On the Feasibility of Perpetual Growth in a Decentralized Economy Subject to Environmental Constraints

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Abstract

We propose an endogenous growth model of a decentralized economy subject to environmental constraints. In a basic version, we consider an economy where final production requires some material input and where research activities allow simultaneously productive firms to reduce the dependency of their production process on this input and to improve the quality of their output. We adopt a material balance approach and, in spite of the optimistic assumption that the material input is perfectly recyclable (and thus never exhausted), we show that material output growth is always a transitory phenomenon. When it exists, a balanced growth path is necessarily characterized by constant values of the material variables, long term economic growth taking exclusively the form of perpetual improvements in the quality of consumption goods. The material resource constraint is not solely a long term issue since it is also shown to affect the whole transitory dynamics of the (material) growth process. Renewable energy is introduced in an extension of our basic model. This extension does not affect qualitatively the features of a feasible balanced growth path but make its conditions of existence more restrictive.

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1 Introduction

An important literature testifies to the continuous debate around the physical limits to growth[1]. If one considers the controversies between economists, one can schematically distinguish two antagonist positions. The tenants of the first and most optimistic position consider that long run economic growth is possible within a finite world thanks to substitutions between natural resources and man-made inputs and to technical progress. This position is best epitomized by the contributions of Dasgupta and Heal, Solow and Stiglitz to the Review of Economic Studies symposium on the Economics of exhaustible resources (1974) but many other contributions followed. This position has been remaining very influential and the vast majority of the contributions to the theory of endogenous growth theory did not incorporate any environmental consideration[2], which reflects indirectly that the environment has not necessarily been considered as a key issue for long term economic growth.

The second position is much more pessimistic about the long run growth prospects in a finite world. It was first built on a critical appraisal of the representation of the production process in neoclassical growth theory. Following Georgescu-Roegen (1971), ecological economists (see e.g. Cleveland and Ruth (1997), Daly (1997)) consider that neoclassical growth models rely on much too optimistic assumptions about substitution possibilities between natural and man-made inputs and about how they can be affected by the technological progress. Ecological economists outline in particular that the neoclassical growth models ignore the physical laws (the conservation laws of matter and energy and the second principal of thermodynamics) that govern the transformation process of matter and energy in all human activities, in particular the production of goods and services[3]. For instance, Islam (1985) and Anderson (1987) illustrate how thermodynamic laws limit the substitution elasticities between natural and man made inputs and exclude in particular a Cobb-Douglas production function of all these inputs. Anderson also shows how the material balance principle constraints the asymptotic behaviour of the production function

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[1]We resume here the title of the well-known controversial book by Meadows et al. (2004), first published in 1972. Another shortcut expression to refer to the constrained capacity of the environment to sustain economic growth is scarcity and growth, following Barnett and Morse (1963).

[2]The bestseller textbooks on economic growth offer a good illustration of this statement: Even in its second edition of 2004, the 600-page book by Barro and Sala i Martin does not content a word on environmental matters of any type. Aghion and Howitt's book only contains a very short chapter on growth and non renewable resources. The recent 900-page book by Daron Acemoglu does not mention the environmental issues.

[3]Already in the eighties and early nineties, some theoretical works in an exogenous growth setting (a.o. Ayres and Miller (1980), Germain (1991), Ruth (1993)) proposed models consistent with the ecological economists' criticisms and all showed that the physical limits to growth then appeared to be much more stringent than in a purely neoclassical setting. Two other contributions using production functions consistent with physical principles are van den Bergh and Nijkamp (1994) and Ayres and van den Bergh (2005).

(i.e. when the man-made input tends to infinity). In the same spirit, Baumgärtner (2004) proves that the Inada conditions for material resource (a usual assumption in growth models that include a material resource as a production factor in an otherwise standard production function) are inconsistent with the law of conservation of mass because this law implies that the marginal and average products of a material resource are bounded from above. In a theoretical general equilibrium setting, Krysiak and Krysiak (2003) show that commonly used production functions (including the CES) are inconsistent with the physical laws of matter and energy conservation.

Progressively, contributions to the theory of endogenous growth have been interested in the question of long term growth in the presence of natural resources and/or pollution. But rather surprisingly, the vast majority of those papers (even among the recent ones) have proposed models that disregard the laws of physics and ignore the ecological economists' criticisms to the representation of the production process in the neoclassical growth theory. For instance, Grimaud and Rougé (2003, 2005), Groth (2004), Groth and Schou (2007) build models in which the natural resource is one of the production factors of a Cobb-Douglas technology; Stockey (1998), Hart (2004) propose growth models with pollution in which no material flow is explicitly described.

Other contributions (see a.o. Bretschger (2005), Smulders (1995a,b, 2003), Bretschger and Smulders (2004), Akao and Managi (2007), Pittel et al (2006)) have aimed at a more thorough representation of the environmental constraints. Taking into account some implications of the physical laws, these authors show that growth can be sustained in the long run thanks to investment in knowledge capital through research and development. Even with constant flows of energy and matters, they suggest that it is possible to derive more productivity and utility thanks to goods and services of a rising quality.

But in spite of their intention to do so, it is not clear that those papers meet completely the critics of ecological economics. In a first instance, Krysiak (2006) underlines that the result of unlimited growth obtained in certain models (a.o. Smulders, 1995a,b) follows from the assumption that human capital and/or knowledge are produced without the use of matter and/or energy. It is however sure that all activities (including R&D) require matter and energy even when these inputs are not as such embodied in the produced output. Akao and Managi (2007) make the assumption of a Cobb Doublas technology in a part of their paper. Pittel *et al.* (2006) make a technological assumption that ignores the results of Anderson or Baumgärtner (op citum). As a result, in Akao and Managi (2007) or Pittel et al (2006), sustainable growth is characterized by a complete dematerialisation of the homogenous final output (the material resource content of a unit of final output becoming infinitely small).

Our paper is concerned with the feasibility of sustained growth in a finite world framework where the

environment is considered as both a source (supplier of raw materials and other services) and a sink (where wastes and pollutants end up) and where the physical laws (in particular the law of mass conservation) and their economic implications are explicitly taken into account. It aims at bridging the gap between ecological economics and endogenous growth models.

In our model, each final good or service is described by four explicit characteristics: on the one hand, quantity and price as usual in economics, on the other hand, quality and physical content (in terms of mass and/or energy). An important part of our contribution relies on the explicit distinction between the two latter characteristics. Obviously enough, quality and physical content are not necessarily orthogonal characteritics: for instance, the mass of a laptop -at given computational power- can be seen as an aspect of its quality. But the quality of a production cannot be summarized to its physical content (two laptops with the same mass are not necessarily equally powerful, ergonomic,...). In our model, these two characteristics will be endogenously affected by research activities from firms. By distinguishing these characteristics explicitly, we can outline that there is a fundamental asymmetry in the way in which technological progress can change them: on the one hand, there is a priori no upper bound on the quality level that a production may reach thanks to perpetual improvements in knowledge; on the other hand, there is a lower bound on the minimal physical content of human productions. All activities (including research itself) need matter and energy. In other words, even though technological progress is a priori an unbounded process from the point of view of the quality of the human productions, there is however an impossibility of a complete dematerialization of the final goods and/or of the production processes of those goods. This is the key assumption of our model. It is supported by both empirical and theoretical arguments. Empirically speaking (or in terms of descriptive realism), final productions have and will always have some material content: even though totally immaterial services may be developed, some productions remain and will always remain partly material, a.o. those productions such as food, clothes, housing, pharmaceuticals,... Furthermore, even the production of purely immaterial services requires (man-made) capital inputs that have a minimum material content (such as tools, machines, vehicles, cables,...). Of course, technological progress as well as the sectoral reallocations of final productions can increase the degree of the dematerialization of human productions but a state of complete dematerialization of aggregate output and of its production process is only an intellectual curiosity. Our assumption also receives some theoretical support as it is the logical consequence of the conclusions of the above quoted works by Anderson (1987) or Baumgärtner (2004).

In the present paper, we do not deal with the exhaustion of non renewable resources. This is certainly an important issue but we think that its impact on economic growth is rather well covered by the economic literature (at least from a theoretical viewpoint): as is well-known, the exhaustion of a non renewable

resource that would remain both necessary and essential to production (in the sense of Dasgupta and Heal (1974)) imposes a physical limit to growth that technological progress can potentially postpone but not remove. More fundamentally, the choice of not dealing with the case of non renewable resources is also a modelling strategy that will allow us to stress more straightforwardly that there are physical limits to growth even in a ideal world where human productions would only used renewable resources[4]. That is the main reason why our basic model relies on an assumption of a perfect recycling of the material input. Even in this optimistic scenario of an economy that has found renewable substitutes to non-renewable resources, the scarcity of the (renewable or recyclable) resource[5] remains a crucial question. Given the impossibility of a complete dematerialization of man-made productions, the finiteness of the resource stock is in itself a limit to growth as fundamental as its possible depletion[6]: perpetual material growth is simply impossible on a finite earth. But the absence of a long term material growth does no mean that no type of economic growth is possible: we show that thanks to research activities, a long term growth path may exist, path along which economic growth can only take the form of perpetual improvements in the quality of the final goods. We establish the existence conditions of such a long term growth path. We show that it may fail to exist in a decentralized framework even though it is quite feasible from a purely physical point of view. Limits to growth do thus not follow exclusively from the sole physical constraints but also from the interactions between these constraints and the economic behaviours of decentralized agents.

Incidentally and in opposition to some growth theorists who argue that environmental constraints have not been an impediment to the economic growth of the OECD countries over the last 2 centuries and might thus only matter for the very long run, we also show that the law of mass conservation does not only shape the type of economic growth that is possible in the long run: it may also matter in the short and medium runs since it can affect the whole transitory dynamics of the growth process[7].

Section 2 presents our basic model. The general equilibrium of the economy and its dynamics are described in section 3. Section 4 deals with the properties and the existence of the balance growth path. In section 5, the transitory dynamics of the economy is illustrated numerically. In section 6, energy is introduced in the model, which does not change the basic conclusion of our first model. The only possible type of long run economic growth is qualitative. However such a long run growth regime is shown to exist under more restrictive conditions than in the basic model.

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[4]Akao and Managi (2007) show that another limit to growth may follow from the environmental degradation caused by production/consumption activities.

[5]At a given point in time, any resource is never available without limit.

[6]The consequences of these two limits to growth are of course not the same.

[7]It is the case in our model with a perfectly renewable resource. Obviously enough, it would be even more the case in a model with non renewable resources which can deplete during the transition process.

2 Agents' Behaviours, Tehnological and Physical Constraints

The economy consists of two types of long-living agents: monopolistic firms and households. Monopolistic firms produce final goods that can be purchased as investment or consumption goods. Their production technology requires a material input and productive capital. Each period the firms take three types of decisions: they choose a production/price policy, a research effort level and an investment level that will determine the available capital stock in the following period. Households receive the whole macroeconomic income, consume and save.

There are two types of markets: the markets for the monopolistic final goods market and a financial market on which monopolistic firms borrow funds from households.

In the first version of the model, the economy is subject to only one environmental constraint: the availability of the material resource. The evolution of the stock resource is the net flow of material following from production on the one hand and recycling on the other hand. Each period indeed, a part of the resource stock is extracted and enters as input into the production process; during the same period, production and consumption activities give rise to a waste of material that can be recycled and will return to the available stock of material at the beginning of the next period. As depicted in figure 1 below, material waste occurs at three levels. First, during the production process, a part of the material input is wasted (only a part of the input being incorporated into the final production). Second, the material content of the consumption goods is wasted after consumption. Similarly, the material content of the investment goods is wasted once capital goods become obsolete. We assume that these wastes of material are perfectly and freely recyclable.

