b.1) If l is only linked to i and j then player l has incentives to delete the link il. (b.2) If l is only linked to i and j and to another weak player then the link il will never be used. (b.3) If l is only linked to i and j and to another strong player $m \in S$ that is only linked to j, then this strong player m has incentives to delete the link lm since player m is in the position of player i in case (a.4). (b.4) If l is only linked to i and j and to another strong player $m \in S$ that is linked to another weak player n ($\neq j$), then the the link lm is never used if $n \neq k$ and player m has incentives to delete the link lm if $n = k$.

(c) In g the strong player i is only linked to one weak player $j \in W$ and to a strong player $k \in S$ that is only linked to i and j. Suppose that we add the link il to g to form $g + il$ where $i, l \in S$. In $g + il$ player k has incentives to delete the link ik since player k is in the position of player i in case (a.4).

(d) In g the strong player i is only linked to one weak player $j \in W$ and to a strong player $k \in S$ that is only linked to i and to another weak player $m \neq j$ ($kj \notin g$). Suppose that we add the link il to g to form $g + il$ where $i, l \in S$. In $g + il$ player l has incentives to delete the link il if he is only linked to j and player l has also incentives to delete the link to another weak player if this link exists in $g + il$.

(e) In g the strong player i is only linked to one weak player $j \in W$ and to a strong player $k \in S$ that is linked to i and j and to another weak player $m \neq j$ ($kj \in g$). Suppose that we add the link il to g to form $g+il$ where $i, l \in S$. (e.1) If l is linked only to j and i then l has incentives to delete the link il to avoid this link being used when l is the seller (notice that l is never intermediary in $g + il$). (e.2) If l is linked only to j and i and to another weak player, then i has incentives to delete the link il either to avoid this link being used when i is the seller or because this link is not used. □

Proof of Lemma 4.

We now show that g cannot be pairwise stable if there is some link between two strong players that are not linked to the same weak player. From Proposition 2, Lemma 1, Lemma 2 and Lemma 3 we know which networks are the candidates for being pairwise stable networks. Hence, take any network g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^W \subseteq g$, (iii) for each $i \in S$ there is $j \in W$ such that $ij \in g$, (iv) $ij \notin g$ if $i, j \in S$ and there is some $k \in W$ such that $ik \in g$ and $jk \in g$.

(a) Suppose that $ik \in g$, $jl \in g$ and $ij \notin g$ where $i, j \in S$ and $k, l \in W$. Suppose that we add the link ij to g to form $g+ij$ where $i, j \in S$. (a.1) If i and j do not have other links then i has incentives to delete the link ij. By deleting the link ij she is only loosing the payoff she obtains as an intermediary for the match (j, l) in $g + ij$. This loss is compensated by the gains she makes by shortening the trading path for the match (i, k) in g. (a.2) If j is linked to another weak player (say $m \in W$) then i would have more incentives than in (a.1) to delete ij since she would earn less from the match (i, k) in $g + ij$ and she would get nothing from the matches (j, l) and (j, m). (a.3) If j is linked to another strong player ($\neq i$) then i has more incentives than in (a.1) to delete ij. (a.4) If i is linked to at least two weak players then (i) if j is also linked to at least two weak players then ij is not used, (ii) if j is linked to one weak player then j has incentives to delete ij since j is in the position of i in case (a.2).

(b) The last case to be considered is when in g strong players are linked to all of them (that is, $g^S \subseteq g$) but each strong player is linked to a different weak player. Suppose that $il \in g$, $jm \in g$ and $kn \in g$ where $i, j, k \in S$ and $l, m, n \in W$. Suppose that we add the link im to g to form $g + im$. For player i the link im only modifies her payoff from the match (i, l). With the link im the trading path is shorter and so, player i has incentives to add the link im. By adding the link im player m makes additional gains from the matches (m, i) and (k, i) for $k \neq j$, $k \in S$, but he makes losses from the matches (i, k) for $k \neq j$, $k \in S$. However, the losses are much smaller than the gains. In all other matches nothing changes for player m. Hence, player m has also incentives to add the link im to g, and so we have that g is not pairwise stable. Once we have added the link im to g, we have obtained a network $g + im$ where two strong players i and j are linked to the same weak player m and we know from Lemma 3 that such network cannot be pairwise stable. □

Proof of Lemma 5.