2.1 Description of the production sector

2.1.1 Technology

There is a continuum of monopolistic firms defined on $[0, 1]$. A firm $j \in [0, 1]$ is the only producer of good j, which can be used for final consumption or investment. This production requires 2 factors: a (natural) material resource, called MR hereafter, and productive capital.

To produce a quantity y_{jt} of good j in period t, firm j thus needs a quantity x_{jt} of natural resource given by:

$$x_{jt} = [\chi_t + \mu_t] \, y_{jt}. \tag{1}$$

$\mu_t > 0$ is a quantity (in mass units) of MR incorporated in a unit of good j; $\chi_t > 0$ is a quantity (in

mass units) of MR that is wasted during the production process. This waste is assumed to be perfectly recyclable. Variables χ_t and μ_t are common to all firms and describe the dependency of the current technology on the natural resource. Both variables are affected by an endogenous technical progress that is exogenous at the firm level and follows from an external effect linked to the past research activities of all firms (see subsection 2.2).

In period t, a unit of good j has a mass m_{jt}. This mass corresponds to the mass of the natural resource incorporated in good j in period t, i.e.,

$$m_{jt} = \mu_t. \tag{2}$$

MR is a free common resource. Its transformation process however requires physical capital. To handle a quantity x_{jt} of RN, firm j needs a quantity of productive capital $_pk_{jt}$ given by

$$_pk_{jt} = \frac{x_{jt}}{1 - \frac{X_t}{R_t}} \tag{3}$$

where R_t is the stock of MR at the beginning of period t and X_t is the total quantity of MR used by all firms during the period. The higher the extraction rate, the more the extraction process is capital intensive. Our technological assumption captures the intuition that the cost of the exploitation of the resource depends on the size of the available stock: extraction costs are likely to increase when a bigger quantity is extracted and a lower stock remains available (Lin et Wagner, 2007). Numerous real examples testify that the exploitation of natural resources (in particular non renewable resources like coal) becomes non profitable largely before the exhaustion of the ressouce[8].

2.1.2 Research and Innovation

Firms may invest in research and development. Research has an instantaneous microeconomic impact: a firm that makes research improves instantaneously[9] the quality of its output, which stimulates final customers' demand at given output price (see sections 2.3 and 3.2). However, a firm active in research during a given period cannot appropriate itself its research results for longer than that period. Research results next become public knowledge.

The research process and the diffusion of its results are formalized as follows. All firms enter a given period t with the same level of knowledge Q_{t-1}, which is the public heritage of the former private research efforts. If a given firm j then invests in research, it raises the quality q_{jt} of its output above Q_{t-1} and can appropriate itself the return on its current quality improvement by enjoying a larger demand from

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[8]This argument is also in line with Meadows and al., (2004) who suggest that the limits to growth due to resource scarcity should be understood as a problem of rising costs and not of a physical exhaustion.

[9]For simplicity, we assume that there is no time-to-build effect in the quality improvement following from research.

costumers during the same period. The research technology is assumed to be deterministic: in order to improve the quality level at a level q_{jt} above Q_{t-1}, the research department of firm j must be endowed with a capital stock $_r k_{jt}$ given by

$$_r k_{jt} = h\left(\frac{q_{jt}}{Q_{t-1}}\right) y_{jt}, \tag{4}$$

where $h(\cdot)$ is an increasing and convex function which satisfies $h(1) = 0$ (no research investment is required to maintain the quality level unchanged).

At the end of period t, the results of individual research activities become a public good: in the next period, all firms will have a free access to a quality level Q_t corresponding to the highest quality level q_{jt} reached in t:

$$Q_t = \max \; \{q_{jt}; j \in [0, 1]\}. \tag{5}$$

Moreover there is a dynamic external effect following from individual research efforts: the past research efforts make the current production process less material resource consuming:

$$\chi_t = \chi(Q_{t-1}) \quad \text{with} \quad \chi'(\cdot) < 0 \tag{6}$$

$$\mu_t = \mu(Q_{t-1}) \quad \text{with} \quad \mu'(\cdot) < 0. \tag{7}$$

However, we assume that the production process cannot reach a state of complete dematerialization in which a unit of good j would be produced from an infinitesimal quantity of MR: both functions χ and μ are thus bounded from below :

$$\lim_{Q \to +\infty} \chi(Q) = \underline{\chi} > 0 \quad \text{and} \quad \lim_{Q \to +\infty} \mu(Q) = \underline{\mu} > 0. \tag{8}$$

2.1.3   Productive capital requirements

Given (1), (3) and (4), the total capital stock requirement of firm j during period t is linked to its production and target quality levels as follows:

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\begin{aligned}
k_{jt} &= {}_p k_{jt} + {}_r k_{jt} \\
&= \left[\frac{\chi(Q_{t-1}) + \mu(Q_{t-1})}{1 - \frac{X_t}{R_t}} + h\left(\frac{q_{jt}}{Q_{t-1}}\right)\right] y_{jt}.
\end{aligned} \tag{9}
$$

2.1.4   Capital stock accumulation

For simplicity we assume a unitary depreciation rate. A given firm j builds its capital stock by purchasing and combining the different final goods produced by the monopolistic firms. There is a one period time-to-build and if firm j purchases ι_{jit} units of each investment good $i \in [0, 1]$ in t, its capital stock in $t + 1$

will be

$$k_{jt+1}^{\alpha} = \int_0^1 [\varphi(q_{it})\iota_{jit}]^{\alpha} \, di, \tag{10}$$

where $0 < \alpha < 1$ and $\varphi(\cdot)$ is a continuous and monotonically increasing function of the quality level of the final goods. We will note $\Phi(q_t)$ the positive elasticity of function $\varphi(\cdot)$ with respect to q_t. The productive capital stock k_{jt+1} thus incorporates the quality of the investment goods used to build it.

We assume that the productive capital stock cannot reach a state of complete dematerialization: it is not possible to build a given level of productive capital stock from an infinitely small quantity of material investment good. That is, the quality function $\varphi(\cdot)$ is bounded from above:

$$\lim_{q \to +\infty} \varphi(q) = \bar{\varphi} < \infty, \tag{11}$$

which also means that $\lim_{q \to +\infty} \Phi(q) = 0$. This assumption reflects the idea that the production process of the material goods cannot itself be completely dematerialized: the productive tool and/or the productive infrastructure must have some material content. Similarly, the research process at the origin of the production of knowledge cannot rely on a purely immaterial input.

If firm j wants to use a given capital stock k_{jt+1} in $t+1$, it will determine its purchases of the different final goods so as minimize its total investment cost, i.e., it will solve

$$\min_{\{\iota_{jit}\}_i} \int_0^1 p_{it}\iota_{jit}di$$

subject to (10) with k_{jt+1} given.

Such a cost minimization problem is fairly standard and leads to the following investment demands:

$$\iota_{jit} = \varphi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{\bar{p}_t}\right]^{-\varepsilon} k_{jt+1}, \quad \forall \, i \tag{12}$$

where $\varepsilon = \frac{1}{1-\alpha}$ and the investment price index (defined as $\bar{p}_t$ such that $\bar{p}_t k_{jt+1} = \int_0^1 p_{it}\iota_{jit}di$) is given by

$$\bar{p}_t^{1-\varepsilon} = \int_0^1 \left[\frac{p_{it}}{\varphi(q_{it})}\right]^{1-\varepsilon} \, di. \tag{13}$$

2.2   Consumer behaviour

We consider a representative and long-living agent who consumes the final goods and accumulates financial wealth. She receives the whole aggregate macroeconomic income under the form of interest rate payments and profits.

The consumer's preferences are representable by the following intertemporal utility function[10]

$$\sum_{t=1}^{T} \beta^t \ln(C_t) \quad \text{with} \quad 0 < \beta < 1$$

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[10]The assumption of a logarithmic utility function is only made for analytical convenience.

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The time horizon T is supposed to very long and possibly infinite. C_t is a final consumption index *à la* Dixit-Stiglitz (1976) defined over the continuum of final goods as follows:

$$C_t^\alpha = \int_0^1 [\psi(q_{it})c_{it}]^\alpha \, di,$$

where $0 < \alpha < 1$. $\psi(\cdot)$ is a continuous and monotonically increasing function of the quality level q_{it}. We will note $\Psi(q_t)$ the positive elasticity of function $\psi(\cdot)$ with respect to q_t. We assume that the welfare impact of a rising quality level is not necessarily bounded: equivalently, the asymptotic value of $\Psi(q_t)$ remains positive, i.e., $\lim_{q_t \to \infty} \Psi(q_t) = \Psi \geq 0$

As the consumer's preferences are time separable, the intertemporal consumption/saving decision can be analysed independently of the choice of the composition of the final consumption bundle in each period.

2.2.1 Intertemporal allocation of income

Let Ω_t be the consumer's financial wealth at the beginning of period $t \geq 1$. The consumer chooses her intertemporal consumption profile so as to solve the following problem where final consumption is used as *numéraire*.

$$\max_{\{C_t\}_{t,\dots,T}} \sum_{t=1}^T \ln(C_t)\beta^t \tag{14}$$

subject to

$$\Omega_{t+1} = \Omega_t [1 + r_t] + \pi_t - C_t, \forall t \geq 1 \; \Omega_1 \text{ given} \tag{15}$$

$$\Omega_{T+1} \geq 0.$$

where r_t is the real interest rate and π_t is the aggregate value of firms' profits.

The optimal consumption path must satisfy the following first-order condition describing the usual consumption smoothing behaviour:

$$\frac{1}{C_t} = \beta [1 + r_{t+1}] \frac{1}{C_{t+1}}, \; t \geq 1, \tag{16}$$

and last period consumption must satisfy the terminal condition $\Omega_{T+1} = 0$, i.e. $C_T = (1 + r_T)\Omega_T + \pi_T$.

2.2.2 Intratemporal consumption choice

For a given level of the consumption index, C_t, the consumer chooses her consumption bundle so as to minimize its cost:

$$\min_{c_{it}} \int_0^1 p_{it} c_{it} di$$

subject to

$$\int_0^1 [\psi(q_{it})c_{it}]^\alpha \, di = C_t^\alpha$$

where p_{it} is the price of good $i \in [0,1]$ in t.

Solving this problem is fairly standard and leads to the following optimality conditions:

$$c_{it} = \psi^{\varepsilon-1}(q_{it}) \left[\frac{p_{it}}{\bar{p}_{ct}}\right]^{-\varepsilon} C_t, \forall \, i \in \ [0,1], \tag{17}$$

where $\bar{p}_{ct}$ is the consumption price index given by

$$\bar{p}_{ct}^{1-\varepsilon} = \int_0^1 \left[\frac{p_{it}}{\psi(q_{it})}\right]^{1-\varepsilon} di = 1. \tag{18}$$

This price index is normalized to 1 since the final consumption index is used as numéraire.

2.3   Price and Research Decisions of Monopolistic Firms

2.3.1   Total demand for good j

Given (12), the total investment demand for good j is

$$\begin{aligned}
d_{jt} &= \int_0^1 \iota_{ijt} di \\
&= \int_0^1 \varphi^{\varepsilon-1}(q_{jt}) \left[\frac{p_{jt}}{\bar{p}_t}\right]^{-\varepsilon} k_{it+1} di \\
&= \varphi^{\varepsilon-1}(q_{jt}) \left[\frac{p_{jt}}{\bar{p}_t}\right]^{-\varepsilon} I_t
\end{aligned} \tag{19}$$

where $I_t =_{def} \int_0^1 k_{it+1} di$.

Given (17) and (19), total consumption and investment demand for good j is thus

$$y_{jt} = c_{jt} + d_{jt} = \psi^{\varepsilon-1}(q_{jt}) \left[\frac{p_{jt}}{1}\right]^{-\varepsilon} C_t + \varphi^{\varepsilon-1}(q_{jt}) \left[\frac{p_{jt}}{\bar{p}_t}\right]^{-\varepsilon} I_t. \tag{20}$$

2.3.2   Profit maximization of firm j

During period t, firm j uses a predetermined capital stock level k_{jt}. It must choose its price policy p_{jt} and the quality level q_{jt} of its current output (which amounts to determining what parts of the existing capital stock are allocated respectively to production activities and to R&D). Moreover, firm j must decide on its current investment level, which will determine its next period capital stock $k_{j,t+1}$.

Firm j makes those decisions so as to maximize the following intertemporal profit function

$$\max_{\{y_{jt}, q_{jt}, p_{jt}, k_{j,t+1}\}_{t \geq 1}} \sum_{t=1}^T \frac{p_{jt} y_{jt} - \bar{p}_t k_{j,t+1}}{\Pi_{\tau=1}^t [1 + r_\tau]}$$

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subject, $\forall t \geq 1$, to (20) and

$$y_{jt} \leq \left[\frac{\chi(Q_{t-1}) + \mu(Q_{t-1})}{1 - \frac{X_t}{R_t}} + h\left(\frac{q_{jt}}{Q_{t-1}}\right) \right]^{-1} k_{jt} \tag{21}$$

$$q_{jt} \geq Q_{t-1} \tag{22}$$

$$k_{j1}, \quad Q_0 \quad \text{given}^{11}.$$

(21) (following from (9)) states simply that the firm's output cannot be larger than what can be produced with the productive capital stock (i.e., the capital stock that is not allocated to R&D)[12].

Let MC_{jt} denote the marginal cost of production in t. A marginal increase in output requires more MR (which is free) and more capital, the user cost of which is equal to $\bar{p}_{t-1} \cdot (1 + r_t)$: productive capital in t is purchased in $t-1$ at a price $\bar{p}_{t-1}$ and is financed by borrowing, which implies in t a total debt service of $\bar{p}_{t-1} \cdot (1 + r_t)$ per unit of capital purchased in $t-1$. The marginal production cost is the product of the user cost of capital by the marginal capital intensiveness of output $\partial y_{jt}/\partial k_{jt}$ (which is the inverse of the marginal productivity of capital): i.e.,

$$MC_{jt} = \bar{p}_{t-1} \cdot (1 + r_t) \cdot \left[\frac{\chi(Q_{t-1}) + \mu(Q_{t-1})}{1 - \frac{X_t}{R_t}} + h\left(\frac{q_{jt}}{Q_{t-1}}\right) \right] \tag{23}$$

where the term between brackets is also equal to k_{jt}/y_{jt} (see (9)).

Appendix 1 consists of the detailed resolution of the above problem. When constraint (22) is not binding (which will always be the case under the assumption that $h'(1) = 0$), the optimality conditions on the price and quality levels can be written respectively as follows

$$p_{jt} = \frac{\varepsilon}{\varepsilon - 1} \cdot MC_{jt} \tag{24}$$

$$(p_{jt} - MC_{jt}) \cdot \frac{\partial y_{jt}}{\partial q_{jt}} = \bar{p}_{t-1} \cdot (1 + r_t) \cdot \frac{\partial k_{jt}}{\partial q_{jt}}. \tag{25}$$

(24) corresponds to the standard monopolistic pricing behaviour: the monopolistic firm sets its price by marking up its marginal cost of production. (25) states that the optimal quality level must equalize the marginal benefit and cost of a quality improvement. The left-hand-side of (25) represents the marginal income following from a marginal increase in quality: a higher quality level increases demand for good j and thus firm j output and revenue at given price. The right-hand side of (25) represents the marginal cost of this quality improvement: to increase quality, the firm needs to allocate more capital to its research department and must thus support the user cost of capital for each additional unit of capital.

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[12]Note that an optimal choice of firm j requires that constraint (21) is always satisfied with strict equality (even in $t = 1$ with an exogenously given k_1). Obviously enough, should the capital stock be larger than what requires production, it would always be profitable to allocate the idle capital stock to research: this extra research effort would increase the output quality and would thereby allow the firm to sell profitably the same quantity of output at a higher price.

The values of p_{jt} and q_{jt} that are solutions to (24)-(25) next determine the corresponding output and investment choices. Moreover (and obviously enough), a profit maximizing firm makes no investment in T, i.e., chooses $k_{jT+1} = 0$.

For the sequel, it is useful to rewrite (25) in a more explicit way. After using (23), multiplying (25) by q_{jt}/y_{jt} gives

$$(p_{jt} - MC_{jt}) \cdot \eta^{jt}_{y \cdot q} = MC_{jt} \frac{y_{jt}}{k_{jt}} \cdot h' \left(\frac{q_{jt}}{Q_{t-1}} \right) \frac{q_{jt}}{Q_{t-1}} \tag{26}$$

where $\eta^{jt}_{y \cdot q}$ is the elasticity of demand for good j with respect to its quality level. Using (20), $\eta^{jt}_{y \cdot q}$ can be shown to be a weighted average of the elasticities on functions $\psi(\cdot)$ and $\varphi(\cdot)$ with respect to q_{jt}, i.e.

$$\eta^{jt}_{y \cdot q} = (\varepsilon - 1) \cdot \left[\Psi(q_{jt}) \frac{c_{jt}}{y_{jt}} + \Phi(q_{jt}) \frac{d_{jt}}{y_{jt}} \right]. \tag{27}$$

Using (24), (26) becomes:

$$\frac{\eta^{jt}_{y \cdot q}}{\epsilon - 1} \frac{k_{jt}}{y_{jt}} = h' \left(\frac{q_{jt}}{Q_{t-1}} \right) \frac{q_{jt}}{Q_{t-1}}. \tag{28}$$

2.4 Dynamics of the material resource stock

During a given period, the change in the available stock of the material resource is the net flow of material following from recycling activities on the one hand and the production process on the other hand.

As explained earlier, the production process and the use of the produced good lead to a waste of material:

- Material residuals are a by-product of the **production process**: when producing y_{jt}, firm j generates a quantity of residuals equal to $\chi(Q_{t-1})y_{jt}$. During a period t, all the production activities thus generate a total waste given by:

$$_pZ_t = \int_0^1 \chi(Q_{t-1})y_{jt}dj = \chi(Q_{t-1}) \int_0^1 y_{jt}dj. \tag{29}$$

- **Consumption** goods are non durable goods and their consumption leads instantaneously to a material residual. Given that the mass per unit of good i is m_{it}, the total mass of residuals following from consumption at time t is :

$$_cZ_t = \int_0^1 m_{it}c_{it}di = \mu(Q_{t-1}) \int_0^1 c_{it}di. \tag{30}$$

- A third flow of waste follows from **capital obsolescence**: At the end of period t, the productive capital stocks of the monopolistic firms get physically obsolete. Since each k_{jt} consists of investment goods bought in $t-1$ and each investment good i has a unit mass of m_{it-1}, the physical obsolescence

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of all $\{k_{jt}\}_{j\in[0,1]}$ gives rise the following flow of residuals :

$$
\begin{aligned}
{}_k Z_t &= \int_0^1 \int_0^1 m_{it-1} \iota_{jit-1} di dj \\
&= \mu(Q_{t-2}) \int_0^1 \int_0^1 \varphi^{\varepsilon-1}(q_{it-1}) \left[\frac{\bar{p}_{t-1}}{p_{it-1}} \right]^{\varepsilon} k_{jt} di dj
\end{aligned}
\tag{31}
$$

where the last equality follows from (2).

The above flows of material residuals are assumed to be perfectly and freely recyclable: they enter again into the stock of MR at the end of the period during which they have been discharged. The dynamic of the available stock of MR thus obeys to the following equation :

$$
R_{t+1} - R_t = {}_p Z_t + {}_c Z_t + {}_k Z_t - X_t
\tag{32}
$$

where X_t is the total use of the natural resource by final firms: $X_t = \int_0^1 x_{jt} dj$ with x_{it} given by (1).

3 General Equilibrium with Natural Ressource

At given Q_{t-1} and $\{k_{jt}\}_j$, the general equilibrium of period t is a system of prices $(\{p_{jt}\}_j, r_t)$, a vector of quantities $\{c_{jt}, k_{jt+1}, y_{jt}, x_{jt}, R_t\}_j$ and a vector of research efforts $\{q_{jt}\}_j$ such that

- Households' consumption choices satisfy the inter- and intratemporal optimality conditions (16) and (17) with a consumption price index given by (18);

- Firms take price, research and investment decisions (and thereby their output level) that maximize their intertemporal profits, i.e., satisfy (24), (28) and (12) with a capital price index given by (13);

- final goods markets clear and satisfy (20)[13];

- the natural resource stock obeys (32).

3.1 The symmetric equilibrium in the monopolistic sector

If all final firms have the same initial capital stock $k_{j1} = k_1 \ \forall j$, they all take the same decisions in period 1 and in all the subsequent periods. The monopolistic sector is characterized by a symmetric equilibrium:

$$
p_{jt} = p_t, q_{jt} = q_t \quad \text{and} \quad k_{jt+1} = k_{t+1}, \forall j \in [0,1] \quad \text{and} \quad t \geq 1.
$$

[13]By Walras law, the financial market clears as well: one can verify that the financial wealth of households is equal to the debt of the monopolistic firms: in each period, $\Omega_t = \bar{p}_{t-1} k_t$.

Therefore, (18) becomes

$$1(=\bar{p}_{ct}) = \frac{p_t}{\psi(q_t)} \quad \text{or} \quad p_t = \psi(q_t). \tag{33}$$

Similarly, the investment price index (13) becomes:

$$\bar{p}_t = \frac{p_t}{\varphi(q_t)} = \frac{\psi(q_t)}{\varphi(q_t)}. \tag{34}$$

Consumption and investment demands (17) and (19) are then the same for all goods j

$$c_{it} = c_t = \frac{C_t}{\psi(q_t)} \tag{35}$$

$$d_{it} = d_t = \frac{I_t}{\varphi(q_t)} \tag{36}$$

where $I_t = \int_0^1 k_{jt+1}dj = k_{t+1}$.

All firms produce the same output level $y_t = c_t + d_t$, also given by

$$y_t = \left[\frac{\chi(Q_{t-1}) + \mu(Q_{t-1})}{E_t} + h\left(\frac{q_t}{Q_{t-1}}\right) \right]^{-1} k_t \tag{37}$$

They use the same quantity of material resource $x_t = [\chi(Q_{t-1}) + \mu(Q_{t-1})]\, y_t$.

In a symmetric equilibrium, the optimality conditions on p_{jt} and q_{jt}, respectively (24) and (28), become:

$$\alpha\psi(q_t) = \frac{\psi(q_{t-1})}{\varphi(q_{t-1})}(1 + r_t)\frac{k_t}{y_t} \tag{38}$$

$$\frac{\eta_{y\cdot q}^t}{\epsilon - 1}\frac{k_t}{y_t} = h'\left(\frac{q_t}{Q_{t-1}}\right)\frac{q_t}{Q_{t-1}} \tag{39}$$

with $\alpha = 1 - 1/\epsilon$ and

$$\eta_{y\cdot q}^t = (\epsilon - 1) \cdot \left[\Psi(q_t)\frac{c_t}{y_t} + \Phi(q_t)\frac{d_t}{y_t} \right].$$

3.2 Dynamics of the natural resource

The dynamics of MR, (32), can thus be rewritten as:

$$\begin{aligned}
R_{t+1} - R_t &= \chi(Q_{t-1})y_t + \mu(Q_{t-1})c_t + \mu(Q_{t-2})\frac{k_t}{\varphi(q_{t-1})} - x_t \\
&= -\mu(Q_{t-1})\left[y_t - c_t\right] + \mu(Q_{t-2})\frac{k_t}{\varphi(q_{t-1})} \\
&= \mu(Q_{t-2})\frac{k_t}{\varphi(q_{t-1})} - \mu(Q_{t-1})\frac{k_{t+1}}{\varphi(q_t)}
\end{aligned}$$

where the penultimate equality follows from the relationship between x_t and y_t and the ultimate one follows from the identities $y_t = c_t + d_t$ and (36).