From Proposition 2, Lemma 1, Lemma 2, Lemma 3 and Lemma 4 we know that the candidates for being pairwise stable are networks g such that (i) $\#C(g) = 1$ and $N(g) = N$, (ii) $g^W \subseteq g$, (iii) $ij \notin g$ if $i \in S$ and $j \in S$. We now show that g cannot be pairwise stable if some strong player $i \in S$ is linked to more than one weak player. Suppose that in g player $i \in S$ is linked to two weak players $k, l \in W$. When i is the buyer she is indifferent between g and $g - ik$. Notice that player is never an intermediary in g nor in $g - ik$. Suppose now that i is the seller. When she is matched to a weak player $m \neq k$ she is better off by deleting the link ik since the equilibrium trading path is shortened of one link, and when she is matched to the weak player k she is equal off by deleting the link ik. When player i (as a seller) is matched to a strong player that is not linked to player k she is better off by deleting the link ik since the equilibrium trading path is shortened of one link, and when she is matched to a strong player that is linked to the weak player k (and not to player l) she is equal off by deleting the link ik. Finally, when player i (as a seller) is matched to a strong player that is linked to player l she is better off by deleting the link ik since the equilibrium trading path is shortened of one link between two weak players. □

C Core periphery networks

Consider any core periphery network g such that (i) $g^W \subseteq g$, (ii) $d_i(g) = 1$ for all $i \in S$, (iii) $d_i(g) = n - 1$ for some $i \in W$, and (iii) $\#C(g) = 1$. That is, we consider core periphery networks where the core consists of weak players who are linked to each other and the periphery consists of strong players who are only linked to the same weak player. Figure 5 illustrates a core periphery network with three weak players $\{1, 2, 3\}$ and three strong players $\{4, 5, 6\}$ and where all strong players are only linked to the weak player 1.

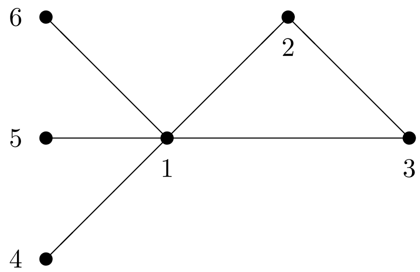

Figure 5: A core periphery network with one weak player linked to all strong players.

The SPE expected payoff for a strong player $i \in S$ in such core periphery trading network g is equal to

$$U_i(g) = \frac{1}{2(n-1)} \frac{r_W}{r_W + r_S} \left(n + \frac{m-1}{2} + (n-m-1) \frac{r_S}{r_W + r_S} \right),$$

and the SPE expected payoff for the weak player $j \in W$ who is linked to all strong players is equal to

$$U_j(g) = \frac{1}{2(n-1)} \left(m - 1 + \frac{r_S}{r_W + r_S} (n-m) \left(m + 1 + 2(n-m-1) \frac{r_S}{r_W + r_S} \right) \right).$$

Suppose $m = 1$. If $2(n-1)r_S > nr_W$ then the SPE expected payoff for the weak player $j \in W$ who is linked to all strong players is greater than the SPE expected payoff for a strong player $i \in S$. However, if $2(n-1)r_S < r_W$ then the SPE expected payoff for the weak player $j \in W$ who is linked to all strong players is smaller than the SPE expected payoff for a strong player $i \in S$. Notice that those core periphery networks give to the strong players (and to player $i \in W$ for which $d_i(g) = n - 1$) their best payoffs among pairwise stable networks.

D Private information and maximum delay

Consider again the path $(i_0, i_1, i_2, ..., i_k, i_{k+1})$ that connects seller s (player i_0) to buyer b (player i_{k+1}). Players negotiate how to split the surplus via successive bilateral bargaining sessions in the following order: (i_k, i_{k+1}), (i_{k-1}, i_k), (i_{k-2}, i_{k-1}), ..., (i_1, i_2), (i_0, i_1). Suppose now that the players have private information. They are uncertain about each others' discount factors. Player i's discount factor lies in the range $[\delta_i^{\mathrm{I}}, \delta_i^{\mathrm{P}}]$, where $0 < \delta_i^{\mathrm{I}} \leq \delta_i^{\mathrm{P}} < 1$. The types are independently drawn from the interval $[\delta_i^{\mathrm{P}}, \delta_i^{\mathrm{I}}]$ according to the probability distribution p_i, $i \in N$.

Lemma 6. *Consider the sequence (i_k, i_{k+1}), (i_{k-1}, i_k), (i_{k-2}, i_{k-1}), ..., (i_1, i_2), (i_0, i_1) of $k+1$ bilateral bargaining sessions with private information in which the probability distributions are common knowledge and in which the period length shrinks to zero. For any perfect Bayesian equilibria, the payoff of player i_{k+1-l} in each bilateral bargaining session (i_{k-l}, i_{k+1-l}) belongs to*

$$\left[\frac{\delta^I_{i_{k+1-l}}\left(1-\delta^P_{i_{k-l}}\right)}{1-\delta^P_{i_{k-l}}\delta^I_{i_{k+1-l}}}y_l, \frac{\delta^P_{i_{k+1-l}}\left(1-\delta^I_{i_{k-l}}\right)}{1-\delta^I_{i_{k-l}}\delta^P_{i_{k+1-l}}}y_l\right],$$

for $l = 0, ..., k$, where y_l is the surplus left to be shared after players i_j $(j > k+1-l)$ have taken their shares.