The ultimate equality translates the law of mass conservation in our framework: the total quantity of material in a closed system is necessarily constant: $\forall t \geq 1$,

$$R_{t+1} + \mu(Q_{t-1})\frac{k_{t+1}}{\varphi(q_t)} = R_t + \mu(Q_{t-2})\frac{k_t}{\varphi(q_{t-1})} = \mathcal{M}, \tag{40}$$

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where $\mathcal{M}$ denotes this constant quantity of material. At the beginning of period t, a part of $\mathcal{M}$ is in the stock of disposable resource R_t, the remaining part being embedded in the productive capital stock k_t.

3.3 Evolution of knowledge

Since firms make identical choices, the max condition (5) becomes simply

$$Q_t = q_t. \tag{41}$$

3.4 The dynamic system

In a symmetric equilibrium in the monopolistic sector and with price levels given by (33) and (34), the whole system of equations characterizing the economy in each period $t \geq 1$ can be written as follows:

$$\frac{1}{C_t} = \beta \left[1 + r_{t+1} \right] \frac{1}{C_{t+1}} \tag{42}$$

$$y_t = c_t + d_t \tag{43}$$

$$c_t = \frac{C_t}{\psi(q_t)} \tag{44}$$

$$d_t = \frac{k_{t+1}}{\varphi(q_t)} \tag{45}$$

$$\frac{k_t}{y_t} = \left[\frac{\chi(q_{t-1}) + \mu(q_{t-1})}{1 - \frac{x_t}{R_t}} + h\left(\frac{q_t}{q_{t-1}} \right) \right] \tag{46}$$

$$x_t = \left[\chi(q_{t-1}) + \mu(q_{t-1}) \right] y_t \tag{47}$$

$$\mathcal{M} = R_t + \mu(q_{t-2}) \frac{k_t}{\varphi(q_{t-1})} \tag{48}$$

$$\alpha \frac{\psi(q_t)}{\psi(q_{t-1})} \frac{\varphi(q_{t-1})}{1 + r_t} = \frac{k_t}{y_t} \tag{49}$$

$$\frac{k_t}{y_t} \left[\Psi(q_t) \frac{c_t}{y_t} + \Phi(q_t) \frac{d_t}{y_t} \right] = h'\left(\frac{q_t}{q_{t-1}} \right) \frac{q_t}{q_{t-1}}. \tag{50}$$

In each period t, the economy is described by 9 equations with 9 unknowns: $C_t, c_t, y_t, d_t, R_t, x_t, q_t, r_t, k_{t+1}$. Initial conditions are k_1, q_0, q_{-1}. The terminal condition is $k_{T+1} = 0$ or $c_T = y_T$ (and $C_T = \psi(q_T)y_T$).

In the sequel, we will consider that $T \to \infty$.

Lemma 1

In an infinite horizon framework, the dynamics of the economy is characterized by the following properties:

1) The material consumption-output ratio (c_t/y_t) and the material investment-output ratio

(d_t/y_t) are constant and respectively equal to

$$\frac{c_t}{y_t} = 1 - \alpha\beta \quad \text{and} \quad \frac{d_t}{y_t} = \alpha\beta, \ \forall t \geq 1. \tag{51}$$

2) Consequently, material output y, consumption c and investment d always grow at the same rate.

3) The growth factors of capital, output, investment and consumption are decreasing functions of the extraction rate x_t/R_t:

$$\frac{k_{t+1}}{k_t} = \frac{\alpha\beta\varphi(q_t)}{k_t/y_t} = \alpha\beta\varphi(q_t)\left[\frac{\chi(q_{t-1}) + \mu(q_{t-1})}{1 - \frac{x_t}{R_t}} + h\left(\frac{q_t}{q_{t-1}}\right)\right]^{-1} \tag{52}$$

$$\frac{y_t}{y_{t-1}} = \frac{d_t}{d_{t-1}} = \alpha\beta\varphi(q_{t-1})\left[\frac{\chi(q_{t-1}) + \mu(q_{t-1})}{1 - \frac{x_t}{R_t}} + h\left(\frac{q_t}{q_{t-1}}\right)\right]^{-1}. \tag{53}$$

Proof: See Appendix 2.

A few comments on Lemma 1 are useful:

- Point 1 of the lemma is a consequence of our assumption of a logarithmic instantaneous utility function. The saving rate would exhibit a transitory dynamics under more general assumptions on the intratemporal utility function. But this would not change the nature of our results.

- Note the negative impact of the extraction rate on the growth rates of the capital stock and output: a higher extraction rate makes the production process more capital intensive, which decreases the marginal productivity of capital and makes physical production (and thereby investment) more costly.

  This point illustrates indirectly that ecological constraints do not only matter in a remote long run: they also affect the whole transitory dynamics of the economy subject to them. In the present model, the material resource constraint (represented by the mass conservation principle) and its impact on the extraction rate affect not only the long run growth possibilities but also the whole transitory dynamics of output, investment,... A numerical exercise in section 5 will show that 2 economies with different material resource endowments (but otherwise structurally identical) will experience very different transitory growth rates of material output and related variables.

- There is an instantaneous negative impact of the research effort q_t on the growth factor of output but also a dynamic positive impact. Indeed, physical production and research are rival activities as far as the use of the existing capital stock is concerned: during a given time period, a bigger research effort implies less physical production. However, past research activities make the present

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production process more efficient (via a larger φ and smaller χ and μ), which stimulates present output growth.

- The instantaneous impact of research activities on productive capital accumulation is *a priori* ambiguous. On the one hand, the instantaneous negative output effect of research lowers physical investment. On the other hand, research improves instantaneously the quality of investment goods, which enhances capital accumulation. In the long run however, the first effect dominates necessarily.

4 Balance Growth Path

4.1 Properties of a BGP

We define a balanced growth path of the economy with a material resource constraint as a growth path characterized by a constant and positive growth rate of the level of knowledge and a constant growth rate of each of the variables y, c, d, k, x, R and C.

Let $\hat{q}$ be the constant growth factor of knowledge along the BGP, i.e., $\hat{q} = (q_t/q_{t-1})_{\text{BGP}}$.

Lemma 2

Along a BGP,

1. Final productions and the productive capital reach their highest level of dematerialization: asymptotically,

$$\mu(q) \xrightarrow{BGP} \underline{\mu}, \chi(q) \xrightarrow{BGP} \underline{\chi} \quad \text{and} \quad \varphi(q) \xrightarrow{BGP} \bar{\varphi}$$

and the elasticity of $\varphi(q)$ with respect to q tends to zero: $\Phi(q) \xrightarrow{BGP} 0$;

2. The asymptotic resource input-output ratio is constant and equal to

$$\left(\frac{x_t}{y_t}\right)_{BGP} = \underline{\chi} + \underline{\mu}. \tag{54}$$

3. Material variables, i.e. output y_t, the natural resource requirement x_t, consumption c_t and investment d_t have the same growth factor as productive capital k_{t+1}. Asymptotically, it is the following decreasing function of the extraction rate $\tilde{E} = \tilde{x}/\tilde{R}$ and the growth factor of knowledge $\hat{q}$:

$$\left(\frac{y_t}{y_{t-1}}\right)_{BGP} = \left(\frac{k_{t+1}}{k_t}\right)_{BGP} = \alpha\beta\bar{\varphi}\left[\frac{\underline{\chi} + \underline{\mu}}{1 - \tilde{E}} + h(\hat{q})\right]^{-1}. \tag{55}$$

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Proof

1. follows directly from (8) and (11): If q exhibits a positive and constant growth rate, $q_t \to +\infty$ and functions $\chi(q)$, $\mu(q)$ and $\varphi(q)$ tend toward their respective asymptotic value.

2. follows obviously from point 1 and (47).

3. Using point 1 of lemma 2, (52) becomes (55) and the growth factor of output (53) has the same BGP value. From lemma 1 and point 2 of Lemma 2, this is also the growth factor of c_t, d_t and x_t.

<div align="right">QED</div>

Lemma 3

A balanced growth path has one of the two following asymptotic features :

- either the growth rate of the material variables (y_t, c_t, d_t, x_t) and the capital stock k_{t+1} is nil and the extraction rate is constant and strictly positive;

- or the growth rate of y_t, c_t, d_t, x_t and k_{t+1} is strictly negative and the extraction rate is nil.

Proof

Assume first that all material variables grow at a strictly positive rate: in particular, d would then become larger and larger, which would violate sooner or later the law of mass conservation (48) $(M = R_t + \underline{\mu}d_{t-1})$. Hence, either d is constant along the BGP or it grows at a strictly negative rate.

- If d is constant, Lemma 2 implies that all material variables are constant as well. (48) then implies a constant available resource stock R_t. The extraction rate is thus constant as well.

- If d grows at a negative rate along the BGP, d tends progressively towards zero and, from lemma 2, k, y, c, x tend towards 0 as well. Asymptotically, (48) implies that $R_t \to M$: the extraction rate thus tends towards 0.

<div align="right">QED</div>

In the sequel, the first (resp. second) possible path will be labeled positive (resp. negative) BGP, in short PBGP (resp. NBGP). Proposition 1 hereafter analyses the properties of a PBGP. Section 4.3 will discuss its feasibility and conditions of existence.

Let $H(\hat{q})$ be defined as

$$H(\hat{q}) = h'(\hat{q}) \cdot \hat{q}.$$

Note that $H(1) = h'(1) = 0$ and $H'(\hat{q}) = h''(\hat{q})\hat{q} + h'(\hat{q}) > 0$ since $h(\cdot)$ is an increasing and convex function.

Proposition 1

In the presence of a limited but perfectly recyclable essential material resource, material output growth can only be a transitory phenomenon: perpetual economic growth can only take the form of perpetual improvements in the quality of the consumption goods. That is, along a PBGP of the decentralized economy:

1. *Material variables y, c, d, R, x and the productive capital stock k are constant*

2. *The growth factor of knowledge is*

$$\hat{q} = H^{-1}\left(\bar{\varphi}\alpha\beta(1 - \alpha\beta)\Psi\right) \tag{56}$$

and implies perpetual improvements in the quality of the consumption goods;

The extraction rate is

$$\tilde{E} = 1 - \frac{\chi + \mu}{\bar{\varphi}\alpha\beta - h(\hat{q})} \tag{57}$$

3. *The perpetual improvements in the quality of the consumption goods are the only source of long term growth of the consumption index C_t and the representative agent's intratemporal welfare.*

Proof

1. follows from lemma 3.

2. Given lemmas 1 and 2, the optimality condition (50) writes as follows along a PBGP:

$$H(\hat{q}_t) = \left(\frac{k_t}{y_t}\right)_{BGP} \left(\frac{c_t}{y_t}\right)_{BGP} \Psi \tag{58}$$

Using successively $k_t = \varphi d_t$, (51) and the first point of this proposition, this last equality becomes

$$H(\hat{q}_t) = \bar{\varphi}\alpha\beta(1 - \alpha\beta)\Psi, \tag{59}$$

which leads to (56). Since function H is nil when $\hat{q} = 1$ and strictly increasing in $\hat{q}$, (56) defines a unique value of $\hat{q} > 1$.