This lemma follows from Watson (1998) Theorem 1. Whether or not all payoffs within the intervals given in Lemma 6 are possible depends on the distributions over types. As Watson (1998) stated "each player will be no worse than he would be in equilibrium if it were common knowledge that he were his least patient type and the opponent were his most patient type. Furthermore, each player will be no better than he would be in equilibrium with the roles reversed". Since we allow for general probability distributions over discount factors, multiplicity of perfect Bayesian equilibria (PBE) is not an exception (even when the game is almost with complete information).

The maximum number of bargaining periods player i_{k+1-l} would spend negotiating in the bilateral bargaining session (i_{k-l}, i_{k+1-l}), $I\left(m(i_{k-l}, i_{k+1-l})\right)$, is given by

$$\frac{\delta^{\mathrm{I}}_{i_{k+1-l}}\left(1-\delta^{\mathrm{P}}_{i_{k-l}}\right)}{1-\delta^{\mathrm{P}}_{i_{k-l}}\delta^{\mathrm{I}}_{i_{k+1-l}}}y_l = \left(\delta^{\mathrm{P}}_{i_{k+1-l}}\right)^{m(i_{k-l}, i_{k+1-l})}\frac{\delta^{\mathrm{P}}_{i_{k+1-l}}\left(1-\delta^{\mathrm{I}}_{i_{k-l}}\right)}{1-\delta^{\mathrm{I}}_{i_{k-l}}\delta^{\mathrm{P}}_{i_{k+1-l}}}y_l,$$

from which we obtain

$$m(i_{k-l}, i_{k+1-l}) = \frac{1}{\log(\delta^{\mathrm{P}}_{i_{k+1-l}})}\log\left[\frac{\delta^{\mathrm{I}}_{i_{k+1-l}}}{\delta^{\mathrm{P}}_{i_{k+1-l}}}\frac{1-\delta^{\mathrm{P}}_{i_{k-l}}}{1-\delta^{\mathrm{I}}_{i_{k-l}}}\frac{1-\delta^{\mathrm{I}}_{i_{k-l}}\delta^{\mathrm{P}}_{i_{k+1-l}}}{1-\delta^{\mathrm{P}}_{i_{k-l}}\delta^{\mathrm{I}}_{i_{k+1-l}}}\right].$$

Notice that $I\left(m(i_{k-l},i_{k+1-l})\right)$ is simply the integer part of $m(i_{k-l},i_{k+1-l})$. It is customary to express the players' discount factors in terms of discount rates, $r_i > 0$, and the length of the bargaining period, Δ, according to the formula $\delta_i = \exp\left(-r_i\Delta\right)$. With this interpretation, player i's type is identified with the discount rate r_i, where $r_i \in [r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$. We thus have that $\delta_i^{\mathrm{I}} = \exp(-r_i^{\mathrm{I}}\Delta)$ and $\delta_i^{\mathrm{P}} = \exp(-r_i^{\mathrm{P}}\Delta)$. Note that $r_i^{\mathrm{I}} \geq r_i^{\mathrm{P}}$ since greater patience implies a lower discount rate. As Δ approaches zero, using l'Hopital's rule we obtain that

$$D(i_{k-l},i_{k+1-l}) = \lim_{\Delta\to 0}\left(m(i_{k-l},i_{k+1-l})\cdot\Delta\right) = -\frac{1}{r_{k+1-l}^{\mathrm{P}}}\cdot\log\left[\frac{r_{k-l}^{\mathrm{P}}}{r_{k+1-l}^{\mathrm{I}}}\cdot\frac{r_{k-l}^{\mathrm{I}}+r_{k+1-l}^{\mathrm{P}}}{r_{k-l}^{\mathrm{P}}+r_{k+1-l}^{\mathrm{I}}}\right],$$

which is a positive, finite number. Notice that $D(i_{k-l},i_{k+1-l})$ converges to zero as r_i^{P} and r_i^{I} become close. Given the equilibrium trading path $(s,i_1,i_2,...,i_k,b)$, the maximum real delay time in reaching a global agreement is $D(s,i_1,i_2,...,i_k,b) = D(s,i_1)+D(i_1,i_2)+...+D(i_k,b)$.

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