(57) follows straightforwardly from equation (52) where the growth factor of productive capital has been set 1.

3. is a mere implication of the first two points of the proposition.

<div style="text-align: right">QED</div>

Intuitively enough, note that along a (P)BGP, the growth of knowledge implied by (56) (and thereby the growth rate of the consumption index) is an increasing function of Ψ and $\bar{\varphi}$:

- Ψ: the more the improvements in the quality of the consumption goods are sensitive to the research effort, the stronger the incentive to invest in it.

- $\bar{\varphi}$: the more physical investment is productive, i.e., the more productive capital is dematerialized (alternatively, the less a given productive capital stock depends on physical investment), the less the research effort is costly (in terms of foregone physical production) and the stronger the incentive to invest in it.

- The growth rate of q is however a non monotonic function of the saving rate $\alpha\beta$ it is increasing in the saving rate if $\alpha\beta$ is not too high (i.e., not larger than $1/2$)[14]; it is decreasing otherwise. This ambiguous impact of the saving rate appears rather clearly in the optimality condition (50). A higher saving rate implies a stronger capital accumulation and a higher capital/output ratio, which contributes to increasing research *ceteris paribus*. However, along a BGP, the only impact of research is the improvement in the quality of consumption goods. From this perspective, the lower the output share allocated to consumption, the weaker the long run incentive to make research. When the consumption/output ratio is low enough (or the saving rate high enough), this second effect dominates and an extra increase in the saving rate further weakens the incentive for research.

Proposition 2

Along a PBGP, material variables are linear functions of the material resource endowment:

---

[14]Indeed,
$$\frac{\partial(\alpha\beta\bar{\varphi}(1-\alpha\beta)\Psi)}{\partial(\alpha\beta)} = \bar{\varphi}(1-2\alpha\beta)\Psi > 0 \Leftrightarrow \alpha\beta < \frac{1}{2}.$$

i.e,

$$\tilde{R} = \Sigma(\tilde{E})\mathcal{M} \quad with \quad 0 < \Sigma(\tilde{E}) = \frac{\underline{\chi} + \underline{\mu}}{\underline{\chi} + \underline{\mu} + \alpha\beta\underline{\mu}\tilde{E}} \leq 1 \tag{60}$$

$$\tilde{x} = \tilde{E}\Sigma(\tilde{E})\mathcal{M} \tag{61}$$

$$\tilde{y} = \tilde{E}\Sigma(\tilde{E})\frac{\mathcal{M}}{\underline{\chi} + \underline{\mu}} < \frac{\mathcal{M}}{\underline{\chi} + \underline{\mu}} \tag{62}$$

and $\tilde{d} = \alpha\beta\tilde{y}$, $\tilde{k} = \bar{\varphi}\alpha\beta\tilde{y}$, $\tilde{c} = (1 - \alpha\beta)\tilde{y}$.

Proof

Using successively $d_t/y_t = \alpha\beta$, (47) and lemma 2, one can rewrite (48) as follows

$$\mathcal{M} = R_t + \underline{\mu}d_t = R_t + \underline{\mu}\alpha\beta\frac{x_t}{\underline{\chi} + \underline{\mu}}$$
$$= R_t\left[1 + \alpha\beta\frac{\underline{\mu}E_t}{\underline{\chi} + \underline{\mu}}\right].$$

Along a PBGP, $E_t = \tilde{E}$ and the last equality above leads straightforwardly to the constant value of R_t given in (60). $\Sigma(\tilde{E})$ is monotonically decreasing in $\tilde{E}$, with $\Sigma(0) = 1$ and $1 > \Sigma(1) > 0$. Using $\tilde{E} = x/R$ (resp. (47)) and (60) leads to (61) (resp. (62)).

<div align="right">QED</div>

4.2   Digression on limit cases with unbounded material growth

Proposition 3

Perpetual growth of material output y, consumption c and investment d would occur in the two following particular cases:

1. The available quantity of material resource is unlimited, i.e., $\mathcal{M} \to \infty$;

2. Technological progress allows firms to produce final goods characterized by a complete dematerialization: i.e., $\underline{\mu} \to 0$ and $\underline{\chi} \to 0$.

Proof

It is easy to verify that in cases 1 and 2, the value of y in proposition 2 tends towards $+\infty$, which reflects that output then grows at a strictly positive rate along a BGP. In both cases, it is not difficult to show that the growth rate of the technological knowledge remains given by (56) whereas $\tilde{E} = 0$. Moreover, in

case 1,

$$\left(\frac{k_{t+1}}{k_t}\right)_{BGP} = \frac{\alpha\beta\bar{\varphi}}{\underline{\chi} + \underline{\mu} + h\left(\hat{q}\right)}. \tag{63}$$

whereas in case 2,

$$\left(\frac{k_{t+1}}{k_t}\right)_{BGP} = \frac{\alpha\beta\bar{\varphi}}{h\left(\hat{q}\right)}. \tag{64}$$

QED

Note the following two points.

1. In these two cases, our model behaves as a fairly standard endogenous growth model. In terms of descriptive realism, these 2 cases seem however unlikely: Case 1 is impossible in a finite world; Case 2 would basically amount to saying that technological progress could ultimately free the production process from the laws of physics.

2. Even though technological progress allowed productive capital to reach a state of complete dematerialization (i.e., $\bar{\varphi} \to \infty$), material variables would nevertheless remain bounded (except in the two particular cases discussed just before). If $\bar{\varphi} \to \infty$, an infinite capital stock could asymptotically be built from an infinitely small quantity of investment good, which would imply that all material production could then be allocated to consumption; moreover, with an infinite capital stock, the extraction could tend to 1. However, final output would remain bounded unless a complete dematerialization of final productions was possible.

4.3 Feasibility of a PBGP and existence of decentralized PBGP

We now analyse the conditions of existence of a PBGP in our decentralized economy. We deal with this issue in two steps: we start by analysing the feasibility of a PBGP from a purely technological/physical point of view. Next, we examine to what extent the decentralized behaviours of the economic agents restrict the existence conditions of a (decentralized) PGBP.

We first define more precisely the concept of feasibility of a BPGP :

Definition

A PBGP is said to be feasible if the following conditions are satisfied :

$$0 \leq \frac{d}{y} \leq 1 \tag{65}$$

$$0 \leq \tilde{E} \leq 1 \tag{66}$$

$$1 \leq \hat{q} \Leftrightarrow 0 \leq h\left(\hat{q}\right) \tag{67}$$

In order to determine the conditions under which a PBGP is technologically/physically feasible, we set aside the model equations describing the decentralized agent's decisions (i.e. the optimality conditions on consumption, price and research) and we focus on the feasibility conditions that follow from the technological and physical constraints (46), (47), (48) and the accounting identities (43), (44), (45). Along a PBGP, this system of 6 equations can easily be reduced to the following relationship:

$$\overline{\varphi}\frac{d}{y} = h\left(\widehat{q}\right) + \frac{\underline{\chi} + \underline{\mu}}{1 - \tilde{E}}. \tag{68}$$

For a given value of the saving rate $d/y \in [0,1]$ (d/y being constant along a PBGP), (68) defines a negative relationship between $\tilde{E}$ and $\hat{q}$ which reflects that research and physical production (or extraction) are rival activities as far as the use of the available capital stock is concerned: the higher the extraction rate, the higher the capital requirement per unit of output (i.e. the higher the ratio $(\underline{\chi} + \underline{\mu})/(1 - \tilde{E})$), so the lower the capital stock available for research and the lower $h\left(\widehat{q}\right)$.

Proposition 4

1. A feasible PBGP can be associated to any couple $(\tilde{E}, \hat{q})$ of the positive orthant that satisfies the inequalities

$$0 \quad < \quad \tilde{E} \le 1 - \frac{\underline{\chi} + \underline{\mu}}{\overline{\varphi}} \tag{69}$$

$$0 \quad < \quad h\left(\widehat{q}\right) < \overline{\varphi} - \frac{\underline{\chi} + \underline{\mu}}{1 - \tilde{E}}. \tag{70}$$

2. A necessary condition for a PBGP to be feasible is

$$\overline{\varphi} > \underline{\chi} + \underline{\mu}. \tag{71}$$

In other words, there is certainly no feasible PBGP if the efficiency of physical investment (equivalently the highest degree of dematerialization of the productive capital stock) is too small relatively to the highest degree of dematerialization of the physical production, i.e. if $\overline{\varphi} < \underline{\chi} + \underline{\mu}$.

Proof

1. The positivity constraints in (69) and (70) are straightforward (see definition of a PBGP). The upper bound on $\tilde{E}$ in (69) follows from (68) where $d/y = 1$ and $\hat{q} = 1$ or $h(\hat{q}) = 0$. The highest feasible extraction rate would indeed be reached if output was fully allocated to investment (unitary saving rate) and if the whole available capital stock was allocated to production/extraction activities (no research investment).

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Since the saving rate is constant and in $[0,1]$, the left hand side of (68) is in $[0,\varphi]$ and the following inequality must thus hold

$$0 < \frac{\chi + \mu}{1 - \tilde{E}} + h\left(\hat{q}\right) < \overline{\varphi}.$$

The left inequality will be necessarily satisfied since $\tilde{E} < 1$ and $h(\hat{q}) > 0$. The right inequality is equivalent to the right inequality in (70).
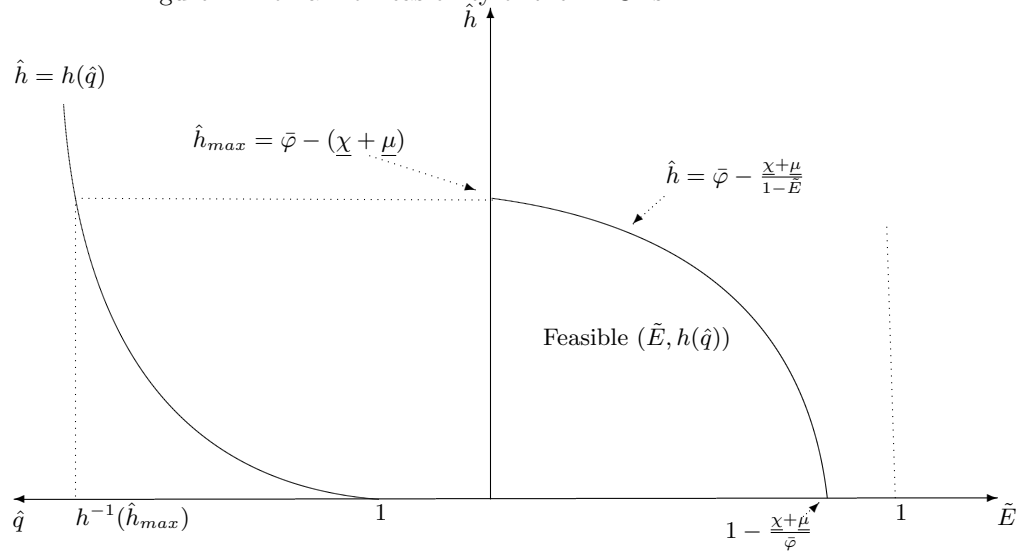
2. If (71) does not hold, the upper bounds on $\tilde{E}$ and $h(\hat{q})$ in (69) and (70) are negative, i.e., no PBGP can exist.

QED

Figure 1 illustrates this proposition graphically. In the case of a unitary saving rate $d/y = 1$, the relationship (68) becomes the curve $h\left(\hat{q}\right) = \overline{\varphi} - \frac{\chi + \mu}{1 - \tilde{E}}$, which determines the frontier of the domain of the feasible values of $\tilde{E}$ and $h(\hat{q})$. This domain consists of all the couples $(\tilde{E}, \hat{h})$ of the positive orthant under this curve. If (71) does not hold, the intercept of the curve is negative and this domain is empty. If it holds, feasible couples $(\tilde{E}, h(\hat{q}))$ exist and a fully informed central planner could choose one of these couples (and the corresponding saving rate) in order to maximize a chosen social objective.

The necessary condition (71) has an intuitive interpretation. If it did not hold, the economy would not be able to sustain a constant capital stock level even in the scenario which is the most favourable to capital accumulation, i.e. a scenario in which 1) the existing stock would be exclusively allocated to productive activities (no research investment), 2) the extraction rate would be nil and 3) final production would only be allocated to investment (unitary saving rate). Assume indeed that a given $\tilde{k}$ could be a stationary state capital stock level. In the scenario just described ($\hat{q} = 1$, $\tilde{E} = 0$, $d/y = 1$), output (and thus investment) would be $\tilde{y} = \tilde{d} = \tilde{k} \cdot (\underline{\chi} + \underline{\mu})^{-1}$. The capital stock of the following period would be $\overline{\varphi}\tilde{d} = \tilde{k} \cdot \overline{\varphi} \cdot (\underline{\chi} + \underline{\mu})^{-1}$ and would thus be necessarily smaller than $\tilde{k}$: if $\overline{\varphi} < (\underline{\chi} + \underline{\mu})$, the capital stock can only decrease through time.

Figure 1: Domain of feasibility of the PBGPs

$\hat{h} = h(\hat{q})$

$\hat{h}_{max} = \bar{\varphi} - (\underline{\chi} + \underline{\mu})$

$\hat{h} = \bar{\varphi} - \frac{\chi + \mu}{1 - \tilde{E}}$

Feasible $(\tilde{E}, h(\hat{q}))$

$\hat{q}$ $h^{-1}(\hat{h}_{max})$ 1 $1 - \frac{\chi + \mu}{\bar{\varphi}}$ 1 $\tilde{E}$

26

We now analyse the existence conditions of the decentralized PGBP characterized in 4.1.

Proposition 5

1. *A decentralized PBGP exists if and only if*

$$(h(\hat{q}) =) h\left(H^{-1}\left(\bar{\varphi}\alpha\beta(1-\alpha\beta)\Psi\right)\right) < \overline{\varphi}\alpha\beta - (\underline{\chi} + \underline{\mu}) \tag{72}$$

2. *A necessary condition for a decentralized PBGP to exist is a sufficiently high saving rate* $\alpha\beta$, *i.e.,*

$$\alpha\beta > \frac{\chi + \mu}{\overline{\varphi}}. \tag{73}$$

Proof

1. The condition on $h(\hat{q})$ follows from the constraint that $\tilde{E}$ must be positive: from (57), this requires that $\underline{\chi} + \underline{\mu} > \bar{\varphi}\alpha\beta - h(\hat{q}) > 0$, which is equivalent to (72).

2. If (73) did not hold, the right-hand-side of (72) would be negative, $h(\hat{q})$ (but also $\tilde{E}$) being constrained to be negative as well.

$$\text{QED}$$

Conditions (72) and (73) are quite intuitive. From lemma 1, we know that the growth factor of the capital stock is given by (52). Along a PBGP, one must have $k_{t+1} = k_t$, i.e. a unitary growth factor of the capital stock. Since the expression of the growth factor of k in (52) is decreasing in the extraction rate, the following inequality must thus hold:

$$\left(\frac{k_{t+1}}{k_t}\right)_{\text{BGP}} = \frac{\bar{\varphi}\alpha\beta}{\frac{\chi+\mu}{1-\tilde{E}} + h(\hat{q})} = 1 < \frac{\bar{\varphi}\alpha\beta}{\underline{\chi} + \underline{\mu} + h(\hat{q})},$$

which is equivalent to (72). If (72) did not hold, the right hand side of this inequality would be smaller than 1, which would imply that the capital stock would decrease through time, the economy moving then along a NBGP. Such a situation would occur if the saving rate of the economy was too small, given the research effort simultaneously developped by the firms.

The necessary condition (73) is equivalent to the inequality (72) in the case where $h(\hat{q})$ is nil: it reflects that the scenario of a NBGP would be unavoidable if the saving rate was too low to maintain the capital stock constant even when no investment in research is made. Indeed, in the absence of any research effort, the investment level $\tilde{d}$ necessary to keep the capital stock constant at a given value $\tilde{k}$ is such that

$$\tilde{k}(= \bar{\varphi}\tilde{d} = \bar{\varphi}\alpha\beta\tilde{y}) = \bar{\varphi}\alpha\beta\left[\frac{\chi + \mu}{1 - \tilde{E}}\right]^{-1}\tilde{k}.$$

27

If (73) does not hold, the term that multiplies $\tilde{k}$ at the right-hand-side is smaller than one even in the most favourable case where $\tilde{E} = 0$ (and thus *a fortiori* for any positive $\tilde{E}$).

The comparison between the necessary conditions (71) and (73) makes clear that a decentralized PBGP may not exist even though the domain of the feasible PBGPs is not empty. If the saving rate in the decentralized framework, $\alpha\beta < 1$, is such that

$$\frac{\underline{\chi} + \underline{\mu}}{\alpha\beta} > \bar{\varphi} > \underline{\chi} + \underline{\mu},$$

no decentralized PBGP exists although there exist feasible PBGPs. One can note here that the likelihood of such a situation is higher

- the more decentralized agents are impatient (the lower β)

- the stronger the goods market imperfections (the lower α).

(72) also shows that an overinvestment in research can be a second motive for which a decentralized PBGP may fail to exist even though there are feasible PBGPs. The existence of a decentralized PGBP does not only require a sufficiently high saving rate: it also needs that a sufficiently large part of the saving be allocated to the accumulation of productive capital.

5 Transitional dynamics

Although some aspects of the transitional dynamics can be characterized analytically, a more thorough exploration of it requires a numerical analysis. Since we only aims at a qualitative exploration, we want to limit the space dedicated to the choice of the calibration.

We choose the following functional forms which satisfy the model assumptions:

$$
\begin{aligned}
h\left(z_t\right) &= \left[z_t - 1\right]^{\gamma}, \ \gamma > 1 \\
\psi(q_t) &= q_t^{\eta}, \ \text{with } \eta > 0 \\
\mu(q_t) &= \underline{\mu} + q_0\frac{\mu_0 - \underline{\mu}}{q_t}, \ \text{with } \mu_0 > \underline{\mu} > 0 \\
\chi(q_t) &= \underline{\chi} + q_0\frac{\chi_0 - \underline{\chi}}{q_t}, \ \text{with } \chi_0 > \underline{\chi} > 0 \\
\varphi(q_t) &= \overline{\varphi} - q_0\frac{\varphi_0 - \overline{\varphi}}{q_t}, \ \text{with } 0 < \varphi_0 < \overline{\varphi}.
\end{aligned}
$$

Hence, $\Psi(q_t) = \eta$ and

$$\Phi(q_t) = \frac{-q_0\frac{\varphi_0 - \overline{\varphi}}{q_t}}{\overline{\varphi} - q_0\frac{\varphi_0 - \overline{\varphi}}{q_t}}.$$

The discount rate β has been set to 0.98 and ϵ (or α) has next been set to obtain a saving rate of 20% (i.e., $\alpha = 0.2/\beta$). The technological parameters γ, η, $\underline{\mu}$, $\underline{\chi}$ and $\bar{\varphi}$ has been chosen so as to satisfy the existence condition of a PBGP and to induce a stationary growth rate of q equal to 3%.

γ	η	$\underline{\mu}$	$\underline{\chi}$	$\bar{\varphi}$
2	1	0.015	0.055	0.386

We set $\mu_0 = 5\underline{\mu}$, $\chi_0 = 5\underline{\chi}$, $\varphi_0 = \bar{\varphi}/5$.

The material resource stock $\mathcal{M}$ has been successively set to 200 and to 300 in order to compare the transitory dynamics of 2 economies that differ only in their resource endowment.

With the chosen calibration, a numerical computation shows that the dynamic model satisfies the Blanchard-Kahn condition for the existence a saddle-path property. We checked numerically that the economy converged towards a PBGP. We also checked numerically that if the parameter values were chosen so as to violate the existence condition of a decentralyzed PBGP, the economy converged towards a NBGP.

The figures below illustrate the transitory dynamics of the material output y_t, the knowledge level q_t and the extraction rate E_t in two economies, one with $\mathcal{M} = 200$ and the other with $\mathcal{M} = 300$. The 2 economies are otherwise identical and have the same initial (predetermined) capital stock and stock of knowledge, both initial stock lying well below their respective PBGP level.

Because of the scale effect linked to the material resource stock, the two economies have different long run output (and capital) levels. In both economies however, the length of the transitory episode during which material output grows is the same, which means that the material growth process is stronger in the more generously endowed economy than in the other one. As the two other figures show, this transitory difference in the material growth rates does not follow from differences in research effort[15] but from differences related to the evolution of the extraction rate. The better endowed economy faces a less severe resource constraint and has therefore a lower extraction rate, which means lower extraction/production costs; accordingly, the marginal productivity of capital is higher during the transition dynamics and the dynamics of the capital accumulation is thus stronger.

In their debates with ecological economists, orthodox growth economists are used to say or write that a growth model that ignores environmental constraints might still provide an acceptable representation of the growth process in the short or medium run[16]. Our simple simulation suggests that this claim is at best

---

[15]The exponential process that q follows (or the negative exponential process of $1/q$ in the figure) is exactly the same in the two economies.

[16]For example, in his reply to a rather polemic paper by Daly (1997), Stiglitz (1997) writes that the criticisms of the

Figure 2: Transitory dynamics and material resource endowment

Transitory dynamics of material output

Transitory dynamics of 1/Q

Transitory dynamics of the extraction rate
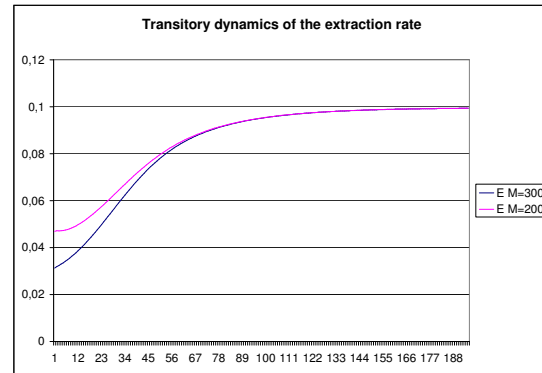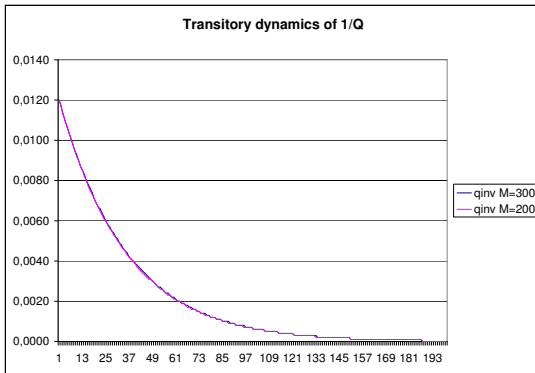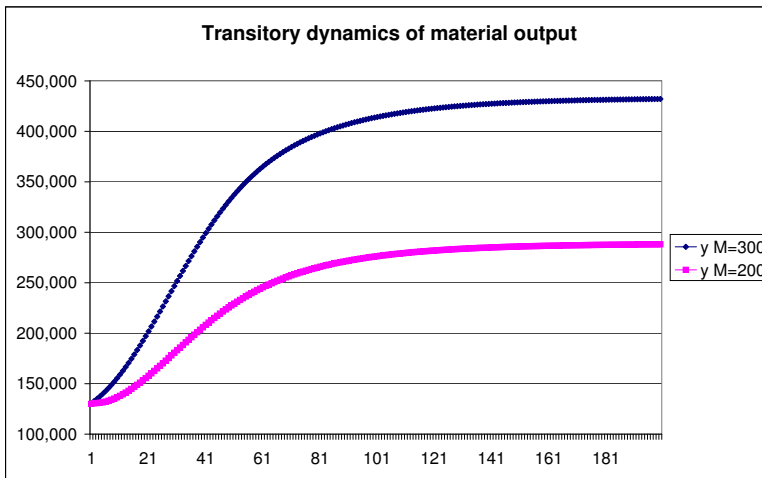
wishful thinking. The environmental constraints (and the related physical laws) do not only shape the very long run growth process: they can also affect the whole transitory dynamics of the economy. Some will argue that environmental considerations do not seem to have constrained very severely the growth process of the Western world during the 19th and 20th centuries. Our analysis suggests however another interpretation, which would certainly seem perfectly trivial to physicists and other natural scientists: past economic growth has not been built out of the physical laws and the environmental constraints and it has thus been affected by them. Had the available environmental resources been different, so would have been the growth process.

6   An extension

Energy has been ignored in our basic model. If energy (more precisely exergy) is as essential to production as matter, a major difference between these two factors is that energy is not recyclable. As a supplier of energy, the environment has also limited capacity.

This section introduces energy in our basic model by modifying the description of the productive technology as follows. The final production process requires a material input as before but also some energy: to produce y_{jt}, a firm j consumes a quantity of energy given by

$$e_{jt} = \zeta(Q_{t-1})y_{jt} \tag{74}$$

where $\zeta(Q_{t-1})$ is the quantity of energy consumed per unit of good j. As χ and μ, ζ is common to all firms and depends on the level of technological knowledge measured by Q. It makes technology less energy depending $(\zeta'(\cdot) < 0)$ but function $\zeta(\cdot)$ is bounded from below:

$$\lim_{Q \to +\infty} \zeta(Q) = \underline{\zeta} > 0.$$

This assumption reflects that production will always remain an energy consuming process, even asymptotically.

During each period, the economy we consider receives a finite flow $\mathcal{F}$ of perfectly renewable energy such as solar energy[17]. Energy is thus immaterial and, when consumed, it is assumed to be completely dissipated

environmental economists to the traditional growth models arise finally "from a lack of understanding of the role of the kind of analytical models that [he and others] have formulated. They are intended to help us to answer questions like *for the intermediate run -for the next 50-60 years, is it possible that growth can be sustained*".

[17]By ignoring non renewable energy resources (such as fossil fuels), we make here a modelling assumption that allows us to disentangle the respective impacts of the availability of material resources and the one of energy. In any case, non renewable energy resource could only play a transitory role in our model.

as heat in the environment without damage. $\mathcal{F}$ is assumed to be a common resource the access to which is free. Its captation and exploitation process however requires productive capital: to transform a quantity e_{jt} of energy, firm j requires a quantity of productive capital ${}_ek_{jt}$ given by

$$
{}_ek_{jt} = g\left(\frac{e_t}{\mathcal{F}}\right)e_{jt} \tag{75}
$$

where $e_t = \int_0^1 e_{jt}\mathrm{d}j$ is the total quantity of energy consumed by all firms. Function $g\left(\frac{e_t}{\mathcal{F}}\right)$ measures the quantity of capital necessary per unit of energy: g is supposed to be positive, increasing and convex: $g(0) = g_0 > 0, g' > 0, g'' \geq 0$ and $g(1) \to +\infty$. Energy costs are thus increasing in both the quantity of energy to capture and the degree of exploitation of the available energy flow $e_t/\mathcal{F}$, denoted F_t hereafter.

The total capital stock used by firm j during period t is now:

$$
\begin{aligned}
k_{jt} &= {}_pk_{jt} + {}_rk_{jt} + {}_ek_{jt} \\
&= \left[\frac{\chi(Q_{t-1}) + \mu(Q_{t-1})}{1 - E_t} + h\left(\frac{q_{jt}}{Q_{t-1}}\right) + \zeta(Q_{t-1})g\left(F_t\right)\right]y_{jt}. \tag{76}
\end{aligned}
$$

Following the same reasoning as for the basic model, it is easy to show that the whole system of equations characterizing the symmetric equilibrium consists of equations (42) to (45), (47) to (50), and the two following new equations:

$$
\frac{k_t}{y_t} = \left[\frac{\chi(q_{t-1}) + \mu(q_{t-1})}{1 - E_t} + h\left(\frac{q_t}{q_{t-1}}\right) + \zeta(q_{t-1})g\left(F_t\right)\right] \tag{77}
$$

$$
e_t = \zeta(q_{t-1})y_t. \tag{78}
$$

Lemmas 1, 2 and 3 and proposition 1 can be reformulated rather straightforwardly in this extended model.

- Lemma 1 holds basically unchanged (to the exception of the expression of the capital growth factor which is modified since k_t/y_t is now given by (77).

- Lemma 2 can simply be supplemented as follows: along a BGP, $\zeta \to \underline{\zeta}$ (point 1); the energy output ratio is constant $(e_t/y_t)_{\mathrm{BGP}} = \underline{\zeta}$ (point 2); the consumed energy e_t grows at the same rate as the material variables (point 3).

- Lemma 3 now integrates that along a PBGP, the consumed energy e_t and the rate of exploitation of the available energy flow, F_t, are constant whereas along a NGBP the two exhibit a negative growth rate.

Along a BGP, the exploitation rate of the energy flow, F_t, can be expressed as an increasing function of

the extraction rate of the natural resource, E_t. Indeed, using successively (78), (48) and (62), one gets

$$
\begin{aligned}
F_t &= \frac{\underline{\zeta}}{\mathcal{F}} y_t \\
&= \frac{\underline{\zeta}}{\underline{\mu} + \underline{\chi}} \frac{\mathcal{M}}{\mathcal{F}} \tilde{E} \Sigma(\tilde{E}) = F(\tilde{E}).
\end{aligned}
\tag{79}
$$

One checks easily that $\tilde{E}\Sigma(\tilde{E})$ is increasing in $\tilde{E}$, so that $F'(\cdot) > 0$ and $F(0) = 0$.

We can reformulate proposition 1 as follows:

Proposition 6

> The PBGP of a decentralized economy with a limited but perfectly recyclable essential material resource and a limited but renewable energy flow is characterized by:
>
> 1. constant material variables y, c, d, R, x, e and productive capital stock k,
>
> 2. a growth factor of knowledge given by (56) and implying perpetual improvements in the quality of consumption goods,
>
> 3. a constant extraction rate, $\tilde{E}$, which is the solution to
>
> $$
> \alpha \beta \overline{\varphi} = \frac{\chi + \underline{\mu}}{1 - \tilde{E}} + h\left(\widehat{q}\right) + \underline{\zeta} g\left(F(\tilde{E})\right),
> \tag{80}
> $$
>
> and a constant exploitation rate of the available energy flow, $\tilde{F} = F(\tilde{E})$.
>
> 4. Material output is given by
>
> $$
> y = \frac{F(\tilde{E})}{\underline{\zeta}} \mathcal{F} = \tilde{E} \Sigma(\tilde{E}) \frac{\mathcal{M}}{\underline{\chi} + \underline{\mu}} < \min\left\{\frac{\mathcal{F}}{\underline{\zeta}}, \frac{\mathcal{M}}{\underline{\chi} + \underline{\mu}}\right\}.
> $$

Proof

The proof follows the same reasoning as for the basic model and proposition1. Point 1 is a mere reformulation. Point 2 states that the growth factor of knowledge is the same as in proposition 1. In our model indeed, the presence of energy does not affect the optimality condition on the individual research effort. Concerning point 3, it is easy to check that the solution to (80) is necessarily unique if it exists (see feasibility condition in the following proposition). The upper bound on y given in point 4 corresponds to what can be at most physically produced per period given on the one hand the available energy flow and the energy intensiveness of the production process and, on the other hand, the available material resource stock[18] and the resource intensiveness of the production process.

---

[18]This second lower bound is actually lower as a fraction of $\mathcal{M}$ is embedded in the existing productive capital stock and is thus not available for current production.

There remains to check whether the PBGP is feasible, i.e., to determine the conditions under which $0 \leq E \leq 1$ and $0 \leq \tilde{F} \leq 1$.

Proposition 7

1. *A PBGP exists if and only if*

$$(h\left(\widehat{q}\right) =) H^{-1}\left(\bar{\varphi}\alpha\beta(1-\alpha\beta)\Psi\right) < \overline{\varphi}\alpha\beta - (\underline{\chi} + \underline{\mu}) - \underline{\zeta}g(0). \qquad (81)$$

2. *A necessary condition for a PBGP to exist is a sufficiently high saving rate, i.e.,*

$$\alpha\beta > \frac{\underline{\chi} + \underline{\mu} + \underline{\zeta}g(0)}{\overline{\varphi}}. \qquad (82)$$

The proof follows the same reasoning as in the basic model. We have also checked numerically that if inequality (81) is satisfied, the model indeed converges to a PBGP.

Note that (82) is a more restrictive condition than (73): the more so, the more production is dependent on energy (the higher ζ) and/or the more energy exploitation is capital intensive (the higher $g(0)$). Similarly, and obviously enough, the conditions for a long run material growth turn out to be even more restrictive than in the basic model:

Proposition 8

Perpetual growth of material output y, consumption c and investment d would occur in each of the two following particular cases:

1. *The available quantity of material resource and the available flow of energy are both unlimited, i.e., $\mathcal{M} \rightarrow \infty$ and $\mathcal{F} \rightarrow \infty$;*

2. *Technological progress allows to develop a production process that does not consume any energy and creates final productions characterized by a complete state of dematerialization: i.e., $\underline{\zeta} \rightarrow 0$, $\underline{\mu} \rightarrow 0$ and $\underline{\chi} \rightarrow 0$.*

If we reconsider the discussion of subsection 4.3 for this model with energy, it appears that the domain of feasibility of the PBGPs in the presence of energy is smaller than in the basic model. Indeed, in the presence of energy, the technological/physical constraints (47), (48), (77), (78) and the accounting identities (43)-(45) imply the following relationship:

$$h(\widehat{q}) = \overline{\varphi}\frac{d}{y} - \frac{\underline{\chi} + \underline{\mu}}{1 - E} - \underline{\zeta}g\left(F(E)\right). \qquad (83)$$

This expression must be compared with the equivalent relationship (68) in the basic model: the curve describing the frontier of the domain of feasible PBGPs in the model with energy is moved downwards (since $\underline{\zeta} g \left(F(E) \right) > 0$) with respect to the equivalent curve in the initial model.

The value of $\hat{q}$ determined by (83) at given $\tilde{E}$ is feasible only if the saving rate is high enough, i.e.

$$\frac{d}{y} > \frac{\underline{\chi} + \underline{\mu} + \underline{\zeta} g(0)}{\bar{\varphi}}. \tag{84}$$

Given that the saving rate is smaller than 1, a necessary condition of the existence of a feasible PBGP is thus

$$\bar{\varphi} > \underline{\chi} + \underline{\mu} + \underline{\zeta} g(0). \tag{85}$$

Once again, the comparison between conditions (85) and (82) shows that a decentralized PBGP may not exist in economies where the set of feasible PBGPs is not empty: if the saving rate $\alpha\beta < 1$ is such that

$$\frac{\underline{\chi} + \underline{\mu} + \underline{\zeta} g(0)}{\alpha\beta} > \bar{\varphi} > \underline{\chi} + \underline{\mu} + \underline{\zeta} g(0),$$

no decentralized PBGP exists although some PBGPs were feasible.

7 Conclusions

We have proposed an endogenous growth model of a decentralized economy where final production requires an essential material input. Following a material balance approach and making technological assumptions fully consistent with the material balance principle, we have shown that material output growth is always a transitory phenomenon even though the material input is perfectly recyclable. When it exists, a balanced growth path is necessarily characterized by constant values of the material variables, long term economic growth taking exclusively the form of perpetual improvements in the quality of the produced goods. Even when balanced growth paths are feasible from a technological/physical point of view, a decentralized growth path may not exist (i) if the saving rate of the decentralized agents is too weak or (ii) if their research effort is too high. Moreover, the material resource constraint has been shown to matter as well in the shorter run since it can also affect the whole transitory dynamics of the (material) growth process. As an extension of our basic model, we have introduced energy as another essential input. This generalization does not affect qualitatively the features of a feasible balanced growth path but makes its conditions of existence more restrictive.

Even though our model describes material growth as a transitory phenomenon, it relies on very optimistic assumptions: we have supposed that recycling is free and perfect and that energy is perfectly renewable;

we have ignored the environmental damage linked to production/consumption activities and the possibility of tresholds in the assimilative capacity of the environment. Less optimistic assumptions about these matters are likely to affect the features of a balanced growth path and to reduce further the domain of feasibility of such growth paths. Such topics are left for future research.

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Appendix 1: Monopolistic Firms' Behaviour

Using (21) and (20) to express k_{jt} and y_{jt} as functions of q_{jt} and p_{jt} and associating the multiplier ν_{jt} to the constraint (22), we write the Lagrangean of the profit maximization problem as follows:

$$\mathcal{L}_j = \sum_{t=1}^{T} \rho_t \left\{ p_{jt} y_{jt} - \bar{p}_t \left(\frac{\chi(Q_t) + \mu(Q_t)}{1 - \frac{X_{t+1}}{R_{t+1}}} + h\left(\frac{q_{jt+1}}{Q_t} \right) \right) y_{jt+1} + \nu_{jt} \cdot (q_{jt} - Q_{t-1}) \right\}$$

$$\text{with } \rho_t = \frac{1}{\Pi_{\tau=1}^{t} (1 + r_\tau)}.$$

It admits the following first order conditions:

$$\frac{\partial \mathcal{L}_j}{\partial p_{jt}} = \rho_{t-1} \left[-\bar{p}_{t-1} \frac{k_{jt}}{y_{jt}} \frac{\partial y_{jt}}{\partial p_{jt}} \right] + \rho_t \left[y_{jt} + p_{jt} \frac{\partial y_{jt}}{\partial p_{jt}} \right] \leq 0;$$

$$p_{jt} \geq 0 \quad \text{and} \quad p_{jt} \frac{\partial \mathcal{L}_j}{\partial p_{jt}} = 0, \ \forall t \geq 1 \tag{86}$$

$$\frac{\partial \mathcal{L}_j}{\partial q_{jt}} = -\rho_{t-1} \bar{p}_{t-1} \left[\frac{k_{jt}}{y_{jt}} \frac{\partial y_{jt}}{\partial q_{jt}} + h'\left(\frac{q_{jt}}{Q_{t-1}} \right) \frac{y_{jt}}{Q_{t-1}} \right] + \rho_t \left[p_{jt} \frac{\partial y_{jt}}{\partial q_{jt}} + \nu_{jt} \right] \leq 0$$

$$q_{jt} \geq 0 \quad \text{and} \quad q_{jt} \frac{\partial \mathcal{L}_j}{\partial q_{jt}} = 0, \ \forall t \geq 1 \tag{87}$$

$$\frac{\partial \mathcal{L}_j}{\partial \nu_{jt}} = q_{jt} - Q_{t-1} \geq 0; \ \nu_{jt} \geq 0 \quad \text{and} \quad \nu_{jt} [q_{jt} - Q_{t-1}] = 0, \ \forall t \geq 1 \tag{88}$$

Obviously enough, $p_{jt} = 0$ cannot be a profit maximizing choice. Hence $\partial \mathcal{L}_j / \partial p_{jt} = 0$. Similarly, (88) and the initial condition $Q_0 > 0$ imply that $q_{jt} \geq Q_{t-1} \geq Q_0 > 0$ for all t, so that $\partial \mathcal{L}_j / \partial q_{jt} = 0$.

After simplifying the discounting factors in (86) and (87) and multiplying (86) (resp. (87)) by p_{jt}/y_{jt}

(resp. q_{jt}/y_{jt}), we thus rewrite the above system as follows: $\forall t \geq 1$,

$$p_{jt}\left(1 + \frac{p_{jt}}{y_{jt}}\frac{\partial y_{jt}}{\partial p_{jt}}\right) = (1+r_t)\bar{p}_{t-1}\frac{k_{jt}}{y_{jt}}\left(\frac{p_{jt}}{y_{jt}}\frac{\partial y_{jt}}{\partial p_{jt}}\right); \qquad (89)$$

$$p_{jt}\frac{q_{jt}}{y_{jt}}\frac{\partial y_{jt}}{\partial q_{jt}} + \nu_{jt}\frac{q_{jt}}{y_{jt}} = (1+r_t)\bar{p}_{t-1}\left[\frac{k_{jt}}{y_{jt}}\frac{q_{jt}}{y_{jt}}\frac{\partial y_{jt}}{\partial q_{jt}} + h'\left(\frac{q_{jt}}{Q_{t-1}}\right)\frac{q_{jt}}{Q_{t-1}}\right] \qquad (90)$$

$$q_{jt} - Q_{t-1} \geq 0; \ \nu_{jt} \geq 0 \quad \text{and} \quad \nu_{jt}\left[q_{jt} - Q_{t-1}\right] = 0, \qquad (91)$$

where the unknowns are q_{jt}, p_{jt}, ν_{jt} for $t \geq 1$ and k_{jt} for $t \geq 2$.

As the price elasticity of demand q_{jt} is $-\epsilon$, (89) describes the usual monopolistic pricing behaviour

$$p_{jt} = \frac{\epsilon}{\epsilon - 1}\bar{p}_{t-1}(1+r_t)\frac{k_{jt}}{y_{jt}}, \qquad (92)$$

which is equation (24) in the main text.

A few simple algebraic manipulations allow ones to write (90) as follows:

$$\left[p_{jt} - \bar{p}_{t-1}(1+r_t)\frac{k_{jt}}{y_{jt}}\right] \cdot \eta_{y\cdot q}^{jt} + \nu_{jt}\frac{q_{jt}}{y_{jt}} = (1+r_t)\bar{p}_{t-1}h'\left(\frac{q_{jt}}{Q_{t-1}}\right)\frac{q_{jt}}{Q_{t-1}} \qquad (93)$$

where $\eta_{y\cdot q}^{jt} > 0$ is the elasticity of y_{jt} with respect to q_{jt}, i.e.,

$$\begin{aligned}
\frac{q_{jt}}{y_{jt}}\frac{\partial y_{jt}}{\partial q_{jt}} &= \frac{q_{jt}}{y_{jt}}\left(\frac{\partial c_{jt}}{\partial q_{jt}} + \frac{\partial d_{jt}}{\partial q_{jt}}\right) \\
&= [\varepsilon - 1]\frac{q_{jt}}{y_{jt}}\left(\psi^{\varepsilon-2}(q_{jt})\psi'(q_{jt})\left[\frac{\bar{p}_{ct}}{p_{jt}}\right]^{\varepsilon}C_t + \varphi^{\varepsilon-2}(q_{jt})\varphi'(q_{jt})\left[\frac{\bar{p}_t}{p_{jt}}\right]^{\varepsilon}I_t\right) \\
&= [\varepsilon - 1]\left[q_{jt}\frac{\psi'(q_{jt})}{\psi(q_{jt})}\frac{c_{jt}}{y_{jt}} + q_{jt}\frac{\varphi'(q_{jt})}{\varphi(q_{jt})}\frac{d_{jt}}{y_{jt}}\right].
\end{aligned}$$

Equation (93) makes obvious that the assumption $h'(1) = 0$ is a sufficient condition for an interior solution for q_{jt} for all $t \geq 1$, i.e. an optimal value of $q_{jt} > Q_{t-1}$. Indeed assume $\nu_{jt} > 0$: (91) then implies $q_{jt} = Q_{t-1}$ (or $q_{jt}/Q_{t-1} = 1$) and the right and side of (93) becomes nil. Since the optimal price behaviour implies that the first term of the left hand side of (93) is strictly positive, a nil value of the left hand side would require that $\nu_{jt} < 0$, which is a contradiction.

Using $\nu_{jt} = 0$ and the optimal price behaviour, (93) may be as follows:

$$\frac{p_{jt}}{\epsilon}\cdot\eta_{y\cdot q}^{jt} = \frac{\epsilon - 1}{\epsilon}p_{jt}\frac{y_{jt}}{k_{jt}}h'\left(\frac{q_{jt}}{Q_{t-1}}\right)\frac{q_{jt}}{Q_{t-1}}$$

which can easily be rewritten as equation (28).

Appendix 2: Proof of Lemma 1

1. Using (44), the optimality condition on consumption (42) can be rewritten as

$$\frac{c_{t+1}}{c_t} = \beta(1+r_{t+1})\frac{\psi(q_t)}{\psi(q_{t+1})}. \qquad (94)$$

Using (94) one-period lagged, the optimal pricing rule (49) becomes

$$\alpha\varphi(q_{t-1})\frac{y_t}{k_t} = \frac{\psi(q_{t-1})}{\psi(q_t)}(1+r_t)$$
$$= \frac{1}{\beta}\frac{c_t}{c_{t-1}}. \tag{95}$$

After substituting k_t by (45) and using next (43), (95) becomes:

$$\alpha\beta\frac{y_t}{c_t} = \frac{y_{t-1}}{c_{t-1}} - 1.$$

Let z_t be the inverse of the consumption-output ratio, i.e. $z_t = y_t/c_t$. The last equation above is a first-order linear autonomous equation in z:

$$z_{t-1} = \alpha\beta z_t + 1.$$

A particular solution of this equation is the constant value

$$\tilde{z} = \frac{1}{1 - \alpha\beta}.$$

The general solution of the autonomous equation is $z_t = A(\alpha\beta)^{-t} + \tilde{z}$ where A is a constant which can be identified using the terminal condition $z_T = 1$:

$$z_T = 1 = A(\alpha\beta)^{-T} + \tilde{z}.$$

Thus, $A = (\alpha\beta)^T(1 - \tilde{z})$. Consequently, the inverse of the average propensity to consume evolves as a monotonically decreasing function of time:

$$z_t = \frac{1 - (\alpha\beta)^{T+1-t}}{1 - \alpha\beta}. \tag{96}$$

If the time horizon of agents is infinite ($T \to \infty$), z_t is always constant and equal to $\tilde{z}$; the investment rate (or saving rate) is then constant as well and equal to $d/y = 1 - c/y = 1 - \alpha\beta$.

2. The second point of lemma 1 follows obviously from 1).

3. Using (45) and (51),
$$\frac{k_{t+1}}{y_t} = \alpha\beta\varphi(q_t). \tag{97}$$

Using the equality
$$\frac{k_{t+1}}{y_t} = \frac{k_{t+1}}{k_t}\frac{k_t}{y_t},$$
and (46), (97) can be recast as (52). Using (45) again, $k_{t+1}/k_t = (\varphi(q_t)d_t)/(\varphi(q_{t-1})d_{t-1})$, which allows to rewrite (52) in order to obtain the growth factor of output (53). Given point 2) of Lemma 1, this is also the growth factor of investment and consumption.